



KEY TO A SCHOOL ARITHMETIC  
FOR INDIAN SCHOOLS  
By HALL, STEVENS AND SIMS



MACMILLAN AND CO., LIMITED  
LONDON BOMBAY CALCUTTA  
MELBOURN

THE MACMILLAN COMPANY  
NEW YORK BOSTON CHICAGO  
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THE MACMILLAN CO OF CANADA, LTD.  
TORO TO

KEY  
TO  
HALL AND STEVENS'  
SCHOOL ARITHMETIC

*ADAPTED FOR USE IN INDIAN SCHOOLS*

BY

REV ANDREW SIMS, B.A.

LONDON MISSIONARY SOCIETY'S INSTITUTION, BHOWANIPUR, CALCUTTA

MACMILLAN AND CO, LIMITED  
ST MARTIN'S STREET, LONDON

1914

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**EXAMPLES I b Page 8**

**EXAMPLES I c Page 11**

$$\begin{array}{r} 857 \\ 21 \ 207 \overline{) 177399} \\ \underline{1179} \\ 1449 \end{array}$$
$$\begin{array}{r} 7126 \\ 24 \overline{) 876513} \\ \underline{155} \\ 324 \\ \underline{783} \text{ rem} \end{array}$$

**MISCELLANEOUS EXAMPLES I**    Page 13

**E**

- 16 If the cart cost 1 share of £52, each horse cost 6 shares of £52,  
 2 horses and cart cost 13 shares of £52,  
 i.e. 13 shares are worth £52, giving cost of each horse as £24

20 Note  $9998 = 10000 - 2$ , see Art 10

- 22 If we divide 987654321 by 164609,  
 we get 6000 as quotient and 321 remainder,  
 987654000 will divide exactly by 164609,  
 hence  $987654000 + 164609$  or 987818609 is req<sup>d</sup> number

27  $(43120 - 55) \times 7$

30 See Art 9

34 Note  $999 = 1000 - 1$ , see Art 10

42  $(76428 - 53) - 611 = 125$

45 Half of  $(£85 - £39)$

- 49  $A$ 's score =  $B$ 's score  $\times 2$ ,  $C$ 's score =  $B$ 's score  $\times 4$ ,  
 total score of  $A$ ,  $B$  and  $C$  =  $B$ 's score  $\times 7 = 168$ ,  
 hence  $B$ 's score = 24,  $A$ 's score = 48,  $C$ 's score = 96  
 Total score of remaining batsmen =  $255 - 168 - 15$

- 51 In skeleton form the two divisions are

$$\begin{array}{r} 8 \overline{) \quad \quad} \\ 9 \overline{) \quad \quad} \end{array} \left. \begin{array}{l} \text{rem } 5 \\ \text{rem } 7 \end{array} \right\} 61$$

$$61 = 7 \times 8 + 5$$

$$\begin{array}{r} 9 \overline{) \quad \quad} \\ 8 \overline{) \quad \quad} \end{array} \left. \begin{array}{l} \text{rem } a \\ \text{rem } b \end{array} \right\} 61$$

$$61 = 9b + a, \text{ whence } b = 6, a = 7$$

- 52 In skeleton form the division is

Divisor | 529565 | Quotient

$$\begin{array}{r} \text{(i)} \\ 246 \overline{) 6} \\ \text{(ii)} \\ 222 \overline{) 5} \\ \text{(iii)} \\ 542 \end{array}$$

The gaps (i), (ii) and (iii) are filled by the nos 5049, 2244, 1683 respectively. Each of these is some multiple of the divisor, which must be greater than 542

By trial,  $5049 = 9 \times 561$ ,  $2244 = 4 \times 561$ ,  $1683 = 3 \times 561$

Thus divisor = 561, and quotient = 943





£	s	d		Rs	a	p		Rs	a	p	
16	46	5	2	17	82	10	3	18	173	11	9
		177				141				193	
	6	177			4	141			4	579	
		29	6d			35	3 p			144	9 p
		885				1410				2123	
	20	914			16	1445			16	2267	
		45	14s			90	5 a			141	11 a
		708				1128				193	
		1062				282				1351	
		<u>£8187</u>				<u>Rs 11652</u>				<u>578</u>	
									<u>Rs 33530</u>		

19      cents  
 4020  
177  
 4020  
 28140  
28140  
 711640 cents  
= 7115 Rs 40 cents

20      cents  
 7655  
307  
 22965  
53585  
 2350085 cents  
= 23500 Rs 85 cents

24 The former dividend =

Rs      a  
 5      3  
 6500  
 16 19500  
 1218      12 a  
32500  
 Rs 33718

The latter dividend =

Rs      a  
 4      6  
 5250  
 8 15750  
 1968      12 a  
21000  
 Rs 22968

Rs      a  
 33718      12  
22968      12  
 the total dividend = Rs 56687      8 a

## EXAMPLES II e Page 34

$$7 \quad \begin{array}{c} \text{Rs} \quad \text{a} \\ (99 \quad 15-10) - (99 \quad 15-12) \end{array} \quad 9 \quad \begin{array}{c} \text{£} \quad \text{s} \quad \text{d} \\ 56 \quad 409 \quad 14 \quad 8 \quad (\text{£}7 \end{array}$$

$$12. \quad \begin{array}{c} \text{cents} \\ 96 \quad 864480 \quad (9005 \text{ cents} \\ \underline{480} \\ 90 \text{ Rs } 5 \text{ cents} \end{array}$$

$$\begin{array}{c} 17 \\ 20 \\ \underline{354} \quad (6s \\ 18 \\ \underline{12} \\ 224 \quad (4d \end{array}$$

$$18 \quad \begin{array}{c} \text{Rs} \\ 407 \quad 8271 \quad (\text{Rs } 20 \\ \underline{131} \\ 16 \end{array}$$

$$2096 \quad (5a$$

$$\underline{61}$$

$$\underline{12}$$

$$732 \quad (1p$$

$$325 p = \text{Re } 1 \quad 11a \quad 1p$$

$$\underline{\text{Rs } 20 \quad 5a \quad 1p,}$$

$$\text{with rem}^r \text{ Re } 1 \quad 11a \quad 1p$$

$$21 \quad \begin{array}{l} 3s \quad 6d = 7 \text{ sixpences,} \\ \text{£}4 \quad 0s \quad 6d = 161 \quad ,, \\ \text{req}^d \text{ no} = \frac{161}{7} = 23 \end{array}$$

$$23 \quad \text{Re } 1 \quad 8a = 3 \text{ half-rupees, } \text{Rs } 148 \quad 8a = 297 \text{ half-rupees} \\ \text{req}^d \text{ no} = \frac{297}{3} = 99$$

$$28 \quad \text{Division is but a short method for successive subtractions}$$

$$\text{Rs } 7 \quad 10a \quad 3p = 1467 p, \quad \text{Rs } 100 = 19200 p$$

$$1467 \quad 19200 \quad (13$$

$$\underline{4530}$$

$$129 p = 10a \quad 9p$$

$$\underline{13 \text{ times, with rem}^r \quad 10a \quad 9p}$$

$$30 \quad \text{No of shares} = \text{total distributed} - \text{share dividend}$$

$$\text{Rs } 29683 \quad 4a = 474932a, \quad \text{Rs } 4 \quad 10a = 74a$$

$$\text{req}^d \text{ no} = \frac{474932}{74} = 6418$$

## EXAMPLES II f Page 35

18 See Art 23, Ex 2

	Rs	a	p		Rs	a	p	
25	5	1	3		483	0	9	975 ) 92745 ) 95
	16				16			<u>4995</u>
	81 a				7728 a			rem <sup>r</sup> 120 = 10 a
	12				12			
	975 p				92745 p			
					<u>95 payments, with rem<sup>r</sup> 10 a</u>			

26 Rs 600 = 60000 cents, Rs 2 55 cents = 255 cents

$$255 \overline{) 60000} \quad (235$$

$$\underline{900}$$

$$\underline{1350}$$

$$75 \text{ cents}$$

$$\underline{235 \text{ days, with rem}^r 75 \text{ cents}}$$

31 £1 + 1s + 1d = 253d, £43 4s 5d = 10373d

$$\text{req}^d \text{ no} = \frac{10373}{25} = 41$$

32 The sum of the 4 profits = Rs 14917 15 a

$$\text{average annual profit} = \frac{\text{Rs } 14917 \text{ } 15 \text{ a}}{4}$$

35

$$A's \text{ share} = B's \text{ share} + \text{Rs } 13$$

$$\text{twice } B's \text{ share} = \text{Rs } (67 - 13) = \text{Rs } 54$$

Thus B gets Rs 27 and A Rs 40

36 B's share = C's share + £15, A's share = C's share + £15 + £40

$$\text{three times } C's \text{ share} + £15 + £55 = £265,$$

$$\therefore C's \text{ share} = \frac{£(265 - 70)}{3} = £65$$

37 18 Rs 20 cents = 1820 cents, 17 Rs 25 cents = 1725 cents,

$$15 \text{ Rs. } 45 \text{ cents} = 1545 \text{ cents}$$

By addition, twice (A's + B's + C's) share = 5090 cents,

A, B, and C together get 2545 cents,

but A and B together get 1820 cents,

$$C \text{ gets } 725 \text{ cents} = 7 \text{ Rs } 25 \text{ cents}$$

$$B's \text{ share} = (1545 - 725) \text{ cents} = 820 \text{ cents} = 8 \text{ Rs } 20 \text{ cents},$$

$$\text{and } A's \text{ share} = (1820 - 820) \text{ cents} = 1000 \text{ cents} = \text{Rs } 10$$

- 38 The total tax  $= (850 \times 11)$  pence  $= \text{£}38 \ 19s \ 2d$
- 39 The no of pounds in his income  $=$  the no of times that  $11d$  is contained in  $\text{£}45 \ 16s \ 8d$ , or 11000 pence Hence the req<sup>d</sup> income  $= \text{£}1000$
- 40 Cost price of each  $= \frac{\text{£}3 \ 7s \ 6d}{20} = 3s \ 4\frac{1}{2}d$   
 Sale „ „  $= \frac{\text{£}2 \ 2s \ 6d}{12} = 3s \ 6\frac{1}{2}d$  } profit  $= 2d$
- 41 Profit per yd  $= 6$  p But the total profit was Rs 2 4 a  $= 432$  p  
 no of yds of cloth bought and sold  $= \frac{432}{6} = 72$
- 42 The cost of 29 mds at 15 Rs 80 cents  $= 458$  Rs 20 cents  
 „ 16 „ 23 Rs 45 cents  $= 375$  Rs 20 cents  
 „ 45 mds  $= 833$  Rs 40 cents  
 „ 1 md  $= 18$  Rs 52 cents
- 43 As in Ex 42, it will be found that 1 lb of the mixture costs  $2s \ 1\frac{1}{4}d$   
 Hence gain per pound  $= 2\frac{3}{4}d$
- 44 The carriage of 1 yd  $= 3\frac{3}{4}d$ , carriage of 1 ton  $= \text{£}200$   
 no of yds to the ton  $= \frac{200 \times 960}{15} = 12800$
- 45 Total takings for 4 months  $= \text{Rs } 1555 \ 14a$   
 takings for remaining 8 months  $= \text{Rs } 5000 - \text{Rs } 1555 \ 14a$   
 $= \text{Rs } 3444 \ 2a$   
 req<sup>d</sup> average  $= \frac{\text{Rs } 3444 \ 2a}{8} = \text{Rs } 430 \ 8a \ 3p$

## EXAMPLES II g Page 39

- 53 There will be 240 intervals between the 1<sup>st</sup> and 241<sup>st</sup> post  
 no of Km  $= \frac{25 \times 240}{1000} = 6$
- 54 The train runs 36 mi in 120 half-minutes  
 no of yds in half a minute  $= \frac{36 \times 1760}{120} = 528$

58 2026 yds 2 ft

$$\begin{array}{r} 18 \\ 3 \overline{) 36 \text{ ft}} \\ 12 \end{array}$$

$$\begin{array}{r} 2026 \\ 16208 \end{array}$$

$$\begin{array}{r} 36480 \text{ yds} \\ 24 \end{array}$$

$$\begin{array}{r} 72960 \\ 145920 \end{array}$$

$$1760 \overline{) 875520} \quad 497 \text{ mi}$$

$$\begin{array}{r} 17152 \\ 13120 \\ \hline 800 \end{array}$$

$$\begin{aligned} \text{The daily run} &= (2026 \text{ yds } 2 \text{ ft}) \times 18 \times 24 \\ &= 497 \text{ mi } 800 \text{ yds} \end{aligned}$$

$$\begin{aligned} 59 \text{ Average advance per day} &= \frac{67 \text{ mi } 500 \text{ yds}}{31} \\ &= 2 \text{ mi } 300 \text{ yds} \end{aligned}$$

$$\begin{array}{r} \text{mi} \quad \text{yds} \\ 31 \overline{) 67 \quad 500} (2 \\ \underline{62} \quad \quad \\ 5 \end{array}$$

$$\begin{array}{r} \text{Km} \quad \text{m} \\ 2 \quad 177 \\ \hline 31 \end{array}$$

$$\begin{array}{r} 65 \quad 31 \\ 2 \quad 177 \\ \hline 67 \quad 487 \end{array}$$

$$\begin{array}{r} 1760 \\ 31 \overline{) 9300} (300 \end{array}$$

$$\begin{aligned} \text{Rem}^r &= 67 \text{ Km } 500 \text{ m} - 67 \text{ Km } 487 \text{ m} \\ &= 13 \text{ m} \end{aligned}$$

60 In 45 secs. the drum makes  $4\frac{1}{2} \times 2$ , or 30 revolutions. Each revolution gives off 44 ft 6 in of rope, hence depth of shaft = 44 ft 6 in  $\times 30 = 435$  yds

### EXAMPLES II h Page 41

$$\begin{aligned} 25 \quad 1 \text{ ton} &= 20 \text{ cwt} = (20 \times 4) \text{ qr} = (20 \times 4 \times 28) \text{ lbs} = 2240 \text{ lbs} \\ 1 \text{ tonne} &= 1000 \text{ Kg} = (1000 \times 2\frac{1}{2}) \text{ lbs} = 2000 \text{ lbs} + \frac{1}{2} \text{ of } 1000 \text{ lbs} \\ &= 2200 \text{ lbs} \end{aligned}$$

$$\begin{aligned} 38 \quad \text{One bottle of each requires } 1\frac{1}{2} \text{ seers} \\ 2 \text{ mds } 1 \text{ seer} &= 162 \text{ half-seers, } 1\frac{1}{2} \text{ seers} = 3 \text{ half seers} \\ \text{req}^d \text{ no of bottles} &= 1\frac{1}{2} \times 2 = 54 \end{aligned}$$

- 40 Total length =  $(4 \text{ ft } 3 \text{ in} \times 48) = 68 \text{ yds}$   
 Weight =  $(3 \times 68) \text{ oz} = 204 \text{ oz} = 12 \text{ lbs. } 12 \text{ oz.}$

ft	in
4	3
8	
34	0
6	
204	0 = 68 yds

41. Weight of rope =  $(2\frac{1}{2} \times 280) \text{ lbs} = (560 + 140) \text{ lbs} = 700 \text{ lbs}$   
 $= 6 \text{ cwt } 1 \text{ qr}$   
 Adding weight of load we get total weight = 2 tons 16 cwt 1 qr

### EXAMPLES II k. Page 46

- 34 Area of the 16 flower beds  
 $= 50 \text{ sq yds } 5 \text{ sq ft} \times 16$   
 $= 808 \text{ sq yds } 8 \text{ sq ft}$   
 Area of lawn is obtained by subtracting the  
 areas of the beds and of the paths from  
 the whole area

sq yds	sq ft
50	5
4	
202	2
4	
808	8

sq yds	sq ft
1300	0
808	8
1	
241	1
0	
250	0

- 41 (i) 1 ac. = 4 r =  $(4 \times 40) \text{ sq p} = 160 \text{ sq p} = (160 \times 30\frac{1}{4}) \text{ sq yds}$   
 $= (4800 + 40) \text{ sq yds}$   
 $= 4840 \text{ sq yds}$   
 (ii) 1 sq mi =  $(1760 \times 1760) \text{ sq yds} = 3097600 \text{ sq yds}$   
 $= (3097600 - 4840) \text{ ac}$   
 $= 640 \text{ ac}$   
 55 6 ac 0 r 25 p = 985 p, 215 ac 1 r 35 p = 34475 p  
 req<sup>d</sup> no =  $\frac{24475}{35} = 35$

### EXAMPLES II l Page 52

- 3 See Art 39  
 11 Each truck carries  $(42 \times 8) \text{ cub ft}$ , or 336 cub ft  
 $112 \text{ cub yds} = (112 \times 27) \text{ cub ft}$ , or 3024 cub ft  
 req<sup>d</sup> no of trucks =  $\frac{3024}{336} = 9$   
 13 Since 7 yds = 21 ft, vol of time =  $(21 \times 1 \times 1) \text{ cub ft} = 21 \text{ cub ft}$   
 its weight =  $(64 \times 21) \text{ lbs} = 1344 \text{ lbs} = 12 \text{ cwt}$

14. Actual volume of water  $= (5 \times 4 \times 3)$  cub ft  $= 60$  cub ft  
its weight  $= (60 \times 1000)$  oz  $= 3750$  lbs  
Estimate  $= 1\frac{1}{2}$  tons  $= 1$  ton 10 cwt  $= 3360$  lbs ,  
so that I overestimate by 390 lbs
- 15 The volume of the cube  $=$  the volume the water is raised  
" "  $= 18 \times 18 \times 18 = 5832$  cub in  
The area of the base of the vessel  $= 72 \times 27 = 1944$  sq in  
the no of inches the water is raised  $= \frac{5832}{1944} = 3$

### EXAMPLES II m Page 53

- 9 The daily no of strokes  $= (1 + 2 + 3 + 4 + \dots + 12) \times 2$   
Feb'y 1912 has 29 days  
no of strokes  $= 78 \times 2 \times 29 = 4524$
- 11 On the last day of 1910 the no of the paper was 5030
- $$\begin{array}{r} 365 \overline{) 5030} \quad (13 \\ \underline{1380} \\ 285 \end{array}$$

So the paper would then have been published over 13 years But  
there were two leap years in these 13 years the paper  
would then have been published 13 years and 283 days,  
i.e. on and from March 24th, 1897

- 12 Had the person been born on Dec 31st, 1891, his age would have  
been 19 years  
These 19 years  $= (19 \times 365 + 4)$  days  $= 6939$  days  
his age was  $6939 - 24 = 6915$  days

### EXAMPLES III a Page 56

- 46 1 lb of tea costs 30d ,                      x lbs of tea cost 30x pence  
1 lb of coffee costs 18d ,                      y lbs of coffee cost 18y pence

$$52 \quad 4x+2p+1-z=4 \quad 3+2 \quad 1+9-0=12+2+9=23$$

$$54. \quad 5xy-y+22-71=5 \quad 3 \quad 7-5+22-7 \quad 9=127-68=59$$

$$55 \quad 93-x^2-3y^2-8p^3=93-3^2-3 \quad 5^2-8 \quad 1^3=93-9-75-8=1$$

$$59 \quad \frac{4r}{z}+5zq^2p-\frac{q^2}{q}=\frac{4}{3}\frac{9}{3}+5 \quad 0 \quad 7^2 \quad 1-\frac{7^2}{7}=12+0-7=5$$

### EXAMPLES III f Page 69

3 Let  $x$  be the number, then  $4(2x+3)=52$

4 Let  $x$  be one part, then  $60-x$  is the other part  
Hence  $3x-100=200-8(60-x)$

5 Let  $\pounds x$  be  $C$ 's share, then  $B$ 's share is  $\pounds(x+8)$ , and  $A$ 's share is  $\pounds(x+8+15)$   
Hence  $x+(x+8)+(x+8+15)=67$

6 Let  $\pounds x$  be  $A$ 's share, then  $B$ 's share is  $\pounds 2x$ , and  $C$ 's is  $\pounds(2x-4)$   
Hence  $x+2x+(2x-4)=66$

7 Let  $A$  have  $\pounds x$ , then  $B$  has  $\pounds(x+10)$ , and  $C$  has  $\pounds 3x$   
Hence  $x+(x+10)+3x=85$

8 Let  $A$  have  $\pounds x$ , then  $B$  has  $\pounds(x+37)$ , and  $C$  has  $\pounds(2x+11)$   
hence  $x+(x+37)+(2x+11)=188$

9 Let  $x$  be number of shillings, then  $67-x$ =number of sixpences  
 $x$  shillings+ $(67-x)$  sixpences= $\pounds 2 \ 13s \ 6d$ ,  
or, reducing to sixpences,  $2x+67-x=107$

10 Let  $A$  have Rs  $x$ , or  $4x$  four-anna pieces,  
then  $B$  has Rs  $(12-x)=(48-4x)$  four-anna pieces  
Hence  $4x+5=7(48-4x-5)$

11 Let  $B$  have  $x$  shillings, then  $A$  has  $3x$  shillings  
hence  $3x-10=2(x+10)$



- 12 Let one have  $x$  shillings, then the other has  $(30-x)$  shillings  
 hence  $x-6=\frac{1}{2}$  of  $(30-x+9)$ ,  
 and, multiplying by 2,  $2(x-6)=39-x$

### EXAMPLES IV a Page 76

- 1-13 See Art 78 In Ex 13 the work is shortened by "casting out nines," as explained in Art 12
- 14 "Casting out nines" from the last example (see Art 12), we get  
 12, 3, 9, 0, 11, 2 Since final rem is 2, the missing digit must be 7
- 15 From Art 77, we have in the last example,  
 sum of odd digits  $= 5+8+6+2=21$ ,  
 sum of *given* even digits  $= 4+7+8=19$ ,  
 missing even digit must be 2

17-34 See Art 82

$$\begin{array}{r} 32 \quad 5 \overline{) 7245} \\ \quad 9 \overline{) 1449} \\ \quad \quad 7 \overline{) 161} \\ \quad \quad \quad 23 \end{array}$$

$$\begin{aligned} \text{No} &= 5 \times 9 \times 7 \times 23, \\ &= 3^3 \times 5 \times 7 \times 23 \end{aligned}$$

$$\begin{array}{r} 33 \quad 2 \overline{) 19206} \\ \quad 9 \overline{) 9603} \\ \quad \quad 11 \overline{) 1067} \\ \quad \quad \quad 97 \end{array}$$

$$\begin{aligned} \text{No} &= 2 \times 9 \times 11 \times 97, \\ &= 2 \times 3^2 \times 11 \times 97 \end{aligned}$$

$$\begin{array}{r} 34 \quad 12 \overline{) 249984} \\ \quad 12 \overline{) 20832} \\ \quad \quad 8 \overline{) 1736} \\ \quad \quad \quad 7 \overline{) 217} \\ \quad \quad \quad \quad 31 \end{array}$$

$$\begin{aligned} \text{No} &= 12 \times 12 \times 8 \times 7 \times 31, \\ &= 2^2 \times 3 \times 2^3 \times 3 \times 2^3 \times 7 \times 31, \\ &= 2^7 \times 3^2 \times 7 \times 31 \end{aligned}$$

- 40 From Art 78, the test for 8 is satisfied,  
 and we get  $712=8 \times 89=2^3 \cdot 89$ , since 89 is prime

- 43 Taking the primes in order, we find that the tests for 2, 3, 5 are not satisfied. We then try 7, 11, 13, 17, 19, 23, and find  $667 = 23 \times 29$ . Since 29 is prime, this is the req<sup>d</sup> result.
- 44 As in no 43, we find that 2, 3, 5, 7 are not factors, but that 11 is a factor, giving  $10681 = 11 \times 971$ . We then find that 971 will not divide by any of the primes 13, 17, , 37, , 971 is a prime (see Art 82, Ex 3), and req<sup>d</sup> result =  $11 \times 971$ .
- 47  $100a + 10b + c$  This is divisible by 2 or 5, when  $c$  is so divisible.
- 49  $\left. \begin{array}{l} 100l + 10m + n \\ 100m + 10n + l \end{array} \right\}$  are any two such numbers  
Their difference is  $99l - 99m - 9n$ , which is obviously divisible by 9.

## EXAMPLES IV b Page 78

18-27 See Art 84, Ex 1, 2

23 Since  $10082 = 2 \times 5041$ , and 5041 is odd,  
2 occurs to an *odd* power, 10082 is not a square number

25 Here  $3136 = 8 \times 8 \times 49,$   $8 \overline{) 3136}$   
 $= 8^2 \times 7^2,$   $8 \overline{) 392}$   
sq root of  $3136 = 8 \times 7 = 56$  49

27 Here  $7056 = 12 \times 12 \times 49,$   $12 \overline{) 7056}$   
 $= 12^2 \times 7^2,$   $12 \overline{) 588}$   
sq root of  $7056 = 12 \times 7 = 84$  49

28  $392 = 4 \times 49 \times 2 = (2^2 \times 7^2) \times 2,$  req<sup>d</sup> factor = 2

36 Here  $46656 = 12 \times 12 \times 12 \times 27,$   $12 \overline{) 46656}$   
 $= 12^3 \times 3^3,$   $12 \overline{) 3888}$   
cube root of  $46656 = 12 \times 3 = 36$   $12 \overline{) 324}$   
27

37 Here  $91125 = 9 \times 9 \times 9 \times 125,$   $9 \overline{) 91125}$   
 $= 9^3 \times 5^3,$   $9 \overline{) 10125}$   
cube root of  $91125 = 9 \times 5 = 45$   $9 \overline{) 1125}$   
125

39 The no must contain  $13^2$ , or 169, as a factor, and every other factor must be twice repeated. Thus we have to select from  $169 \times 2^2$ ,  $169 \times 3^2$ , , and of these  $169 \times 3^2$ , or 1521 is the only one between 1000 and 2000

## EXAMPLES IV d Page 82

$$\begin{array}{r|l} 1 & 117 \quad 221 \quad 1 \\ & \underline{104} \quad \underline{117} \\ & 13 \quad 104 \quad 8 \\ & \underline{104} \end{array}$$

HCF = 13

$$\begin{array}{r|l} 2 & 203 \quad 319 \quad 1 \\ & \underline{116} \quad \underline{203} \\ 3 & \underline{87} \quad \underline{116} \quad 1 \\ & \underline{87} \quad \underline{87} \\ & \underline{\quad} \quad \underline{29} \end{array}$$

HCF = 29

$$\begin{array}{r|l} 3 & 2 \quad 559 \quad 817 \quad 1 \\ & \underline{516} \quad \underline{559} \\ & 43 \quad 258 \quad 6 \\ & \underline{258} \end{array}$$

HCF = 43

4 See Art 91  
 $644 = 4 \times 161$ ,  
 $532 = 4 \times 133$

$$\begin{array}{r|l} 1 & 161 \quad 133 \quad 4 \\ & \underline{133} \quad \underline{112} \\ 4 & \underline{28} \quad \underline{21} \quad 3 \\ & \underline{7} \quad \underline{21} \end{array}$$

HCF =  $4 \times 7 = 28$

$$\begin{array}{r|l} 5 & 1 \quad 255 \quad 391 \quad 1 \\ & \underline{136} \quad \underline{255} \\ & \underline{119} \quad \underline{136} \quad 1 \\ & \underline{119} \quad \underline{119} \\ & \underline{\quad} \quad \underline{17} \end{array}$$

HCF = 17

$$\begin{array}{r|l} 6 & 2 \quad 329 \quad 799 \quad 2 \\ & \underline{282} \quad \underline{658} \\ & 47 \quad 141 \quad 3 \\ & \underline{141} \end{array}$$

HCF = 47

7 See Art 91

$$\begin{array}{r|l} 5 & 527 \quad 1147 \quad 2 \\ & \underline{465} \quad \underline{1054} \\ 2 & \underline{62} \quad \underline{93} \quad 3 \\ & \underline{31} \quad \underline{93} \end{array}$$

HCF = 31

8 See Art 91

$$\begin{array}{r|l} 2 & 623 \quad 833 \quad 1 \\ & \underline{420} \quad \underline{623} \\ 29 & \underline{203} \quad 30 \quad \underline{210} \\ & \underline{203} \quad \underline{\quad} \quad \underline{7} \end{array}$$

HCF = 7

9 See Art 91

$348 = 4 \times 87$ ,  
 $1024 = 4 \times 256$

$$\begin{array}{r|l} 1 & 87 \quad 256 \quad 2 \\ & \underline{82} \quad \underline{174} \\ & 5 \quad 82 \end{array}$$

At this stage we see  
 that 87 and 256 are  
 prime to each other,

HCF req<sup>d</sup> = 4

10 See Art 91

$1702 = 2 \times 851$   
 $1998 = 2 \times 9 \times 111$

$$\begin{array}{r|l} 7 & 851 \quad 111 \quad 3 \\ & \underline{777} \quad \underline{111} \\ 2 & \underline{74} \\ & \underline{37} \end{array}$$

HCF =  $37 \times 2 = 74$

11 1 3451 9367 2

$$\begin{array}{r|l} 2 & 2465 \quad 6902 \quad 2 \\ & \underline{986} \quad \underline{2465} \\ & \underline{986} \quad \underline{1972} \\ & \underline{\quad} \quad \underline{493} \end{array}$$

HCF = 493

12 See Art 91

$$\begin{array}{r|l} 1 & 1379 \quad 2401 \quad 1 \\ & \underline{1022} \quad \underline{1379} \\ 1 & \underline{357} \quad \underline{1022} \quad 2 \\ & \underline{308} \quad \underline{714} \\ 7 & \underline{49} \quad \underline{308} \quad 6 \\ & \underline{49} \quad \underline{294} \\ & 2 \quad \underline{14} \\ & \underline{7} \end{array}$$

HCF = 7

13 See Art 91

$$\begin{array}{r|l} 4 \quad 4199 \quad 5093 \quad | \quad 1 \\ \quad 3536 \quad 4199 \\ \quad 2 \quad 663 \quad 884 \quad | \quad 1 \\ \quad \quad 221 \quad 884 \end{array}$$

H C F = 221

14 See Art 91

$$10520 = 4 \times 10 \times 263$$

$$\begin{array}{r|l} 11 \quad 2893 \quad 263 \\ \quad 2893 \end{array}$$

H C F = 263

15 See Art 91

$$\begin{array}{r|l} 3 \quad 13547 \quad 17081 \quad | \quad 1 \\ \quad 10602 \quad 13547 \\ \quad 5 \quad 2945 \quad 3534 \quad | \quad 6 \\ \quad \quad 589 \quad 3534 \end{array}$$

H C F = 589

16 See Art 91

$$\begin{array}{r|l} 1 \quad 20677 \quad 31279 \quad | \quad 1 \\ \quad 10602 \quad 20677 \\ \quad 5 \quad 10075 \quad 9 \quad 10602 \\ \quad 1 \quad 2015 \quad 1178 \quad | \quad 1 \\ \quad \quad 1178 \quad 837 \\ \quad 9 \quad 837 \quad 341 \quad | \quad 11 \\ \quad \quad 3 \quad 93 \quad 341 \\ \quad \quad \quad 31 \end{array}$$

H C F = 31

17 See Art 91

$$13503 = 9 \times 1507$$

$$\begin{array}{r|l} 7 \quad 10549 \quad 1507 \\ \quad 10549 \end{array}$$

H C F = 1507

18 See Art 91

$$16984 = 4 \times 2 \times 2123,$$

$$5404 = 4 \times 1351$$

$$\begin{array}{r|l} 1 \quad 2123 \quad 1351 \quad | \quad 7 \\ \quad 1351 \quad 1351 \\ \quad 4 \quad 772 \quad \quad \quad | \\ \quad \quad 193 \quad \quad \quad \end{array}$$

H C F =  $193 \times 4 = 772$ 

19 See Art 91

$$16995 = 3 \times 5 \times 11 \times 103,$$

$$64890 = 2 \times 5 \times 3 \times 3 \times 721$$

$$\begin{array}{r|l} 103 \quad 721 \quad 7 \\ \quad 721 \end{array}$$

H C F =  $103 \times 3 \times 5$   
= 1545

$$20 \quad 94248 = 2^3 \quad 3^2 \quad 7 \quad 11 \quad 17,$$

$$504900 = 2^2 \quad 5^2 \quad 3^3 \quad 11 \quad 17,$$

$$\text{H C F} = 2^2 \quad 3^2 \quad 11 \quad 17$$

$$= 6732$$

$$21. \quad 183 = 61 \times 3,$$

$$793 = 61 \times 13,$$

$$976 = 61 \times 16,$$

$$\text{H C F} = 61$$

$$22 \quad 658 = 2 \times 47 \times 7,$$

$$940 = 2 \times 47 \times 10,$$

$$1128 = 2 \times 47 \times 12,$$

$$\text{H C F} = 2 \times 47 = 94$$

$$23 \quad 403 = 31 \times 13,$$

$$744 = 31 \times 6 \times 4,$$

$$1023 = 31 \times 3 \times 11,$$

$$\text{H C F} = 31$$

24 See Arts 91 and 92

$$49237 = 9 \times 4693,$$

$$75582 = 9 \times 2 \times 4199,$$

$$8892 = 9 \times 4 \times 247$$

We now find the H C F of  
4693, 4199, 247

$$\begin{array}{r|l} 1 \quad 4693 \quad 4199 \quad | \quad 17 \\ \quad 4199 \quad 247 \\ \quad 2 \quad 491 \quad 1729 \\ \quad \quad 247 \quad 1729 \end{array}$$

We obtain for this 247,

$$\text{reqd H C F} = 9 \times 247$$

$$= 2223$$

$$25 \quad 3156 = 9 \times 3 \times 27,$$

$$26244 = 9 \times 3 \times 972,$$

$$99225 = 9 \times 3 \times 3675$$

Since 27, 972, and 3675 have  
no factor common to all  
three, it follows that

$$\text{H C F} = 27$$

26 The reqd number must  
divide the numbers  
(14490 - 6) and (31530 - 6)  
without remainder

$$\begin{array}{r|l}
 31 \quad 2 \quad \begin{array}{r} 385 \\ 280 \end{array} & \begin{array}{r} 525 \\ 385 \end{array} \quad 1 \\
 3 \quad \begin{array}{r} (i) 105 \\ 3 \times 35 \end{array} & \begin{array}{r} (ii) 140 \\ 105 \\ 35 \end{array} \quad 1
 \end{array}$$

Starting from the remainder 35, we can build up the work step by step till we arrive at the two numbers required

Thus  $3 \times 35$ , or 105, must equal the remainder (i),  $105 \times 1 + 35$  must equal the remainder (ii), whence we get 140. Then  $2 \times 140 + 105$  gives 385, and  $385 \times 1 + 140$  gives 525

- 32 The required number will divide the difference between any two of the three numbers and leave no remainder

Now  $153599 - 142408 = 11191$ ,  
and  $166402 - 153599 = 12803$ ,  
hence req<sup>d</sup> no is the H C F of 11191 and 12803

$$\begin{array}{r}
 33 \quad (iii) \quad 40051 \quad (iv) \\
 \quad \quad (i) \\
 \quad \quad 1731 \\
 \quad \quad (i) \\
 \quad \quad 294
 \end{array}$$

The gaps are filled as follows

$$(i) = 1731 - 294 = 1437,$$

$$(ii) = 4005 - 173 = 3832$$

Now 1437 (or  $479 \times 3$ ) and 3832 (or  $479 \times 8$ ) are multiples of the req<sup>d</sup> divisor

Hence divisor = 479, and quotient = 83

### EXAMPLES IV f Page 86

- 1-7 See Art 97      8    2s 6d = 5 sixpences, 10s 6d = 21 sixpences,  
LCM of 5 and 21 = 105

- 11 The lowest number which each of the four given numbers will divide *without remainder*, is their LCM

This will be found to be 720, hence (720 ÷ 9) is req<sup>d</sup> number

- 14 Reduce to threepences See Ex 8

- 15 The answer must be the *least number*, which will contain 105, 112, 126, and 168 miles an exact number of times

- 16 As Ex 15

- 17 The req<sup>d</sup> time in secs must divide exactly by each of the numbers 252, 308, and 198, since by then, A, B, and C must each have completed an exact number of rounds  
they will *next* be all together in the LCM of 252 secs, 308 secs, and 198 secs

$$18 \text{ From Art 97, } \text{LCM} = \frac{A \cdot B}{X}, \text{ or } 11781 = \frac{1071 \times B}{119}$$

## EXAMPLES V d Page 93

$$12 \quad \frac{315}{420} = \frac{3 \times 105}{3 \times 140} = \frac{3 \times 5 \times 21}{3 \times 5 \times 28} = \frac{3 \times 5 \times 7 \times 3}{3 \times 5 \times 7 \times 4} = \frac{3}{4}$$

$$16 \quad \frac{231}{3003} = \frac{3 \times 77}{3 \times 1001} = \frac{3 \times 11 \times 7}{3 \times 11 \times 91} = \frac{3 \times 11 \times 7 \times 1}{3 \times 11 \times 7 \times 13} = \frac{1}{13}$$

$$20 \quad \frac{2508}{3036} = \frac{11 \times 228}{11 \times 276} = \frac{11 \times 12 \times 19}{11 \times 12 \times 23} = \frac{19}{23}$$

$$27 \quad \frac{391}{408} = \frac{391}{3 \times 8 \times 17} \quad (a) \\ = \frac{17 \times 23}{3 \times 8 \times 17} \\ = \frac{23}{24}$$

(a) At this stage we see neither 3 nor 8 divides 391. Hence, if the fraction admits of reduction, 17 must be a factor of 391.

By division we obtain the other factor 23.

28-29 See Art 107, Ex 1

$$34 \quad \frac{572}{1287} = \frac{4 \times 11 \times 13}{9 \times 11 \times 13} = \frac{4}{9}$$

$$35 \quad \frac{954}{2544} = \frac{2 \times 3 \times 3 \times 53}{2 \times 2 \times 8 \times 53} = \frac{3}{8}$$

## EXAMPLES V g Page 98

$$47 \quad \text{LCD of } 39, 65, 15 = 3 \times 5 \times 13 = 195,$$

$$\exp^n = \frac{5+21-13}{195} = \frac{13}{195} = \frac{13 \times 1}{13 \times 15} = \frac{1}{15}$$

$$48 \quad \text{LCD of } 35, 21, 15 = 3 \times 5 \times 7 = 105,$$

$$\exp^n = \frac{36-20+49}{105} = \frac{65}{105} = \frac{5 \times 13}{5 \times 21} = \frac{13}{21}$$

$$49 \quad \text{LCD of } 12, 36, 27 = 4 \times 3^3 = 108,$$

$$\exp^n = \frac{99-15-8}{108} = \frac{76}{108} = \frac{4 \times 19}{4 \times 27} = \frac{19}{27}$$

## EXAMPLES V h Page 100

$$17 \quad \text{Exp}^n = 1 + \frac{5}{5 \times 2} + \frac{10}{3 \times 6} + \frac{7}{6 \times 6} + \frac{11}{2 \times 6} = 1 + \frac{15+7+6}{13 \times 3 \times 4} + \frac{7+13}{6 \times 6} \\ = 1 + \frac{91}{13 \times 12} + \frac{2}{3} = 1 + \frac{7}{12} + \frac{2}{3} = 2\frac{1}{4}$$

$$18 \quad \text{Exp}^n = 9 + \frac{9}{7 \times 7} + \frac{7}{3 \times 3} + \frac{10}{1 \times 1} + \frac{48}{6 \times 3} = 9 + \frac{27+49+210}{21 \times 11} + \frac{16}{21} \\ = 9 + \frac{286}{21 \times 11} + \frac{16}{21} = 9 + \frac{26+16}{21} = 11$$

$$19 \quad \text{Exp}^n = \frac{21-8}{13 \times 6} + \frac{1}{6} = \frac{13}{13 \times 6} + \frac{1}{6} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$20 \quad \text{Exp}^n = 3\frac{16-9}{7 \times 3 \times 8} - \frac{1}{24} = 3\frac{7}{7 \times 24} - \frac{1}{24} = 3\frac{1}{24} - \frac{1}{24} = 3$$

$$21 \quad \text{Exp}^n = \frac{2}{5} - \frac{7}{55} - \frac{3}{11} = \frac{22-7-15}{55} = 0$$

$$22 \quad \text{Exp}^n = 1\frac{7}{7} - \frac{2}{55} - \frac{1}{35} = 1\frac{25-14}{11 \times 5 \times 7} - \frac{1}{35} \\ = 1\frac{11}{11 \times 5 \times 7} - \frac{1}{35} = 1\frac{1}{35} - \frac{1}{35} = 1$$

### EXAMPLES V k Page 101

$$38 \quad 3 - (2\frac{2}{5} - \frac{5}{8}) = 3 - 2\frac{2}{5} + \frac{5}{8} = \frac{7}{5} + \frac{5}{8} = \frac{28+10}{38} = 1\frac{1}{19}$$

$$39 \quad 3\frac{1}{2} - 1\frac{1}{2} = 2\frac{1}{2} - \frac{1}{8} = 2 + \frac{3-4}{24} = 1 + \frac{24+1-4}{24} = 1\frac{21}{24} \\ 2 - \frac{1}{12} = 1\frac{11}{12} = 1\frac{21}{24}$$

first exp<sup>n</sup> is greater, by  $\frac{1}{24}$

$$41 \quad 2\frac{1}{2} - \frac{2}{5} + \frac{9}{10} = 2 + \frac{7-2+9}{10} = 2\frac{8}{10} = 2\frac{4}{5}, \quad \frac{1}{5} \text{ must be added}$$

$$42 \quad 10 - (1\frac{1}{3} - \frac{7}{7}) = 10 - 1\frac{1}{3} + \frac{7}{7} = 10 - 3\frac{2}{3} + \frac{7}{7} = 7 + \frac{7}{7} - \frac{2}{3} \\ = 7 + \frac{17-4}{21} = 7\frac{13}{21} \quad \text{Hence reqd diff} = \frac{1}{21}$$

$$43 \quad (i) 6\frac{2}{5} - x = 1\frac{7}{18}, \quad (ii) x + 1\frac{1}{8} - \frac{7}{10} = 1\frac{3}{8}, \\ \text{or } -6\frac{2}{5} + x = -1\frac{7}{18}, \quad x = 1\frac{3}{8} + \frac{7}{10} - 1\frac{1}{8}, \\ x = 6\frac{2}{5} - 1\frac{7}{18}, \quad \text{or } x = \frac{8}{18} \\ \text{or } x = 4\frac{4}{9}$$

### EXAMPLES V l Page 103

$$11 \quad \text{Exp}^n = 3\frac{10}{11} + 3\frac{2}{5} - 1\frac{9}{12} - 4\frac{9}{10} = 1 + (1\frac{10}{11} - \frac{9}{12}) - (\frac{9}{10} - \frac{2}{5}) \\ = 1 + \frac{11}{22} - \frac{7}{10} = 1 + \frac{1}{2} - \frac{1}{2} = 1$$

$$12 \quad \text{Exp}^n = 8 + (\frac{1}{2} + \frac{3}{8}) - (\frac{1}{2} + \frac{1}{30}) = 8 + \frac{7}{8} - \frac{1}{6} \\ = 8 + \frac{7}{8} - \frac{5}{24} = 8\frac{18-5}{24} = 8\frac{13}{24}$$

$$13 \quad \text{Exp}^n = \frac{8}{5} - \frac{1}{7} + \frac{2}{35} + \frac{1}{6} = \frac{56-45+18+65}{6 \times 7 \times 5} = \frac{84}{210} = \frac{4}{10}$$

$$14 \quad \text{Exp}^n = 1 - (\frac{9}{34} - \frac{5}{51}) + (\frac{7}{18} - \frac{2}{9}) = 1 - \frac{17}{102} + \frac{3}{18} = 1 - \frac{1}{6} + \frac{1}{6} = 1$$

$$17 \quad (\frac{3}{4} + \frac{9}{7} + \frac{7}{12}) - \frac{4}{21} = \frac{6}{7} - \frac{4}{21} + \frac{9+7}{12} = \frac{2}{3} + 1\frac{1}{3} = 2$$

$$18 \quad (8\frac{5}{11} - 3\frac{2}{9}) - (10\frac{9}{34} - 5\frac{1}{18}) = 8\frac{5}{11} - 3\frac{2}{9} - 10\frac{9}{34} + 5\frac{1}{18} \\ = (\frac{17}{18} - \frac{2}{9}) - (\frac{9}{34} - \frac{5}{18}) = \frac{1}{18} - \frac{17}{102} = \frac{1}{18} - \frac{1}{6} = \frac{5}{18}$$

$$\begin{aligned}
 19 \quad (2\frac{2}{3}x - 3\frac{4}{5}x) - (1\frac{2}{10}x - \frac{1}{7}x) &= 2\frac{2}{3}x - 3\frac{4}{5}x - 1\frac{2}{10}x + \frac{1}{7}x \\
 &= 1 + (\frac{1}{7}x + \frac{2}{5}x) - (\frac{4}{5}x + 1\frac{2}{10}x) = 1 + \frac{8x+14}{5 \times 7 \times 11} - \frac{1x+8}{106} \\
 &= 1 + \frac{9}{7}x - \frac{2}{5}x = 1 + \frac{7x-2x}{5 \times 7 \times 11} = 1\frac{1}{11}
 \end{aligned}$$

$$\begin{aligned}
 20 \quad \text{Reqd amount} &= \text{Rs } (2\frac{2}{5} + 1\frac{1}{4}) - \text{Rs } (3\frac{1}{8} + 1\frac{1}{2}) \\
 &= \text{Rs } (2\frac{2}{5} + 1\frac{1}{4} - 3\frac{1}{8} - 1\frac{1}{2}) = \text{Rs } (\frac{2}{5} + \frac{1}{4} - \frac{1}{8} - \frac{1}{2}) \\
 &= \text{Rs } (\frac{1}{4} - \frac{1}{4}) = \text{Rs } \frac{1}{4} \text{ or } 8d
 \end{aligned}$$

$$\begin{aligned}
 21 \quad \text{Reqd amount} &= £10 - £(2\frac{1}{5} + 4\frac{7}{10} + 1\frac{1}{10}) \\
 &= £4 - £(\frac{1}{5} + \frac{7}{10} + \frac{1}{10}) = £4 - £\frac{9}{10} = £3\frac{2}{5} = £3 \ 5s \ 3d
 \end{aligned}$$

$$\begin{aligned}
 22 \quad \text{Reqd fraction} &= (1 - \frac{5}{7} - \frac{7}{10} - \frac{1}{2}) = \frac{1}{7} \\
 \text{we get first sum} &= \frac{5}{7} \text{ of Rs } 240 = \text{Rs } 50, \\
 \text{second sum} &= \frac{7}{10} \text{ of Rs } 240 = \text{Rs } 72, \\
 \text{third sum} &= \frac{1}{2} \text{ of Rs } 240 = \text{Rs } 48, \\
 \text{fourth sum} &= \frac{1}{7} \text{ of Rs } 240 = \text{Rs } 70
 \end{aligned}$$

$$\begin{aligned}
 23 \quad A \text{ owes } B \ 4s \ 7\frac{3}{4}d \text{ in all} \quad \text{This to the nearest penny is } 4s \ 8d \\
 A \text{ pays } B \ 5s, \quad B \text{ pays } A \ 4d
 \end{aligned}$$

$$\begin{aligned}
 24. \quad \text{Length of rod} &= (10\frac{5}{8} + 40\frac{3}{8} + 38\frac{1}{4} - 5\frac{1}{2}) \text{ in} \\
 &= (81 + 38\frac{1}{4} - 5\frac{1}{2}) \text{ in} = 113\frac{1}{4} \text{ in} = 111 \text{ in to nearest inch}
 \end{aligned}$$

$$\begin{aligned}
 25 \quad \text{Residue} &= (1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}) = \frac{1}{16} \text{ of property} \\
 \frac{1}{16} \text{ of property} &= \text{Rs } 1200, \text{ and whole property} = \text{Rs } 14400
 \end{aligned}$$

$$\begin{aligned}
 26 \quad \frac{2}{7} + \frac{1}{2} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} &= \frac{5}{6} \\
 \text{"the rest" consist of } \frac{1}{6} \text{ of } 350000, \text{ or } 58333
 \end{aligned}$$

$$\begin{aligned}
 27 \quad (i) \quad \frac{a+b}{a} + \frac{a+b}{b} - 2 &= \frac{(a+b) \times b + (a+b) \times a - 2ab}{ab} \\
 &= \frac{ab + b^2 + a^2 + ab - 2ab}{ab} \\
 &= \frac{a^2 + b^2}{ab} \\
 (ii) \quad \frac{a-b}{a^2b} - \frac{1}{ab^2} + \frac{1}{a^2b} &= \frac{a-b-a+b}{a^2b^2} \\
 &= 0
 \end{aligned}$$

If  $a=2$  and  $b=3$ ,

$$\begin{aligned}
 \text{then } \frac{a+b}{a} + \frac{a+b}{b} - 2 &= \frac{5}{2} + \frac{5}{3} - 2 = 2\frac{1}{6} \\
 \text{Also } \frac{a^2+b^2}{ab} &= \frac{2^2+3^2}{6} = 2\frac{1}{6}
 \end{aligned}$$

If  $a=2$  and  $b=3$ ,

$$\begin{aligned}
 \text{then } \frac{a-b}{a^2b^2} - \frac{1}{ab^2} + \frac{1}{a^2b} &= \frac{2-3}{3^2 \cdot 2^2} - \frac{1}{2 \cdot 3^2} + \frac{1}{2^2 \cdot 3} \\
 &= \frac{1}{12} - \frac{1}{18} - \frac{1}{18} = 0
 \end{aligned}$$



## EXAMPLES V n Page 109

- 15 The daughter's share is  $(1 - \frac{5}{8})$ , or  $\frac{3}{8}$  of the estate  
 req<sup>d</sup> value =  $\frac{3}{8}$  of £5650 = £2118 15s
- 16  $(1 - \frac{11}{16})$  or  $\frac{5}{16}$  of the distance remains  
 req<sup>d</sup> no of Km =  $\frac{5}{16}$  of 896 = 280
- 17 I possess  $\frac{7}{8}$  of Rs 3,00,000 or Rs. 1,75,000  
 req<sup>d</sup> amt = Rs 1,75,000 - Rs 25,000 = Rs 1,50,000
- 18  $(1 - \frac{13}{18})$  or  $\frac{5}{18}$  of the amount remains  
 Since  $\frac{5}{18}$  of 1620 tons = 450 tons,  
 req<sup>d</sup> time =  $\frac{450}{90}$  days = 5 days
- 19 The senior partner gets  $\frac{4}{11}$  of £10043 or £3652,  
 „ second „ „  $\frac{3}{11}$  of £10043 or £2739,  
 „ third „ „  $\frac{2}{11}$  of £10043 or £1826  
 The reserve fund = £(10043 - 3652 - 2739 - 1826) = £1826
- 20  $(1 - \frac{3}{8})$  or  $\frac{5}{8}$  of the cargo remained good. Now  $\frac{5}{8}$  of 12000 = 7500,  
 and req<sup>d</sup> profit = Rs  $2 \times \frac{7500}{100} =$  Rs 150

## EXAMPLES V p Page 113

- 36 A major's pay =  $\frac{8}{9}$  of 18s = 16s ,  
 hence a captain's pay =  $\frac{7}{9}$  of 16s = 11s 6d
- 37 B's age =  $\frac{4}{5}$  of C's =  $\frac{4}{5}$  of 15 yrs = 20 yrs ,  
 A's age =  $\frac{7}{4}$  of B's =  $\frac{7}{4}$  of 20 yrs = 25 yrs ,  
 M's age = (25 + 20 + 15) yrs = 60 yrs
- 38 In Dec. 1905 its value =  $\frac{5}{9}$  of Rs 1,20,000 = Rs 1 00,000  
 In Dec. 1906 „ „ =  $\frac{5}{9}$  of Rs 1,00,000 = Rs 83,333 5a 4p
- 39 Since the rule is  $\frac{1}{12}$  ft too short,  
 the apparent length of 1 ft is really  $\frac{11}{12}$  ft ,  
 „ „ 24 ft „  $\frac{11}{12}$  of 24 ft, or 22 ft ,  
 and „ „ 20 ft „  $\frac{11}{12}$  of 20 ft, or 18  $\frac{2}{3}$  ft  
 Had each foot of the rule been  $\frac{1}{12}$  ft too long,  
 a real length of  $1\frac{1}{12}$  ft would have appeared to be 1 ft  
 „ „ 1 ft „ „ „  $\frac{13}{12}$  ft  
 „ „ 22 ft „ „ „  $\frac{13}{12}$  of 22 ft,  
 or  $20\frac{4}{3}$  ft  
 „ „ 18  $\frac{2}{3}$  ft „ „ „  $\frac{13}{12}$  of 18  $\frac{2}{3}$  ft,  
 or  $16\frac{1}{3}$  ft

- 40 Total amount cut off =  $\frac{7}{15} + \frac{3}{15}$  of  $\frac{8}{15} = \frac{10}{15}$ ,  
 fraction of stick left is  $\frac{5}{15}$
- 41 Total distance he rides =  $\frac{1}{5} + \frac{1}{4}$  of  $\frac{7}{5} = \frac{3}{5}$ ,  
 distance he walks =  $\frac{1}{5}$
- 42 The first has  $\frac{7}{4}$  of 48s, or 14s,  
 the second has  $\frac{8}{7}$  of 34s, or 16s,  
 and the third has  $\frac{2}{5}$  of 18s, or 4s,  
 whence the fourth has  $(48 - 14 - 16 - 4)s$ , or 14s

+

## EXAMPLES V q Page 116

- 25 Let  $x$  be the req<sup>d</sup> fraction, then  $4\frac{7}{8} \times x = 4\frac{1}{2}$ , or  $x \times \frac{39}{8} = \frac{9}{2}$  Multi-  
 plying both sides by  $\frac{8}{39}$ , we have  $x \times \frac{8}{39} \times \frac{39}{8} = \frac{9}{2} \times \frac{8}{39}$ , or  
 $x = \frac{12}{13}$
- 26 Req<sup>d</sup> no =  $\frac{\text{dividend}}{\text{quotient}} = 4\frac{7}{2} - 1\frac{1}{3} = \frac{51}{2} - \frac{4}{3} = \frac{153}{2} \times \frac{2}{4} = 3\frac{7}{8}$
- 27 Req<sup>d</sup> no of times =  $15 - (4\frac{1}{8} \text{ of } \frac{4}{11}) = 15 - \frac{1}{2} = 15 \times \frac{2}{2} = 10$
- 28 We have  $10\frac{2}{3} - (2\frac{2}{7} \times 2\frac{4}{7}) = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \times \frac{1}{8} = 2$
- 29 (i)  $\frac{5}{8} \text{ mi} = 1 \text{ Km}$ , (ii) From (i)  
 $\frac{5}{8} \times \frac{8}{5} \text{ mi} = \frac{8}{5} \text{ Km}$ ,  $1 \text{ mi} = 1\frac{3}{5} \text{ Km}$ ,  
 or  $1 \text{ mi} = 1\frac{3}{5} \text{ Km}$   $1 \text{ mi} = 1\frac{3}{5} \times 1000 \text{ m} = 1600 \text{ m}$
- (iv)  $39\frac{1}{8} \text{ in} = 1 \text{ m}$ , (v)  $39\frac{3}{8} \text{ in} = 1 \text{ m}$ ,  
 hence  $1 \text{ in} = (1 - 39\frac{3}{8}) \text{ m}$  or  $39\frac{3}{8} \text{ in} = 100 \text{ cm}$ ,  
 $36 \text{ in} = \frac{36}{39\frac{3}{8}} \text{ m} = \frac{32}{35} \text{ m}$  whence  $1 \text{ in} = \frac{100}{39\frac{3}{8}} \text{ cm} = 2\frac{34}{83} \text{ cm}$
- 30 (ii)  $1 \text{ cwt} = 112 \text{ lbs} = 112 \times \frac{5}{11} \text{ Kg} = 50\frac{1}{11} \text{ Kg}$   
 (vi)  $1 \text{ litre} = \frac{7}{4} \text{ pints}$ , (vii)  $1 \text{ gram} = \frac{1}{1000} \text{ of } 2\frac{1}{5} \text{ lbs}$   
 $1 \text{ Kl} = \frac{7}{4} \times 1000 \text{ pints}$   $= \frac{1}{1000} \times \frac{11}{5} \times 7000 \text{ grs}$   
 $= \frac{7 \times 1000}{4 \times 8} \text{ gals}$   $= 15\frac{1}{5} \text{ grs}$   
 $= 218\frac{3}{4} \text{ gals}$

## EXAMPLES V r Page 119

$$4 \quad \text{Exp}^n = 3\frac{1}{2} + \frac{2}{8} \times \frac{3}{1} = 3\frac{1}{2} + 1\frac{1}{6} = 4 + \frac{5+2}{10} = 4\frac{7}{10}$$

$$5 \quad \text{Exp}^n = \frac{7}{2} \times \frac{1}{2} + \frac{1}{3} = 8\frac{1}{4} + \frac{1}{3} = 9\frac{1}{12}$$

$$6 \quad \text{Exp}^n = 3\frac{1}{2} + (\frac{2}{8} \times \frac{3}{1}) = 3\frac{1}{2} + 1\frac{2}{8} = 3\frac{1}{2} \frac{2}{8}$$

$$13 \quad \text{Exp}^n = \frac{2}{8} \times \frac{5}{3} - \frac{1}{2} = 8 - \frac{1}{2} = 7\frac{1}{2}$$

$$14 \quad \text{Exp}^n = (5\frac{2}{8}) \times \frac{1}{2} = 2\frac{7}{10}$$

$$15 \quad \text{Exp}^n = 4\frac{4}{8} + (\frac{3}{10}) = 5\frac{1}{10}$$

$$22 \quad \text{Exp}^n = (4 + \frac{2}{7} \frac{8}{8}) - (2 + \frac{4}{1} \frac{1}{8} \frac{1}{1}) = 4\frac{1}{8} \frac{3}{8} - (1 + \frac{18}{1} \frac{4}{8} - \frac{1}{8}) \\ = 4\frac{1}{8} - 1\frac{7}{8} = \frac{2}{8} \times \frac{1}{8} = 3$$

$$23 \quad \text{Exp}^n = (\frac{1}{4} \times \frac{1}{3}) - (\frac{2}{6}) \times (\frac{3}{4}) = \frac{1}{4} \times \frac{1}{3} \times \frac{6}{13} \times \frac{1}{4} = 21\frac{1}{8}$$

$$24 \quad \text{Exp}^n = (\frac{5}{2} \frac{1}{8}) (\frac{3}{4} \frac{3}{8}) - (\frac{6}{7} \frac{3}{8} \frac{5}{8}) = \frac{1}{8} \times \frac{1}{4} \times \frac{1}{7} = 1\frac{2}{8}$$

$$25 \quad \text{Exp}^n = (\frac{9}{8} \frac{1}{8}) - (2\frac{3}{8}) \times \frac{8}{18} = \frac{2}{8} \times \frac{1}{8} \times \frac{8}{18} = \frac{1}{18}$$

$$26 \quad \text{Exp}^n = \frac{4}{8} + (\frac{1}{8} \text{ of } \frac{1}{8}) + (\frac{9}{17} \text{ of } \frac{3}{8}) = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8} + \frac{1}{8} + \frac{2}{8} = 1\frac{1}{8}$$

$$27 \quad \text{Exp}^n = (\frac{5}{3} \times \frac{1}{8}) + 2\frac{3}{8} - (\frac{7}{8} \times \frac{3}{8}) = \frac{3}{8} + 2\frac{3}{8} - \frac{2}{8} = 2\frac{1}{8}$$

$$28 \quad \text{Exp}^n = (\frac{1}{3} \times \frac{9}{2}) - (\frac{1}{4} \times \frac{1}{8}) - 1\frac{3}{8} = 24 - 10\frac{2}{8} - 1\frac{3}{8} = 24 - 11 - \frac{1}{8} = 12$$

$$29 \quad \text{Exp}^n = 2\frac{1}{3} + (\frac{4}{8} \times \frac{1}{8} \times \frac{3}{4}) - 2\frac{1}{2} + \frac{5}{11} = 2\frac{1}{3} + 2 - 2\frac{1}{2} + \frac{5}{11} \\ = 2 + \frac{1}{3} + \frac{5}{11} - \frac{1}{2} = 2 + \frac{22+30-11}{66} = 2\frac{1}{6}$$

$$30 \quad \text{Exp}^n = \frac{2}{10} \frac{9}{8} + (\frac{4}{8} \times \frac{7}{4}) - (\frac{7}{11} \times \frac{1}{3} \times \frac{6}{40}) = \frac{2}{10} \frac{9}{8} + \frac{1}{8} - \frac{7}{40} = \frac{1}{10} \frac{9}{8} = \frac{4}{11}$$

$$31 \quad \text{Req}^d \text{ value} = (1\frac{1}{4} - \frac{3}{8}) \times (7\frac{9}{10}) - (1\frac{3}{8} \frac{3}{8} - \frac{1}{4}) \\ = \frac{7}{8} \times \frac{1}{10} \frac{1}{8} - 1\frac{7}{8} = \frac{7}{8} \times \frac{1}{10} \frac{1}{8} \times \frac{4}{8} = 5\frac{3}{10}$$

$$32 \quad \text{Req}^d \text{ value} = 2\frac{1}{8} + (6\frac{1}{4} \times 1\frac{3}{8}) - (2\frac{1}{2} \times 3\frac{2}{8} - 1\frac{2}{8}) \\ = 2\frac{1}{8} + (\frac{2}{4} \times \frac{8}{8}) - (\frac{5}{2} \times \frac{1}{8} \times \frac{1}{8}) \\ = 2\frac{1}{8} + 10 - 7\frac{1}{8} = 5 + \frac{1}{8} - \frac{1}{8} = 4 + \frac{18}{18} = 4\frac{1}{8}$$

$$33 \quad \text{Req}^d \text{ value} = [7\frac{1}{2} \text{ of } (9\frac{1}{2} - 5\frac{1}{4})] - [(2\frac{1}{2} \text{ of } 9\frac{1}{2}) - 5\frac{1}{4}] \\ = [7\frac{1}{2} \text{ of } 4\frac{3}{4}] - [1\frac{1}{2} \times \frac{1}{2} - 5\frac{1}{4}] \\ = (\frac{1}{2} \text{ of } \frac{1}{4}) - (\frac{1}{4} \times \frac{1}{2}) + 5\frac{1}{4} \quad (\text{See Art 51}) \\ = (\frac{1}{4} \times 31) - (\frac{1}{4} \times 19) + 5\frac{1}{4} \\ = \frac{1}{4} (31 - 19) + 5\frac{1}{4} \quad (\text{See Art 55}) \\ = \frac{1}{4} \times 12 + 5\frac{1}{4} = 12\frac{1}{4} + 5\frac{1}{4} = 17\frac{1}{4}$$

## EXAMPLES V s Page 121

$$22 \text{ Fraction} = \frac{3\frac{1}{2} - 1\frac{7}{8}}{3\frac{1}{2} \times 2\frac{1}{2}} = \frac{2\frac{4}{8} - \frac{7}{8}}{\frac{1}{4} \times \frac{5}{2}} = \frac{1 + \frac{12}{8} - \frac{7}{8}}{\frac{5}{8}} = 1\frac{5}{8} - \frac{6}{8} = \frac{1}{8}$$

$$23 \text{ Fraction} = (\frac{6}{7} \times \frac{2}{10}) - (\frac{5}{8} \times \frac{1}{17}) = \frac{9}{10} \times \frac{5}{7} = 2\frac{1}{4}$$

$$24 \text{ Fraction} = (\frac{3}{4} \times \frac{2}{5}) - (\frac{3}{4} \times \frac{8}{5}) = 7 - 14 = \frac{1}{2}$$

$$25 \text{ Fraction} = (\frac{9}{10} \text{ of } \frac{2}{3}) - (2 + \frac{20}{10} - \frac{11}{1}) = \frac{1}{4} - 2\frac{9}{10} = 1\frac{1}{4}$$

$$26 \text{ Exp}^n = \frac{1\frac{1}{2} \times 2}{22\frac{1}{2} \times 2} - \frac{21\frac{1}{2} \times 3}{1\frac{1}{2} \times 3} = \frac{3}{4} - \frac{6}{4} = \frac{1}{4}$$

$$27 \text{ Fraction} = \frac{\frac{5}{4} \times \frac{2}{3}}{\frac{1}{16} + \frac{1}{16}} = \frac{\frac{5}{4} \times \frac{2}{3}}{\frac{2}{16}} = \frac{5}{4} \times \frac{2}{3} \times \frac{8}{1} = 5$$

$$28 \text{ Fraction} = \frac{(\frac{1}{2} + \frac{3}{4} + \frac{5}{6}) \times 12}{(\frac{2}{3} + \frac{1}{4} + \frac{1}{6}) \times 12} = \frac{6+9+10}{8+3+14} = 1$$

$$29 \text{ Exp}^n = \frac{3}{14} \text{ of } (\frac{4}{9} \times \frac{3}{7}) - (\frac{8}{7} \times \frac{1}{10}) = \frac{1 \times 4 \times 1 \times 6 \times 7 \times 6}{14 \times 9 \times 3 \times 7 \times 8 \times 11} = \frac{6}{14}$$

$$30 \text{ Exp}^n = \frac{7}{15} - \frac{\frac{7}{4}}{\frac{2}{5}} = \frac{7}{15} - (\frac{7}{4} \times \frac{5}{2}) = \frac{7}{15} - \frac{35}{8} = 1\frac{2}{3}$$

$$31 \text{ Exp}^n = \frac{3\frac{8}{17} - \frac{5}{9}}{2\frac{4}{6} - 2\frac{1}{3}} + \frac{1\frac{1}{6}}{11\frac{1}{6} - 2\frac{1}{3}} = \frac{(3\frac{8}{17} - \frac{5}{9}) \times 45}{2\frac{4}{6} \times 45} + \frac{1\frac{1}{6} \times 15}{(11\frac{1}{6} - 2\frac{1}{3}) \times 15}$$

$$= \frac{150 - 25}{135} + \frac{15}{15} = 1\frac{1}{3}$$

$$32 \text{ Exp}^n = \frac{3 + \frac{2}{3} + \frac{4}{7} - \frac{3}{5}}{\frac{1}{3} + \frac{1}{5} + \frac{1}{7}} - (3 - \frac{1}{10}) = \frac{(3 + \frac{2}{3} + \frac{4}{7} - \frac{3}{5}) \times 105}{(\frac{1}{3} + \frac{1}{5} + \frac{1}{7}) \times 105} - \frac{105}{10}$$

$$= \frac{315 + 70 + 60 - 63}{35 + 21 + 15} \times \frac{70}{105} = \frac{382}{71} \times \frac{70}{105} = 1\frac{6}{7}$$

## EXAMPLES V t Page 122

11 Multiply by 6, then  $3x + 2v = 30$

13 Multiply by 4, then  $4x = v + 24$

18 Multiply by 6, then  $2(v+1) + 3(z-1) = 24$

21 Let  $z$  be the number, then  $z + \frac{x}{3} = 20$

22 Let  $x$  be the number, then  $x = 20 - \frac{7}{4}$

23 Let  $x$  be the number, then  $\frac{x}{3} - \frac{x}{5} = 2$

24  $\frac{1}{7}x = \frac{8}{5}$ ,  $x = \frac{8}{5} \times \frac{7}{1}$       25  $\frac{3}{7}x = \frac{14}{5} \times \frac{7}{8}$ ,  $x = \frac{14}{5} \times \frac{5}{8} \times \frac{7}{3}$

26 Let  $x$  and  $50 - x$  be the two parts, then  $50 - x = \frac{3x}{7}$

27 Let  $x$  and  $x + 6$  be the two numbers, then  $x + (x + 6) = 28$

28 Let  $A$  have  $x$  rupees and  $B$   $(20 - x)$  rupees, then  $(20 - x) + \frac{x}{3} = x$

29 Equation becomes  $\frac{7}{2} = \frac{21}{x}$  Multiply by  $2x$ , then  $7x = 42$

30 (i) Equation becomes  $\frac{3x}{2} = \frac{6 \times 4 \times 7}{14}$ , or  $\frac{3x}{2} = 12$

Multiply by 2, then  $3x = 24$

(ii) Equation becomes  $\frac{6 \times 16 \times x}{8} = 7$ , or  $12x = 7$

Divide by 12, then  $x = \frac{7}{12}$

(iii) Equation becomes  $\frac{x}{7} = 17$  Multiply by 7, then  $x = 119$

### EXAMPLES VI a Page 126

8 Exp<sup>n</sup> = Rs  $4\frac{1}{10} \times \frac{3}{2} = \text{Rs } \frac{7}{10} \times \frac{3}{2} = \text{Rs } \frac{21}{20} = 8 \text{ a } 3 \text{ p}$

9 Exp<sup>n</sup> = Rs  $6\frac{1}{4} \times \frac{1}{2} = \text{Rs } \frac{3}{2} \times \frac{1}{2} = \text{Rs } \frac{3}{4}$

10 Exp<sup>n</sup> =  $81\frac{1}{3} \times \frac{7}{10} = \frac{244}{3} \times \frac{7}{10} = \frac{4727}{10} = \text{£}1 \text{ } 15 \text{ } 7 \text{ d}$

11 Exp<sup>n</sup> = Rs  $4\frac{3}{8} \times \frac{2}{1} = \text{Rs } \frac{35}{8} \times \frac{2}{1} = \text{Rs } \frac{7}{1} = 9 \text{ a } 4 \text{ p}$

12 Exp<sup>n</sup> = 6 tons  $\times 3\frac{4}{5} = 18 \text{ tons} + 6 \text{ tons} \times \frac{4}{5}$

Now  $6 \text{ tons} \times \frac{4}{5} = 6 \times 20 \times \frac{4}{5} \text{ cwt} = 13\frac{2}{5} \text{ cwt}$ ,

Again  $\frac{2}{5} \text{ cwt} = \frac{2 \times 4}{7} \text{ qrs} = 2\frac{2}{7} \text{ qrs}$ ,

and  $\frac{2}{7} \text{ qrs} = \frac{2 \times 28}{7} \text{ lbs} = 24 \text{ lbs}$

Exp<sup>n</sup> = 18 tons 13 cwt 2 qrs 24 lbs

13 Exp<sup>n</sup> =  $105 \text{ s} \times \frac{1}{1} = \frac{175}{1} \text{ s} = \text{£}1 \text{ } 9 \text{ s } 2 \text{ d}$

14 Exp<sup>n</sup> =  $24 \text{ ft} \times 1\frac{7}{10} = 24 \text{ ft} \times \frac{17}{10} = \frac{408}{10} \text{ ft} = 9 \text{ yds } 1 \text{ ft } 8 \text{ in}$

15 Exp<sup>n</sup> = Rs  $12 \times \frac{1}{10} = \text{Re } \frac{12}{10} = \text{Re } 1\frac{2}{5} = \text{Re } 1 \text{ } 6 \text{ a}$

16 Exp<sup>n</sup> =  $18 \text{ mds} \times 2\frac{5}{4} = 18 \text{ mds} \times \frac{13}{4} = \frac{117}{4} \text{ mds} = 39 \text{ mds } 30 \text{ srs}$

17 Since  $5\frac{3}{4} = 6 - \frac{1}{4}$ ,  $\text{exp}^n = \text{Rs. } 11 \text{ 7a } 8\text{p} \times (6 - \frac{1}{4})$

$$= \text{Rs } 11 \text{ 7a } 8\text{p} \times 6 - \text{Rs } 11 \text{ 7a } 8\text{p} \times \frac{1}{4}$$

$$= \text{Rs } 68 \text{ 14a} - \text{Rs } 2 \text{ 13a } 11\text{p} = \text{Rs } 66 \text{ 0a } 1\text{p}$$

18  $\frac{5}{8} = \frac{4+1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{1}{2} + \frac{1}{8}$

Rs	a	p	
3	7	4	(a)
		2	

$\frac{1}{2}$	6	14	8	
$\frac{1}{8}$	1	11	8	$= \frac{1}{8}$ of (a)
		6	11	$= \frac{1}{8}$ of (a)
	9	1	3	

19  $\frac{3}{8} = \frac{2+1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{1}{4} + \frac{1}{8}$

tons	cwt	qrs	lbs	oz	
1	7	3	0	0	(a)
				5	

$\frac{1}{4}$	6	18	3	0	0	
$\frac{1}{8}$		6	3	21	0	$= \frac{1}{8}$ of (a)
		3	1	24	8	$= \frac{1}{8}$ of (a)
	7	9	0	17	8	

20  $\text{Exp}^n = \text{£}4 \text{ 3s } 8\text{d} \times \frac{2}{3}$ , we divide  $\text{£}4 \text{ 3s } 8\text{d}$  by 16 and multiply the result by 23

21 Since  $3\frac{7}{8} = 4 - \frac{1}{8}$ ,  $\text{exp}^n = \text{Rs } 9 \text{ 6a } 8\text{p} \times (4 - \frac{1}{8})$

$$= \text{Rs } 9 \text{ 6a } 8\text{p} \times 4 - \text{Rs } 9 \text{ 6a } 8\text{p} \times \frac{1}{8}$$

$$= \text{Rs } 37 \text{ 10a } 8\text{p} - \text{Rs } 1 \text{ 2a } 10\text{p} = \text{Rs } 36 \text{ 7a } 10\text{p}$$

22  $\text{Exp}^n = 288 \text{ p} \times \frac{1\frac{1}{2}}{2} = \frac{108}{2} \text{ p} = 211\frac{1}{2} \text{ p} = 1 \text{ ac } 1 \text{ 11}\frac{1}{2} \text{ p}$

23  $\text{Exp}^n = \text{Rs } 24 \text{ 11a} \times \frac{5}{12} = \text{Rs } 123 \text{ 7a} + \text{Rs } 24 \text{ 11a} \times \frac{1}{12}$

Now  $\text{Rs. } 24 \text{ 11a} \times \frac{1}{12} = (\frac{1}{12} \text{ of Rs } 24 \text{ 11a}) \times 7$

$$= \text{Rs } 2 \text{ 0a } 11\text{p} \times 7 = \text{Rs } 14 \text{ 6a } 5\text{p}$$

$$\text{exp}^n = \text{Rs } 123 \text{ 7a} + \text{Rs } 14 \text{ 6a } 5\text{p} = \text{Rs } 137 \text{ 13a } 5\text{p}$$

24  $\text{Exp}^n = 2 \text{ mds } 30 \text{ srs } 4 \text{ chks} \times \frac{4}{5} = 1764 \text{ chks} \times \frac{4}{5}$

$$= 980 \text{ chks} = 1 \text{ md } 21 \text{ srs } 4 \text{ chks}$$

25  $\text{Req}^d \text{ value} = (\frac{2}{3} \text{ of } 7 + \frac{1}{3} \text{ of } 11) \text{ half-annas}$

$$= (3 + 3\frac{1}{2}) \text{ half annas} = 3 \text{a } 3\text{p}$$

26  $\text{Req}^d \text{ value} = (\frac{2}{3} \text{ of } 126 + \frac{1}{3} \text{ of } 33) \text{ sixpences} = (28 + 12) \text{ sixpences}$   
 $= \text{£}1$

27  $\text{Req}^d \text{ value} = (\frac{2}{5} \text{ of } 90 - \frac{1}{5} \text{ of } 192) \text{ in} = (126 - 120) \text{ in} = 6 \text{ in}$

28  $\text{Req}^d \text{ value} = (\frac{2}{3} \text{ of } 1760 - \frac{1}{3} \text{ of } 220) \text{ yds} = (330 - 100) \text{ yds}$   
 $= 230 \text{ yds}$

29  $\text{Req}^d \text{ value} = (\frac{5}{8} \text{ of } 63 + \frac{1}{8} \text{ of } 55) \text{ p}$

$$= (\frac{315}{8} + \frac{55}{8}) \text{ p} = \text{Rs } 1 \text{ 9a } 5\text{p}$$

30  $\text{Req}^d \text{ value} = (\frac{1}{2} \text{ of } 1980 + \frac{1}{4} \text{ of } 34) \text{ ft.} = 2886 \text{ ft} = 962 \text{ yds}$

$$31 \text{ Req}^d \text{ value} = \left( \frac{5}{14} \text{ of } 112 + \frac{3}{13} \text{ of } 52 + \frac{8}{3} \text{ of } 12 \right) \text{ lbs} = (40 + 12 + 32) \text{ lbs} \\ = 3 \text{ qrs}$$

$$32 \text{ Req}^d \text{ value} = \left( \frac{3}{8} \text{ of } \frac{4}{11} \text{ of } 256 + \frac{1}{2} \text{ of } 504 + \frac{6}{11} \text{ of } 352 \right) \text{ p} \\ = (384 + 384 + 192) \text{ p} = \text{Rs } 5$$

$$33 \text{ Req}^d \text{ value} = \left( \frac{7}{6} \text{ of } \frac{1}{4} \text{ of } 120 - \frac{1}{8} \text{ of } \frac{5}{4} \text{ of } 24 + \frac{3}{11} \text{ of } 462 \right) \text{ pence} \\ = (12 - 18 + 126) \text{ pence} = 10s$$

### EXAMPLES VI b Page 128

$$22 \text{ Fraction} = \frac{23\frac{1}{4} \text{ a}}{31 \text{ a}} = \frac{93}{4 \times 31} = \frac{3}{4} \qquad 24 \text{ Fraction} = \frac{476 \text{ cm}}{544 \text{ cm}} = \frac{7}{8}$$

$$26 \text{ Fraction} = \frac{323 \text{ sq ft}}{68 \times 9 \text{ sq ft}} = \frac{17 \times 19}{17 \times 4 \times 9} = \frac{1}{4}$$

$$27 \text{ Fraction} = \frac{\frac{3}{4} \text{ of } 37\frac{1}{2}s}{\frac{4}{9} \text{ of } 135s} = \frac{\frac{3}{4} \text{ of } \frac{75}{2}}{\frac{4}{9} \text{ of } \frac{135}{1}} = \left( \frac{3 \times 75}{4 \times 2} \right) - \left( \frac{4 \times 135}{9 \times 1} \right) \\ = \frac{3 \times 75 \times 9}{4 \times 2 \times 135} = \frac{3}{8}$$

$$31 \text{ Fraction} = \frac{2244 \text{ yds}}{2 \times 1760 \text{ yds}} = \frac{3}{8}$$

$$32 \text{ See Art 137 We have } \frac{4d}{5\frac{3}{4}d} = \frac{1}{5} \quad \text{Also } \frac{5 \text{ yds } 1 \text{ ft}}{7 \text{ yds } 2 \text{ ft}} = \frac{16 \text{ ft}}{23 \text{ ft}} = \frac{16}{23}$$

$$33 \text{ From Art 137 } \frac{x \text{ lbs}}{28 \text{ lbs}} = \frac{5\frac{1}{4} \text{ a}}{73\frac{1}{2} \text{ a}}, \text{ or } \frac{x}{28} = \frac{21}{294}, \\ \text{whence } x = \frac{21}{294} \times 28 = 2$$

$$34 \text{ A saves } \frac{2}{5} \text{ of Rs } 2000, \text{ or Rs } 800 \text{ per annum}$$

$$\text{B saves } \frac{3}{8} \text{ of Rs } 1440, \text{ or Rs } 540 \text{ per annum}$$

$$\text{req}^d \text{ ratio} = \frac{\text{Rs } 800}{\text{Rs } 540} = \frac{40}{27}$$

$$35 \text{ Rem}^r = \frac{1}{12} \text{ of } \frac{5}{9} \text{ of } £15 - \frac{7}{15} \text{ of a guinea} \\ = \left( \frac{1}{12} \text{ of } \frac{5}{9} \text{ of } 300 - \frac{7}{15} \text{ of } 21 \right) \text{ shillings} \\ = \left( \frac{1}{9} \times 5 - \frac{49}{5} \right) \text{ shillings} = \frac{184}{45}s$$

$$\text{req}^d \text{ fraction} = \frac{\frac{184}{45}s}{£3 \text{ } 9s} = \frac{\frac{184}{45}s}{69s} = \frac{184}{45 \times 69} = \frac{8}{135}$$

## EXAMPLES VI c. Page 131

- 7 Since 6% are cherry trees, 94% are trees of other kinds,

$$\text{req'd number} = \frac{94}{100} \text{ of } 350 = 329$$

- 9 Let  $r$  = the rate per cent, then  $\frac{r}{100} = \frac{18}{20}$ , and  $r = 5$

- 10 Let  $r$  be the req'd percentage. He saves Rs 2400,

$$\frac{r}{100} = \frac{2400}{7500}, \text{ and } r = 32$$

- 12 Let  $r$  be the req'd number, then  $\frac{r}{100}$  of 2 = 1603,

$$\text{whence } r = 1603 \times \frac{100}{2} = 80150$$

- 15 If Rs  $r$  be the req'd price, then  $\frac{61}{100}$  of 2 = 9307,

$$r = 9307 \times \frac{100}{61} = 15273.77$$

- 16 Since 6 1/2% are lost, 93 1/2% are left fit for service

If  $x$  be original number, then  $\frac{93\frac{1}{2}}{100}$  of  $x = 13132$ ,

$$x = 13132 \times \frac{100}{93\frac{1}{2}} = 14070$$

- 17 If £ $x$  be the income, then the deficit is  $\frac{£7\frac{1}{2}}{100}$

But this is the difference between the expenses and the income

$$\text{or } £(4837\frac{1}{2} - x), \quad \frac{7\frac{1}{2}}{100}x = 4837\frac{1}{2} - x, \quad x + \frac{7\frac{1}{2}}{100}x = 4837\frac{1}{2}$$

$$\text{or } \frac{107\frac{1}{2}}{100}x = 4837\frac{1}{2}, \text{ whence } x = 4837\frac{1}{2} \times \frac{100}{107\frac{1}{2}} = 4500$$

- 19 The Welsh boys =  $(100 - 45 - 20 - 13)\% = 22\%$  of req'd number

$$\text{If this be } x, \text{ then } \frac{22x}{100} = 44, \quad x = 44 \times \frac{100}{22} = 200$$

- 20 He takes off 3a 9p. If  $x$  be the req'd amount per cent,

$$\frac{x}{100} = \frac{3a \ 9p}{\text{Rs } 1 \ 15s \ 3p}, \text{ or } \frac{x}{100} = \frac{45p}{375p} \text{ and } x = 12$$



## EXAMPLES VI e Page 134

8 Re 1 5a 4p = Rs 1½

9 £3 2s 6d = £3½

10 £4 1s 3d = £4 1½

11 Re 1 12a = Rs 2 - Re ½

12 Rs 2 14a = Rs (3 - ½)

14 10a 8p = Re (1 - ½)

	Rs	a	p
	425	0	0
8a = ½ of Re 1	212	8	0
2a = ¼ of 8a	53	2	0

	Rs	a	p
	960	0	0
2a = ½ of Re 1	120	0	0
3p = ½ of 2a	15	0	0

	Rs	a	p
	324	0	0
5a 4p = ½ of Re 1	108	0	0
4a = ¼ of Re 1	81	0	0

	Rs	a	p
	357	0	0
			2
5a 4p = ½ of Re 1	714	0	0
1a 4p = ¼ of 5a 4p	119	0	0
	29	12	0

	£	s	d
	464	0	0
			4
5s = ½ of £1	1856	0	0
1s 3d = ¼ of 5s	116	0	0
	29	0	0

	£	s	d
	425	0	0
			3
10s = ½ of £1	1275	0	0
1s 8d = ⅙ of 10s	212	10	0
	35	8	4

	Rs	7	p
	723	0	0
			3
8a = 1 of Re 1	2169	0	0
2a = ½ of 8a	361	8	0
1a = ¼ of 2a	90	6	0
	45	3	0

	Rs	7	p
	355	0	0
			3
8a = 1 of Re 1	1065	0	0
5a 4p = ½ of Re 1	177	8	0
	118	5	4

	£	s	d
	632	0	0
10s = 1 of £1	316	0	0
2s = ½ of 10s	63	4	0
6d = ¼ of 2s	15	16	0
2d = ½ of 6d	5	5	4
½d = ¼ of 2d	1	6	4

	£	s	d
	6113	0	0
5s = ½ of £1	1528	5	0
2s = ¼ of £1	611	6	0
1s = ½ of 2s	305	13	0
1d = ½ of 1s	25	9	5
½d = ¼ of 1d	12	14	8½

	Rs	a	p		Rs	a	p
27	427	0	0	28	288	0	0
			6				3
5a 4p = $\frac{1}{2}$ of Re 1	2562	0	0	8a = $\frac{1}{2}$ of Re 1	864	0	0
1a 4p = $\frac{1}{4}$ of 5a 4p	142	5	4	4a = $\frac{1}{4}$ of 8a	144	0	0
1p = $\frac{1}{16}$ of 1a 4p	35	9	4	3p = $\frac{1}{16}$ of 4a	72	0	0
	2	3	7		4	8	0

	£	s	d		£	s	d
29	6184	0	0	30	1260	0	0
			5				
5s = $\frac{1}{4}$ of £1	30920	0	0	5s = $\frac{1}{4}$ of £1	315	0	0
2s = $\frac{1}{10}$ of £1	1546	0	0	2s 6d = $\frac{1}{10}$ of 5s	157	10	0
3d = $\frac{1}{8}$ of 2s	618	8	0	1s 3d = $\frac{1}{6}$ of 2s 6d	78	15	0
	77	6	0	2 $\frac{1}{2}$ d = $\frac{1}{6}$ of 1s 3d	13	2	6

## MISCELLANEOUS EXAMPLES II Page 135

- 6 He sells  $\frac{195}{1\frac{1}{2}}$  lbs, he adds  $\left(\frac{195 \times 2}{3} - 112\right)$  lbs
- 10 If  $x$  be the req<sup>d</sup> no, then Rs  $24\frac{7}{8} \times \frac{x}{2} + \text{Rs } 26\frac{5}{8} \times \frac{x}{2}$   
 $= \text{Rs } 25 \times x + \text{Rs } 15$
- 11 Let  $x$  be the req<sup>d</sup> number, then  $\frac{1}{25}x = 36084$ ,  
 $x = 36084 \times 25 = \frac{36084 \times 100}{4} = 902100$
- 14 The total wages in the req<sup>d</sup> time must be an exact multiple both of 10s 6d and 16s 4d, i.e. of 126d and 196d. The L.C.M. of 126d and 196d, or  $(14 \times 14 \times 9)d$ , will therefore be the total wages earned in the req<sup>d</sup> time  
 This will take  $\frac{14 \times 14 \times 9}{196}$  weeks, or 9 weeks to earn
- 15 Train travels  $35 \times 1760 \times 3$  ft in  $60 \times 60$  secs
- 16 If Rs  $x$  be req<sup>d</sup> sum, then  $\frac{3x}{5} + \frac{4x}{25} + 750 = x$
- 17
- |  | £     | s  | d  |
|--|-------|----|----|
|  | 1746  | 0  | 0  |
|  |       |    | 14 |
| 5s = $\frac{1}{4}$ of £1                   | 24444 | 0  | 0  |
| 1s 3d = $\frac{1}{4}$ of 5s                | 436   | 10 | 0  |
| 7 $\frac{1}{2}$ d = $\frac{1}{2}$ of 1s 3d | 109   | 2  | 6  |
|  | 54    | 11 | 3  |

- 20 The train travels  $\frac{14 \times 1760}{80}$  yds or 1584 yds in 1 min  
 the number of spaces, each equal to 66 yds, passed over in  
 1 min will be  $\frac{1584}{66}$  or 24. If at the beginning of the minute  
 the traveller is opposite a post, he will count that post as 1, and  
 will afterwards count 24 more posts in 1584 yds. In any  
 other case he will count 24

Thus 25 is the greatest number and 24 the least  
 [The student should draw a diagram]

- 22 From Art 97,  $LCM = \frac{A \times B}{X}$ , i.e.  $9100 = \frac{260B}{65}$

- 26 (i)  $237160 = 2^4 \cdot 5^2 \cdot 7^2 \cdot 11^2$

(ii)  $60825 = 3^2 \cdot 5^2 \cdot 11$  Thus (i) and (ii) are perfect squares.

- (iii)  $571536 = 2^4 \cdot 3^6 \cdot 7^2$

The sq root of (i) is  $2^2 \cdot 5 \cdot 7 \cdot 11$ , of (ii) is  $2 \cdot 3 \cdot 7$

Hence HCF =  $2^2 \cdot 7$  or 28, LCM =  $2^4 \cdot 3^6 \cdot 5^2 \cdot 7^2 \cdot 11^2$  or 11580

- 29 21 posts have 20 spaces of 11 yds each between them,  
 the train travels  $41 \times 20$  yds or  $\frac{1}{2}$  mi per 1 min,  
 its speed is 30 mi per hour

- 33 Since  $\frac{1}{8}$  mi = 1 Km,  $\frac{1}{8} \times 1760$  yds or 1100 yds = 1000 m,  
 $100 \text{ yds} = \frac{1000}{9} \text{ m} = 91 \text{ m (nearest)}$

- 34 Since the length and breadth must divide 72 and 57 exactly, both  
 length and breadth must be equal to the H.C.F. of 72 and 57,  
 i.e. 3 ft

$$\text{area reqd} = (3 \times 3) \text{ sq ft, i.e. } 1 \text{ sq yd}$$

- 35 By the method of Art 82 we find  
 the number =  $2^7 \cdot 3^2 \cdot 7 \cdot 31 = (2 \cdot 31) \times (3^2 \cdot 7) \times 2^6 = 62 \times 63 \times 64$

- 37 (i)  $\text{Exp}^n = 2^{\frac{1}{2}} + (\frac{1}{2})^{\frac{1}{2}} + (\frac{1}{2})^{\frac{1}{2}} = 2^{\frac{1}{2}} + (1 - \frac{1}{2})^{\frac{1}{2}} + (\frac{1}{2})^{\frac{1}{2}} + (1 - \frac{1}{2})^{\frac{1}{2}}$   
 $= 9 + (\frac{1}{8} - \frac{1}{24}) + (\frac{1}{2} - \frac{1}{24}) = 9 + \frac{1}{4} + \frac{1}{4} = 9\frac{1}{2} + \frac{1}{4} = 10\frac{1}{4}$

- (ii)  $\text{Exp}^n = 1 + \frac{1}{16} - \frac{1}{16} - (\frac{1}{16} - \frac{1}{16}) = 1 + \frac{1}{16} - \frac{1}{16} = 1 + \frac{11-10}{16 \times 11} = 1 + \frac{1}{16}$

- 39 11 tons =  $(11 \times 2240) \text{ lbs} = (\frac{11 \times 2240}{16}) \text{ cwt}$

$$= (\frac{11}{16} \times 112 \times \frac{2240}{16}) \text{ mds} = 299\frac{1}{2} \text{ mds}$$

- 42 Number = (L.C.M. of 15, 35, 42) + 6

- 43 Req<sup>d</sup> value = Rs  $33\frac{1}{3} \times \frac{1}{4} = \text{Rs } 8\frac{1}{4} \times \frac{1}{4} = \text{Rs } 2\frac{1}{4} \times 17$

46 Req<sup>d</sup> number =  $\frac{18 \times 8}{82\frac{2}{7}} = 1\frac{3}{4}$

47. Each fraction is greater than  $\frac{1}{2}$ ,

then sum is greater than  $\frac{1}{2} \times 4$  or 2

For the second part of the question, we find by method of

Art 113 the two fractions to be  $\frac{1}{2}$  and  $\frac{2}{3}$

Req<sup>d</sup> difference =  $\frac{1}{2} - \frac{2}{3} = \frac{3-4}{6} = -\frac{1}{6}$

49 Fraction =  $\frac{\frac{1}{2} \text{ of } 20 - \frac{1}{4} \text{ of } 21}{21} = (\frac{1}{2} - \frac{1}{4}) \times \frac{2}{3} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

52 Cost at pit's mouth =  $5a \times 230 = \text{Rs } 71 \ 14a$

Cost at pit's mouth + carriage for 60 miles =  $\text{Rs } 143 \ 12a$

carriage for 60 mi =  $\text{Rs } 143 \ 12a - \text{Rs } 71 \ 14a$

=  $\text{Rs } 71 \ 14a$  for 230 mds,

hence carriage per md per mile =  $\frac{1150a}{230 \times 60}$

=  $\frac{1150 \times 12}{230 \times 60} p = 1p$

54 See Art 130

55 Walking to and riding from he takes  $3\frac{1}{2}$  hrs

Riding to and riding from he takes  $2\frac{1}{2}$  hrs

by riding one way instead of walking he saves  $1\frac{1}{2}$  hrs,

and by riding both ways instead of walking he would save  $2\frac{1}{2}$  hrs

he would walk both ways in  $(2\frac{1}{2} + 2\frac{1}{2})$  or 5 hrs

57 (i) 1 yd 2 ft 3 in = 63 in      req<sup>d</sup> value =  $63 \text{ in} \times \frac{25}{7}$   
= 225 in = 6 yds 9 in

(ii) Req<sup>d</sup> value =  $\text{Rs } 8 \ 7a \ 4p \times (10 - \frac{1}{5})$   
=  $\text{Rs } 84 \ 9a \ 4p - \text{Rs } 1 \ 0a \ 11p$

58 The first remainder is  $\frac{2}{3}$ , the second is  $\frac{3}{4}$  of  $\frac{2}{3}$  or  $\frac{1}{2}$   
if I had Rs 2 at first,  $(1 - \frac{1}{3} - \frac{1}{4} \text{ of } \frac{2}{3} - \frac{1}{6} \text{ of } \frac{1}{2})x = 100$

59 £3 7s 10½d = 1629 halfpence, £1 = 480 halfpence

The least sum of money whose value can be reckoned exactly in gold or or in sovereigns, will be the LCM of 1629 halfpence and 480 halfpence This is  $(160 \times 543 \times 3)$  halfpence, or  $(160 \times 1629)$  halfpence, and would require 160 oz of gold

- 60 Req<sup>d</sup> diff =  $\frac{5}{48}$  of £3 -  $\frac{3}{14}$  of 17s 6d =  $(\frac{5}{48}$  of 120 -  $\frac{3}{14}$  of 35) sixpences  
 =  $(12\frac{1}{2} - 7\frac{1}{2})$  sixpences = 2s 6d
- 63 If a gallery ticket cost  $x$  rupees, then  
 $50x + 30(x+2) + 20(x+3) = 16 \times 20$
64. (i) Fraction =  $\frac{1\frac{2}{3} \times 6 + 6\frac{1}{2}}{5\frac{1}{3} - \frac{1}{6}}$  Multiply numerator and denominator by 36
- 66 (i) No of metres =  $\frac{84 \times 12}{39\frac{3}{8}} = \frac{84 \times 12 \times 8}{315} = 25\frac{2}{5}$   
 (ii) No of cwt =  $\frac{280 \times 1\frac{1}{2}}{112} = \frac{280 \times 11}{112 \times 5} = 5\frac{1}{2}$   
 (iii) 9 gals =  $(9 \times 8)$  pints =  $\frac{9 \times 8}{1\frac{3}{4}}$  litres =  $41\frac{1}{2}$  litres  
 (iv) 2 tons 4 cwt =  $2\frac{1}{2}$  tons =  $2\frac{1}{2} \times 2240$  lbs  
 $= \frac{2\frac{1}{2} \times 2240}{2\frac{1}{2}} \text{ Kg} = 2240 \text{ Kg}$
- 67 (i) Fraction =  $\frac{315 \text{ p}}{855 \text{ p}} = \frac{7}{18}$  (ii) Fraction =  $\frac{77\frac{1}{2} \text{ n}}{87\frac{1}{2} \text{ n}} = 3\frac{1}{2}$
- 68 (i) Exp<sup>n</sup> =  $7 + \frac{7+2+9}{1\frac{1}{8}} + \frac{14+11}{1\frac{1}{8}} = 8\frac{1}{2} + 1\frac{7}{8} = 9\frac{9}{8} = 9\frac{5}{8}$   
 (ii) Exp<sup>n</sup> =  $1\frac{8}{9} \times \frac{7}{3} + \frac{2 \times 1\frac{1}{2} \times 1}{3 \times 3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$
- 69 1 lt =  $1\frac{3}{4}$  pts =  $(1\frac{3}{4} \times 2)$  tumblers =  $(\frac{1\frac{3}{4} \times 2}{\frac{1}{2}})$  teacups, or  $5\frac{1}{2}$  teacups  
 5 teacups can be exactly filled Now  $\frac{5}{8}$  teacup is left  
 This =  $(\frac{1}{8} \times \frac{1}{5\frac{1}{2}})$  litre or  $\frac{1}{7}$  litre
- 70 40 Km per hr =  $40 \times \frac{5}{18}$ , or 25 mi per hr, or 5 mi in 12 min
- 71 3 ac 2 r 20 p =  $3\frac{2}{3}$  ac 1 sq mi = 640 ac  
 Req<sup>d</sup> no =  $(\frac{2}{3} \times 7 \times 640) - 3\frac{2}{3}$
- 72 1 Kg costs  $\frac{51\frac{1}{2} \text{ s}}{20}$ ,  
 1 lb costs  $\frac{51\frac{1}{2} \text{ s}}{20 \times 2\frac{1}{2}}$  and 1 cwt costs  $\frac{51\frac{1}{2} \text{ s}}{20 \times 2\frac{1}{2}} \times 112$
- 73 Exp<sup>n</sup> =  $\frac{15}{20} - \frac{7\frac{1}{2}}{15} = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

74. Let  $9x$  miles be the distance by sea, then  $3x$  miles and  $x$  miles are the distances by rail and coach respectively,

$$9x + 3x + x = 520, \text{ whence } x = 40$$

$$\text{Cost by coach} = 9p \times 40 = 360p, \text{ cost by rail} = 4p \times 120 = 480p$$

$$\text{Cost by sea} = 2p \times 360 = 720p,$$

$$\text{total cost} = (360 + 480 + 720)p = 1560p,$$

$$\text{hence average cost per mile for whole journey} = \frac{1560p}{520} = 3p$$

75.  $55 \text{ Kg} = (55 \times 2\frac{1}{2}) \text{ lbs} = 121 \text{ lbs}$ . cask holds  $(121 - 11) \text{ lbs}$  or  $110 \text{ lbs}$  of water, i.e.  $1\frac{1}{10}$  gals or  $11 \text{ gals}$  of water

76. 2 tons on 1 ac =  $2 \times 2240 \text{ lbs}$  on  $4840 \text{ sq yds}$

$$= \frac{2 \times 2240}{2\frac{1}{2}} \text{ Kg on } 4840 \times 36^2 \text{ sq m}$$

$$= \frac{2 \times 2240}{2\frac{1}{2}} \times 1000 \text{ gm on } \frac{4840 \times 36^2}{(39\frac{3}{8})^2} \text{ sq m}$$

$$\text{This gives } \frac{2 \times 2240 \times 1000 \times 39\frac{3}{8} \times 39\frac{3}{8}}{2\frac{1}{2} \times 4840 \times 36 \times 36} \text{ gm on 1 sq m}$$

$$\text{This fraction} = \frac{2 \times 2240 \times 1000 \times 315 \times 315}{11 \times 4840 \times 8 \times 8 \times 36 \times 36} = \frac{10 \times 35 \times 100 \times 35 \times 35}{11 \times 484 \times 4 \times 4} \\ = \frac{42875000}{88184} = 503 \text{ to nearest integer}$$

77.  $\frac{7}{18} = \frac{4}{18} + \frac{2}{18} + \frac{1}{18} = \frac{1}{4} + \frac{1}{9} + \frac{1}{18}$  of  $\frac{1}{8} = \frac{1}{4} + \frac{1}{9}$  of  $\frac{1}{4} + \frac{1}{18}$  of  $(\frac{1}{4} \text{ of } \frac{1}{4})$

For second part of question we have  $\frac{2}{9}$  of  $2\frac{2}{3} = \frac{1}{9}$

we have to multiply £33 2s 8d by  $\frac{8}{10}$  or  $5\frac{7}{10}$

$$\text{Reqd value} = £33 \text{ 2s } 8d \times 5\frac{7}{10}$$

$$= £33 \text{ 2s } 8d \times [5 + \frac{1}{4} + \frac{1}{20} \text{ of } \frac{1}{4} + \frac{1}{20} \text{ of } (\frac{1}{20} \text{ of } \frac{1}{4})]$$

$$= £165 \text{ 13s } 4d + £8 \text{ 5s } 8d + £4 \text{ 2s } 10d + £2 \text{ 1s } 5d = £180 \text{ 3s } 3d$$

78.  $50 \text{ Km} = 50 \times \frac{5}{8} \text{ mi} = 31\frac{1}{4} \text{ mi}$

the slower train is  $1\frac{1}{4} \text{ mi}$  behind in 1 hr,

i.e. it will be 5 mi behind in 4 hrs

### EXAMPLES VII a Page 143

3. 1 sheep will cost  $£18 \times \frac{1}{2}$       4. 1 man will take  $9 \text{ days} \times 10$

7. 1 cwt should be carried  $30 \text{ mi} \times 7$ , 10 cwt should be carried  $30 \text{ mi} \times \frac{7}{10}$

9. I pay (i) Rs  $12 \times \frac{1}{8}$  for 1 lb, (ii) Rs  $12 \times \frac{6}{8}$  for 6 lbs, (iii) Rs  $12 \times \frac{x}{8}$  for  $x \text{ lbs}$

- 10 1 lb costs Rs  $15 \times \frac{1}{x}$ ,  $y$  lbs cost Rs  $15 \times \frac{y}{x}$
- 11 £68 will be earned in 12 wks  $\times \frac{6}{12}$
- 13 In 100 days 125 men  $\times \frac{100}{125}$  are wanted
- 14 35 horses will eat it in 14 days  $\times \frac{35}{14}$
- 17 At 24 Km an hour I should take 16 hrs  $\times \frac{24}{16}$
- 20 5 loads would bring  $15\frac{3}{4}$  tons  $\times \frac{5}{15\frac{3}{4}}$
- 21 In 43 wks I should save Rs  $62\frac{7}{10} \times \frac{43}{57}$
- 23 (ii)  $x$  men would take  $p$  hrs  $\times \frac{x}{y}$
- 24 (i) Req<sup>d</sup> distance =  $13\frac{1}{2} \times \frac{8}{3}$  mi (ii) Req<sup>d</sup> distance =  $m$  miles  $\times \frac{x}{p}$
- 25 In 17 min it will travel  $30$  mi  $\times \frac{17}{60}$  In  $r$  min it will travel  
 $30$  mi  $\times \frac{x}{60}$
- 26 Req<sup>d</sup> speed = (i)  $\frac{1}{17} \times \frac{9}{100}$  mi  $\times 60$  per hr (ii)  $\frac{a}{1760}$  mi  $\times 60$  per hr
- 27 (i) Req<sup>d</sup> distance =  $(90 \times 1760 \times 3)$  ft  $\times \frac{1}{60 \times 60}$  per sec  
(ii) Req<sup>d</sup> speed =  $(m \times 1760)$  yds  $\times \frac{1}{60}$  per min
- 28 Req<sup>d</sup> time = (i)  $\frac{3}{4}$  min  $\times \frac{60}{100}$ , (ii)  $a$  min  $\times \frac{x}{y}$
- 35 It loses 133 min  $\times \frac{30}{24 \times 7}$  37 Req<sup>d</sup> time = 55 months  $\times \frac{4}{70}$
- 40 Req<sup>d</sup> time = 42 days  $\times \frac{4000}{4000 - 1700}$

### EXAMPLES VII b Page 146

- 1 The whole =  $\frac{8}{9} = \frac{8}{9}$  of  $\frac{3}{8} = \frac{8}{9}$  of Rs 930
- 3 The whole =  $\frac{7}{8} = \frac{7}{8}$  of  $\frac{9}{7} = \frac{7}{8}$  of 54 in = 63 in
- 4  $(1 - \frac{1}{10})$  or  $\frac{9}{10}$  of journey = 6 mi,  $\frac{1}{10} = \frac{1}{10}$  of  $\frac{3}{10} = \frac{1}{10}$  of 6 mi  
= 34 mi
- 5  $\frac{2}{7}$  of inc =  $\frac{2}{7}$  of  $\frac{5}{4}$  of  $\frac{1}{2}$  of inc =  $\frac{2}{7}$  of  $\frac{5}{4}$  of Rs 3080 = Rs 1100
- 6 and 7 As No 5 8 Req<sup>d</sup> part =  $\frac{7}{13} \times \frac{6}{7} \times \frac{9}{10}$  9 As No 8
- 10 1 Km =  $\frac{5}{8}$  mi and costs  $\frac{5}{8}$  of Rs 2400, or Rs 1500
- 11 1 mi =  $\frac{5}{8}$  Km and costs  $\frac{5}{8}$  of Rs 311 $\frac{1}{2}$ , or Rs 498
- 12 1 lb =  $\frac{5}{11}$  Kg and costs  $\frac{7}{11}$  of 44d, or 1s 8d

- 13 1 lb costs  $\frac{1}{11}$  of 110d or  $1\frac{1}{2}$ d  
 1 Kg, or  $2\frac{1}{2}$  lbs costs  $1\frac{1}{2}$ d  $\times 2\frac{1}{2}$ , or  $2\frac{3}{4}$ d
- 14 1 lit =  $\frac{1}{4}$  pts =  $\frac{1}{4}$  of  $\frac{1}{2}$  gal, and costs  $\frac{1}{4}$  of  $\frac{1}{2}$  of 40d, or  $8\frac{1}{2}$ d
- 19 We have to find which is the greater, 96 Km per hr,  
 or 60 mi in hr  
 Since 60 mi =  $60 \times \frac{5}{8}$  Km = 96 Km, they are approx<sup>y</sup> equal
- 20 A speed of 119 mi in  $2\frac{1}{2}$  hrs = a speed of  $119 \times \frac{2}{5}$  Km in  $\frac{5}{2}$  hrs  
 = a speed of  $119 \times \frac{2}{5}$  Km  $\times 2$  in 1 hr
- 23 56a buy 36 m, 56a buy  $\frac{36}{39\frac{1}{2}}$  m or  $\frac{32}{35}$  m,  
 hence Rs 30 10a or 190a buy  $\frac{3}{4}\frac{2}{5}$  m  $\times \frac{4}{5}\frac{5}{8}$ , i.e. 8 metres
- 24  $8\frac{1}{2}$ a per 1 m =  $8\frac{1}{2}$ a per  $\frac{39\frac{1}{2}}{36}$  yds =  $\frac{35}{1}$ a per  $\frac{35}{32}$  yds,  
 i.e. 8a per 1 yd gain on 1 yd is  $\frac{1}{2}$ a, and gain  
 on 48 yds is Rs 1 8a
- 26 Corn land =  $(1 - \frac{1}{4})$ , or  $\frac{3}{4}$  of acreage  
 the whole =  $\frac{1}{3}$  =  $\frac{1}{3}$  of  $\frac{4}{3}$  =  $\frac{1}{3}$  of 216 acres = 468 acres
- 27  $(1 - \frac{1}{8})$ , or  $\frac{7}{8}$  of battalion = 718 men,  
 816 men are  $\frac{1}{8}$  of  $\frac{1}{4}$  of battalion, or  $\frac{1}{32}$  of battalion,  
 hence req<sup>d</sup> fraction =  $\frac{1}{32}$
- 28  $(1 - \frac{1}{8} - \frac{1}{4})$ , or  $\frac{1}{4}$  of journey = 8 mi, whole journey = 128 mi
- 29 Master's share =  $(1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8})$ , or  $\frac{1}{8}$  of vessel  
 Whole vessel =  $\frac{60}{8} = \frac{15}{2}$  of  $\frac{1}{6}$  and is worth  $\frac{60}{15}$  of Rs 5858 $\frac{1}{2}$ ,  
 or Rs 27037 8a
- 30  $(1 - \frac{1}{4} - \frac{1}{4})$ , or  $\frac{1}{2}$  of inc. = Rs. 4927 $\frac{1}{2}$ ,  
 total income =  $\frac{60}{2} = \frac{30}{1}$  of  $\frac{4}{3}$  =  $\frac{40}{1}$  of Rs 1927 $\frac{1}{2}$   
 = Rs  $\frac{60 \times 1927\frac{1}{2}}{40 \times 1} = \text{Rs } \frac{24111}{4} = \text{Rs } 6033 \frac{1}{4}$  12a

## EXAMPLES VIII g Page 163

- |    |           |                             |    |           |                          |
|----|-----------|-----------------------------|----|-----------|--------------------------|
| 27 | See Art 5 | $\frac{\text{Km}}{90 \ 00}$ | 29 | See Art 5 | $\frac{\text{yds}}{100}$ |
|    |           | 15 26                       |    |           | 14 2                     |
|    |           | 73 50                       |    |           | 17 4                     |
|    |           | 0 74                        |    |           | 3 5                      |
|    |           | 0 50 = 500 m                |    |           | 64 9                     |
- error  $\approx 0 \frac{1}{2}$  yd = 36 in



- 30 Perimeter  $= (4\ 64 + 5\ 02 + 6\ 7)\text{ in} = 16\ 36\text{ in}$   
 $\text{req}^d \text{ error} = 0\ 06\text{ in}$ , shewing  $16\ 4\text{ in}$  to be a nearer estimate
- 33  $1\ 93 - 2\ 4 = -(2\ 4 - 1\ 93) = -0\ 47$
- 36  $\text{Exp}^a = 5\ 99 - 6 = -(6 - 5\ 99) = -0\ 01$
- 40 Transposing  $v = 16\ 821 - (4\ 04 + 11\ 4 + 0\ 291) = 1\ 09$
- 43 
$$\begin{array}{r} 3\ 92 \\ 14 \\ \hline 0\ 876 \\ 5\ 61 \\ 0\ 0003 \\ 1\ 4037 \\ \hline 25\ 81 \end{array} = 26 \text{ correct to nearest unit}$$

### EXAMPLES VIII k Page 168

- 7 To multiply  $86\ 54$  by  $2\ 37$ , add the results of (i), (ii) and (iii)
- 17  $1\text{ yd} = 2\ 54\text{ cm} \times 36 = 91\ 44\text{ cm} = 0\ 9144\text{ m}$
- 26  $16 \times 2\ 5 = 40$ ,  $1\ 6 \times 0\ 25 = (16 \times 2\ 5) - 100 = 0\ 4$   
 Also  $160 \times 0\ 025 = 16 \times 2\ 5 \times 10 - 100 = 4$
- 31  $2\text{ Km } 375\text{ m} = 2\ 375\text{ Km}$ ,  $1\text{ Km } 800\text{ m} = 1\ 8\text{ Km}$
- $$\begin{array}{r} 2\ 375 \\ 1\ 8 \\ \hline 2\ 375 \\ 1\ 9000 \\ \hline 4\ 275 \end{array} \quad \text{Area} = 4\ 275\text{ sq Km}$$
- 32 Area  $= (4\ 6)^2\text{ sq in} = 21\ 16\text{ sq in}$ ,  
 total pressure  $= (21\ 16 \times 15)\text{ lbs} = 317\ 4\text{ lbs}$
- 35 (i) 
$$\begin{array}{r} 3\ 1416 \\ 28 \\ \hline 62\ 832 \\ 25\ 1328 \\ \hline 87\ 9648 \end{array} \quad \text{(ii) } \begin{array}{r} 3\ 1416 \\ 450 \\ \hline 1256\ 64 \\ 157\ 08 \\ \hline 1413\ 72 \end{array} \quad \begin{array}{l} \text{(iii) } 3\ 1416 \times 15 \times 40 \\ = 3\ 1416 \times 600 \\ = 314\ 16 \times 6 = 1884\ 96 \\ \text{ie } 1885\text{ ft} \end{array}$$
- ie  $87\ 9\text{ in}$       ie  $1414\text{ ft}$
- 37  $\text{Req}^d \text{ wt} = (0\ 28 \times 1000 \times 13\ 5)\text{ gm} = (28 \times 135)\text{ gm} = 3780\text{ gm}$
39.  $5\text{ cu m} = (5 \times 1000 \times 1000)\text{ cu cm}$   
 This weighs  $(5000 \times 1\ 15)\text{ Kg}$ , or  $(50 \times 115)\text{ Kg}$ , or  $5750\text{ Kg}$
- 41 Value  $= 0\ 0875 \times 40 = 3\ 5$

$$\begin{array}{r}
 42 \quad 1 \ 25 \\
 \quad \underline{3 \ 2} \\
 3 \ 75 \\
 \underline{0 \ 250} \\
 4 \ 000
 \end{array}
 \quad 4 \times 2 \ 375 = 9 \ 5$$

$$43 \quad \text{Value} = 0 \ 04 \times 0 \ 125 \times 200 = 8 \times 0 \ 125 = 1$$

$$44 \quad \text{Value} = 1 \ 625 \times 0 \ 16 \times 0 \ 5 = 1 \ 625 \times 0 \ 08 = 0 \ 13$$

### EXAMPLES VIII 1. Page 171

$$20 \quad \text{Each share} = \pounds \frac{4 \ 8}{8} = \pounds 0 \ 6 = 6 \text{ tenths of } \pounds 1 = 12s$$

$$21-49 \quad \text{See Art 167}$$

$$52 \quad \text{Circumference} = \frac{1001}{400} \text{ yds} = \frac{1001 \times 36}{400} \text{ in} = \frac{36036}{400} \text{ in} = 90 \ 09 \text{ in}$$

This = 90 1 in (to nearest tenth) and error = 0 01 in

$$57 \quad \frac{86}{9} = 0 \ 9555 \dots = 0 \ 9556 \text{ correct to 4 decimal places}$$

$$65 \quad 135 \left\{ \begin{array}{l} 5 \overline{) 9 \ 016} \\ 9 \overline{) 1 \ 8092} \\ 3 \overline{) 0 \ 2010} \\ \hline 0 \ 0670 \end{array} \right.$$

giving 0 067

$$66 \quad 385 \left\{ \begin{array}{l} 5 \overline{) 349 \ 968} \\ 11 \overline{) 69 \ 9936} \\ 7 \overline{) 6 \ 3630} \\ \hline 0 \ 9090 \end{array} \right.$$

giving 0 909

### EXAMPLES VIII m Page 174

$$19 \quad \frac{30 \ 26}{89} = \frac{3 \ 026}{8 \ 9} = 0 \ 34 \quad \frac{302 \ 6}{8 \ 9} = \frac{30 \ 26 \times 10}{89 \times 10} = \frac{30 \ 26}{89} \times 100 = 34$$

$$\begin{array}{r}
 0 \ 34 \\
 8 \ 9 \overline{) 3 \ 026} \\
 \hline
 356
 \end{array}$$

$$\text{Also } \frac{3 \ 026}{890} = \frac{30 \ 26 - 10}{89 \times 10} = \frac{30 \ 26}{89} - 100 = 0 \ 0034$$

$$22 \quad (iv) \quad r = 4 \ 8 \times \frac{1}{2} = 4 \ 8 \times \frac{5}{4} = 6$$

$$\begin{array}{r}
 214 \\
 23 \ 565 \overline{) 1210} \\
 \hline
 800 \\
 \hline
 2350 \\
 \hline
 90
 \end{array}$$

remainder = 90c

$$29 \quad \text{Exp}^n = 72 \times \frac{84}{8} = 72 \times \frac{3}{2} = 108$$

$$31 \quad \text{Exp}^n = \frac{0.017}{0.34 \times 0.25} = \frac{0.017 \times 4}{0.34} = \frac{0.068}{0.34} = 0.2$$

$$35 \quad \frac{0.08571}{25.603} = \frac{0.008571}{2.5603} = 0.0033$$

$$\begin{array}{r} 0.00334 \\ 25603 \overline{) 0.0085710} \\ \underline{89010} \\ 12201 \end{array}$$

$$40 \quad (i) \text{ diameter} = \frac{49.6 \text{ in}}{3.1416} = 15.8 \text{ in}$$

$$\begin{array}{r} 15.78 \\ 3.1416 \overline{) 49.6} \\ \underline{18.1840} \\ 2.47600 \\ \underline{27.688} \end{array}$$

$$\begin{aligned} (ii) \text{ diameter} &= \frac{1760 \times 3 \text{ ft}}{500 \times 3.1416} = \frac{3520 \times 36 \text{ in}}{1000 \times 3.1416} \\ &= \frac{3.52 \times 36 \text{ in}}{0.2618} = \frac{10.56}{0.2618} \text{ in} = 40.34 \text{ in} \end{aligned}$$

$$\begin{array}{r} 40.336 \\ 2.618 \overline{) 105.60} \\ \underline{8800} \\ 9460 \\ \underline{16060} \end{array}$$

$$41 \quad \frac{19}{0.42} = \frac{190}{4.2} = 45 \frac{1}{2} = 45 \frac{0.1}{0.42} \quad \text{rem}^r = 0.1 \text{ in}$$

$$\begin{aligned} 42 \quad (iii) \quad \frac{37.8241}{293} &= \frac{0.378241}{2.93} \\ &= 0.1290 \frac{0.000271}{2.93} = 0.1290 \frac{0.0271}{293} \\ \text{rem}^r &= 0.0271 \end{aligned}$$

$$\begin{array}{r} 0.1290 \\ 293 \overline{) 0.378241} \\ \underline{852} \\ 2664 \\ \underline{271} \end{array}$$

$$\begin{aligned} (iv) \quad \frac{37.8241}{90.7} &= \frac{3.78241}{9.07} \\ &= 0.417 \frac{0.00022}{9.07} = 0.417 \frac{0.0022}{90.7} \\ \text{rem}^r &= 0.0022 \end{aligned}$$

$$\begin{array}{r} 0.417 \\ 907 \overline{) 3.78241} \\ \underline{1544} \\ 6371 \\ \underline{22} \end{array}$$

### EXAMPLES VIII n. Page 179

$$30. \quad \begin{array}{r} 0.478 \\ 23 \overline{) 110} \\ \underline{180} \\ 190 \\ \underline{6} \end{array}$$

$$\frac{11}{23} = 0.478 \frac{6}{23} = 0.478$$

$$48 \quad 0.03125 = 0.031\frac{1}{8} = \frac{31}{100} = \frac{31}{800} = \frac{1}{25}$$

$$49 \quad 0.071875 = 0.071\frac{7}{8} = \frac{71\frac{7}{8}}{1000} = \frac{575}{8000} = \frac{23}{320}$$

$$50 \quad (i) \frac{3}{5^2} = \frac{3 \times 2^2}{10} = \frac{12}{100} = 0.12, \quad (iii) \frac{7}{5^3 \times 2} = \frac{7 \times 2^3}{10^3 \times 2} = \frac{14}{1000} = 0.014$$

### EXAMPLES IX a Page 181

$$23 \quad \begin{array}{r} 12 \overline{) 9 \text{ p}} \\ 16 \overline{) 875 \text{ a}} \\ \hline 0.546875 \text{ Re} \end{array}$$

$$24 \quad \begin{array}{r} 12 \overline{) 9 \text{ p}} \\ 16 \overline{) 775 \text{ a}} \\ \hline 8.484375 \text{ Rs} \end{array}$$

$$25 \quad \begin{array}{r} 12 \overline{) 15 \text{ p}} \\ 16 \overline{) 8125 \text{ a}} \\ \hline 5.5078125 \text{ Rs} \end{array}$$

$$26 \quad \begin{array}{r} 12 \overline{) 105d} \\ 20 \overline{) 13875\text{¢}} \\ \hline 11.694\text{£} \end{array}$$

$$27 \quad \begin{array}{r} 12 \overline{) 75d} \\ 20 \overline{) 16625\text{¢}} \\ \hline 0.831\text{£} \end{array}$$

$$28 \quad \begin{array}{r} 12 \overline{) 4d} \\ 20 \overline{) 153333\text{¢}} \\ \hline 8.767\text{£} \end{array}$$

$$29 \quad \begin{array}{r} 12 \overline{) 825d} \\ 20 \overline{) 106875\text{¢}} \\ \hline 2.5344\text{£} \end{array}$$

$$30 \quad \begin{array}{r} 12 \overline{) 55d} \\ 20 \overline{) 745833\text{¢}} \\ \hline 5.3729\text{£} \end{array}$$

$$31 \quad \begin{array}{r} 12 \overline{) 425d} \\ 20 \overline{) 335416\text{¢}} \\ \hline 0.1677\text{£} \end{array}$$

$$32 \quad \begin{array}{r} 16 \overline{) 12 \text{ chks}} \\ 40 \overline{) 1375 \text{ sps}} \\ \hline 5.34375 \text{ mds} \end{array}$$

$$33 \quad \begin{array}{r} 16 \overline{) 14 \text{ chks}} \\ 20 \overline{) 5875 \text{ cot}} \\ \hline 4.20375 \text{ big} \end{array}$$

$$34 \quad \begin{array}{r} 3 \overline{) 15 \text{ ft}} \\ 11 \overline{) 55 \text{ yds}} \\ 10 \overline{) 05} \\ 16 \overline{) 005} \\ \hline 0.003125 \text{ mn} \end{array}$$

$$35 \quad \begin{array}{r} 4 \overline{) 025 \text{ q1}} \\ 20 \overline{) 30625 \text{ cwt}} \\ 5 \overline{) 2153125 \text{ tons}} \\ \hline 0.430625 \text{ of 5 tons} \end{array}$$

$$36 \quad \begin{array}{r} 12 \overline{) 9 \text{ p}} \\ 16 \overline{) 975 \text{ a}} \\ 5 \overline{) 19609375 \text{ Rs}} \\ 5 \overline{) 3921875} \\ \hline 0.784375 \text{ of Rs 25} \end{array}$$

## EXAMPLES IX b Page 183

- |    |  |     |  |    |  |
|----|--|-----|--|----|--|
| 13 | Rs 3 <u>21875</u><br>35 <u>r</u><br>60 p                   | 14  | Rs 5 <u>1875</u><br>30 a   | 15 | Rs 9 <u>875</u><br>140 a   |
| 16 | Rs 2 <u>078125</u><br>125 <u>r</u><br>30 p                 | 17. | Rs 11 <u>5625</u><br>90 a  | 18 | Rs 0 <u>40625</u><br>65 a<br>60 p                                      |
| 19 | £4 <u>63</u><br>12 <u>6s</u><br>7 <u>2d</u>                | 20  | £5 <u>72</u><br>14 <u>4s</u><br>4 <u>8d</u>                                    | 21 | £14 <u>823</u><br>16 <u>46c</u><br>5 <u>52d</u>                        |
| 22 | £9 <u>89</u><br>17 <u>8s</u><br>9 <u>6d</u><br>2 <u>4f</u> | 23  | £0 <u>634</u><br>12 <u>68s</u><br>8 <u>16d</u><br>0 <u>64f</u>                 | 24 | £11 <u>047</u><br>0 <u>91s</u><br>11 <u>28d</u><br>1 <u>12f</u>        |
| 25 | Re 0 <u>28125</u><br>2<br>Re 0 <u>3625</u><br>90 r         | 26  | Rs 1 <u>578125</u><br>4<br>Rs 6 <u>3125</u><br>50 a                            | 27 | Rs 3 <u>11875</u><br>5<br>Rs 15 <u>59375</u><br>95 a.<br>60 p          |
| 28 | Re 0 <u>53125</u><br>10<br>Rs 5 <u>3125</u><br>50 a        | 29  | Rs 7 <u>304</u><br>8<br>Rs 58 <u>432</u><br>6912 a<br>10944 p<br>Rs 58 6a 11 p | 30 | Re 0 <u>78</u><br>9<br>Rs 7 <u>02</u><br>032 a<br>384 p<br>Rs 7 0a 3 p |
| 31 | 12   45d<br>20   15375c<br>4   379875£<br>0 94218 of £4    | 32  | 12   4d<br>20   933333c<br>8   746666£<br>0 93333 of £8                        | 33 | 12   8d<br>20   1766666s<br>40   1588333£<br>0 39708 of £40            |

$$\begin{array}{r}
 34 \quad 12 \overline{) 9 \, 75d} \\
 \underline{20 \overline{) 7 \, 8125s}} \\
 11 \, 390625\text{£} \\
 \underline{\phantom{11} 0 \, 008} \\
 0 \, 091125 \text{ of } \text{£}125
 \end{array}$$

$$\begin{array}{r}
 35 \quad 1 \, 8875 \text{ mds} \\
 35 \, 5 \text{ srs} \\
 8 \, 0 \text{ chks}
 \end{array}$$

$$\begin{array}{r}
 36 \quad 6 \, 475 \text{ big} \\
 9 \, 5 \text{ cot} \\
 8 \, 0 \text{ chk}
 \end{array}$$

$$\begin{array}{r}
 37 \quad 0 \, 895 \text{ day} \\
 21 \, 48 \text{ hours} \\
 28 \, 8 \text{ min} \\
 48 \text{ secs}
 \end{array}$$

$$\begin{array}{r}
 38 \quad 2 \, 03125 \text{ m1} \\
 = 2 \, 03\frac{1}{8} \text{ m1} \\
 = 2\frac{1}{3} \text{ m1}
 \end{array}$$

$$\begin{array}{r}
 39 \quad 0 \, 0425 \text{ md.} \\
 \underline{\phantom{0} 10} \\
 0 \, 425 \text{ md} \\
 17 \, 0 \text{ srs}
 \end{array}$$

$$\begin{array}{r}
 40 \quad 0 \, 9375 \\
 \underline{\phantom{0} 7} \\
 6 \, 5625 \text{ tons} \\
 11 \, 25 \text{ cwt} \\
 1 \, 0 \text{ qr}
 \end{array}$$

$$\begin{array}{r}
 41 \quad 9 \, 9375 \text{ of } \text{£}3 \, 3s \\
 = 9\frac{1}{8} \text{ of } \text{£}3 \, 3s \\
 = (10 - \frac{1}{8}) \text{ of } \text{£}3 \, 3s
 \end{array}$$

$$\begin{array}{r}
 42 \quad 16 \overline{) 12 \, 75a} \\
 \text{Re } 0 \, 79687 \\
 = \text{Re } 0 \, 7969
 \end{array}$$

$$\begin{array}{r}
 43 \quad 4 \overline{) 3 \, 25 \text{ r}} \\
 \underline{\phantom{4} 0 \, 8125 \text{ ac}} \\
 \underline{\phantom{4} 0 \, 4} \\
 0 \, 325
 \end{array}$$

$$\begin{array}{r}
 44 \quad 7 \overline{) 5 \, 25 \text{ d.}} \\
 52 \overline{) 9 \, 75 \text{ wks}} \\
 5 \overline{) 0 \, 1875 \text{ yrs}} \\
 0 \, 0375 \text{ of } 5 \text{ yrs}
 \end{array}$$

$$\begin{array}{r}
 45 \quad \text{£}4 \, 325 \\
 6 \, 5s
 \end{array}$$

$$\begin{array}{r}
 46 \quad \text{£}3 \, 072 \\
 1 \, 44s \\
 5 \, 28d \\
 1 \, 12f
 \end{array}$$

$$\begin{array}{r}
 47 \quad \text{£}0 \, 71 \\
 14 \, 2s \\
 2 \, 4d \\
 1 \, 6f
 \end{array}$$

$$\begin{array}{r}
 48 \quad \text{£}0 \, 099872 \\
 = \text{£}0 \, 1 \\
 = 2s
 \end{array}$$

$$\begin{array}{r}
 49 \quad \text{£}7 \, 778 \\
 15 \, 56s \\
 6 \, 72d \\
 2 \, 88f
 \end{array}$$

$$\begin{array}{r}
 50 \quad \text{£}0 \, 754 \\
 15 \, 08s \\
 096 \, d \\
 3 \, 84f
 \end{array}$$

51-54 See Arts 178, 179

For 57-60, see Art 179

Thus in Ex 57 we require  $0 \, 017 \times 640$ , or  $0 \, 17 \times 64$ , and in Ex 58 we require  $0 \, 2190 \times 1760$ , or  $2 \, 19 \times 176$

For Ex 59, see Ex 54, and for Ex 60, see Ex 53

## EXAMPLES IX d Page 188

$$25 \quad 24f = 25 \text{ mils}, \quad 1f = \frac{25}{24} \text{ mils} = (1 + \frac{1}{24}) \text{ mils} = £0.001 \times (1 + \frac{1}{24})$$

$$26 \quad \text{By Ex 25, } 1f = £0.001 \times \frac{25}{24} = \frac{£0.025}{24} = \frac{£0.00625}{6} = £0.001042$$

$$1 \text{ mil} = £0.001 \quad \text{req'd diff} = £0.000042$$

$$37 \quad 12 \text{ half-cents} = \text{Re } 0.06$$

$$\underline{0.96 \text{ a}}$$

$$11.52 \text{ p}$$

$$\text{req'd diff} = 12 - 11.52 = 0.48 = \frac{48}{100} = \frac{12}{25}$$

$$38 \quad \text{Rs } 2.624$$

$$\underline{5}$$

$$\text{Rs } 13.12$$

$$\text{Rs } 13.12$$

$$\underline{1.92 \text{ a}}$$

$$11.04 \text{ p}$$

## EXAMPLES X a Page 193

$$38 \quad (1) \text{ Absolute error} = £(37.5 - 37.482) = £0.018$$

$$\text{Relative error} = \frac{0.018}{37.482} = \frac{3}{6247} = 0.00048$$

Percentage error If  $x$  denote the req'd rate per cent

$$\text{then } \frac{x}{100} = 0.00048 \text{ and } x = 0.048$$

$$38 \text{ (ii)-44} \quad \text{As No 38 (i)}$$

## EXAMPLES X b Page 195

$$1 \quad \begin{array}{r|l} 41.04 & 62 \\ 0.00 & 17 \\ \hline 21.89 & 2 \\ \hline 62.93 & 99 \end{array}$$

$$2 \quad \begin{array}{r|l} 1.35 & 64 \\ 0.04 & 02 \\ \hline 4.50 & 19 \\ \hline 5.89 & 85 \end{array}$$

$$3 \quad \begin{array}{r|l} 5.08 & 19 \\ 1.66 & 66 \\ \hline 0.03 & 33 \\ \hline 6.78 & 18 \end{array}$$

$$4 \quad \begin{array}{r|l} 0.000 & 71 \\ 0.008 & 5 \\ \hline 0.000 & 08 \\ \hline 0.009 & 29 \end{array}$$

$$5 \quad \begin{array}{r|l} 0.285 & 71 \\ 0.833 & 33 \\ \hline 3.256 & 14 \\ \hline 4.375 & 18 \end{array}$$

$$6 \quad \begin{array}{r|l} 0.3 & \\ 0.016 & \\ \hline 3.682 & 82 \\ \hline 0.000 & 99 \\ \hline 3.999 & 81 \end{array}$$

$$7 \quad \begin{array}{r|l} 26 & 09 \\ 306 & 9 \\ \hline 0 & 00 \\ \hline 332 & 99 \end{array}$$

$$8 \quad \begin{array}{r|l} 41.6 & 82 \\ 0.1 & 00 \\ \hline 41.5 & 82 \end{array}$$

$$\begin{array}{r|l} 9 & 37 \quad 26 \\ & 982 \quad 8 \\ \hline & 1020 \quad 06 \\ & 19 \quad 9 \\ \hline & 1000 \quad 16 \end{array}$$

$$\begin{array}{r|l} 10 & 0 \ 210 \quad 52 \\ & 0 \ 352 \quad 94 \\ & 0 \ 384 \quad 61 \\ \hline & 0 \ 948 \quad 07 \end{array}$$

11 Expressing each item in millions, we have

$$\begin{array}{r} 4 \ 683 \\ 0 \ 807 \\ 17 \ 493 \\ 0 \ 094 \\ \hline 23 \ 077 \end{array}$$

12 Expressing each item in millions, we have

$$\begin{array}{r} 0 \ 971 \\ 8 \ 792 \\ 0 \ 093 \\ 17 \ 828 \\ \hline 27 \ 684 \end{array}$$

giving as answer 27,700,000

14 (i)  $x + x^2 + x^3$   
 $= 0 \ 08 + 0 \ 0064 + 0 \ 00051$

(ii)  $x^4 = 0 \ 00004$

15  $\begin{array}{r|l} 0 \ 2 & \\ 0 \ 04 & \end{array}$  This result differs  
 $\begin{array}{r|l} 0 \ 00 & 8 \text{ from } \frac{1}{4} \text{ or } 0 \ 25 \text{ by} \\ 0 \ 00 & 16 \ 0 \ 00008, \text{ which proves} \\ 0 \ 00 & 032 \text{ the second part of the} \\ 0 \ 24 & 992 \text{ question} \end{array}$

13 Expressing each item in millions, we have

$$\begin{array}{r} £ \\ 65 \ 409 \\ 30 \ 814 \\ 15 \ 659 \\ 28 \ 458 \\ 8 \ 474 \\ 0 \ 460 \\ 0 \ 936 \\ 0 \ 675 \\ \hline 150 \ 885 \end{array}$$

giving as answer £150,900,000

16 Rs  $\begin{array}{r} 350 \ 87 \\ 23 \ 68 \\ 8 \ 33 \\ \hline 663 \ 94 \\ 1046 \ 82 \end{array}$   
 Rs 1047

17  $\begin{array}{r|l} 361 \ 4 & \\ 28 \ 7 & 95 \\ 0 \ 6 & 3 \\ 1 \ 7 & 64 \\ \hline 392 \ 5 & 89 \end{array}$

18 £  $\begin{array}{r} 3 \ 7292 \\ 0 \ 7917 \\ 12 \ 7273 \\ \hline 17 \ 2482 \end{array}$

19 Rs  $\begin{array}{r|l} 2 \ 43 & \\ 7 \ 149 & 2 \\ 0 \ 421 & 1 \\ \hline 10 \ 000 & 3 \end{array}$   
 Rs 10

20 Rs  $\begin{array}{r|l} 5 \ 486 & 2 \\ 3 \ 625 & \\ 888 & 8 \\ \hline 10 \ 000 & 0 \end{array}$   
 Rs 10



## EXAMPLES X c Page 198

$$\begin{array}{r|l} 1 & 562 \\ \hline & 327 \\ 1586 & \\ 112 & 4 \\ 39 & 3 \\ \hline 1837 & 7 \end{array}$$

$$\begin{array}{r|l} 2 & 070 \\ \hline & 28 \\ & 473 \\ 281 & 1 \\ 49 & 1 \\ 02 & 1 \\ \hline 332 & 3 \end{array}$$

$$\begin{array}{r|l} 3 & 943 \\ \hline & 57 \\ & 525 \\ 4717 & 9 \\ 188 & 7 \\ 47 & 2 \\ \hline 4953 & 8 \end{array}$$

$$\begin{array}{r|l} 4 & 003 \\ \hline & 175 \\ & 4116 \\ 012 & 7 \\ 00 & 3 \\ 00 & 0 \\ 00 & 0 \\ \hline 013 & 0 \end{array}$$

$$\begin{array}{r|l} 5. & 502 \\ \hline & 81 \\ & 317 \\ 1508 & 4 \\ 50 & 3 \\ 35 & 1 \\ \hline 1593 & 8 \end{array}$$

$$\begin{array}{r|l} 6 & 70284 \\ \hline & 0 \\ & 0541 \\ 3514 & 2 \\ 281 & 1 \\ 7 & 0 \\ \hline 3802 & 3 \end{array}$$

$$\begin{array}{r|l} 7 & 078 \\ \hline & 95 \\ & 909 \\ 710 & 6 \\ 07 & 1 \\ \hline 717 & 7 \end{array}$$

$$\begin{array}{r|l} 8 & 357 \\ \hline & 82 \\ & 2376 \\ 715 & 6 \\ 107 & 3 \\ 25 & 0 \\ 02 & 1 \\ \hline 850 & 0 \end{array}$$

$$\begin{array}{r|l} 9 & 028 \\ \hline & 947 \\ & 8643 \\ 231 & 5 \\ 17 & 3 \\ 01 & 1 \\ 00 & 1 \\ \hline 250 & 0 \end{array}$$

$$\begin{array}{r|l} 10 & 46 \\ \hline & 20 \\ & 8136 \\ 369 & 6 \\ 4 & 6 \\ 1 & 4 \\ & 2 \\ \hline 375 & 8 \end{array}$$

$$\begin{array}{r|l} 11. & 7 \\ \hline & 0928 \\ & 428 \\ 28 & 4 \\ 1 & 4 \\ & 6 \\ \hline 30 & 4 \end{array}$$

$$\begin{array}{r|l} 12 & 0 \\ \hline & 8631 \\ & 9602 \\ 7 & 7 \\ & 4 \\ \hline 8 & 1 \end{array}$$

$$\begin{array}{r|l} 13 & 40 \\ \hline & 0404 \\ & 606 \\ 240 & 2 \\ 2 & 4 \\ \hline 242 & 6 \end{array}$$

$$\begin{array}{r|l} 14 & 0561 \\ \hline & 023 \\ & 597001 \\ 2805 & 1 \\ 504 & 9 \\ 039 & 3 \\ \hline 3349 & 3 \end{array}$$

$$\begin{array}{r|l} 15 & 1414 \\ \hline & 21 \\ & 141421 \\ 1414 & 2 \\ 565 & 7 \\ 014 & 1 \\ 005 & 6 \\ & 3 \\ \hline 1999 & 9 \end{array}$$

$$\begin{array}{r}
 16 \quad 2400 \overline{) 89} \\
 \underline{4237} \\
 9603 \overline{) 6} \\
 \underline{480} \quad 2 \\
 \underline{072} \quad 0 \\
 \underline{016} \quad 8 \\
 10172 \overline{) 6}
 \end{array}$$

$$\begin{array}{r}
 19. \quad 00204 \overline{) 76} \\
 \underline{2406} \\
 0409 \overline{) 5} \\
 \underline{0081} \quad 9 \\
 \underline{0001} \quad 2 \\
 00492 \overline{) 6}
 \end{array}$$

$$\begin{array}{r}
 22 \quad 0786 \overline{) 66} \\
 \underline{421} \\
 3146 \overline{) 6} \\
 \underline{157} \quad 3 \\
 \underline{79} \\
 3311 \overline{) 8}
 \end{array}$$

$$\begin{array}{r}
 25 \quad 0563 \overline{) 26} \\
 \underline{3277} \\
 1689 \overline{) 8} \\
 \underline{112} \quad 6 \\
 \underline{39} \quad 4 \\
 \underline{39} \quad 4 \\
 \underline{4} \\
 1846 \overline{) 1}
 \end{array}$$

$$\begin{array}{r}
 28 \quad 79 \overline{) 364} \\
 \underline{6024} \\
 476 \overline{) 2} \\
 \underline{15} \quad 3 \\
 478 \overline{) 0 \text{ millions}}
 \end{array}$$

giving answer as 478,000,000.

$$\begin{array}{r}
 17 \quad 1732 \overline{) 05} \\
 \underline{173205} \\
 1732 \overline{) 1} \\
 \underline{1212} \quad 4 \\
 \underline{052} \quad 0 \\
 \underline{003} \quad 5 \\
 \underline{000} \quad 1 \\
 3000 \overline{) 1}
 \end{array}$$

$$\begin{array}{r}
 20 \quad 41 \overline{) 0208} \\
 \underline{7305} \\
 287 \overline{) 1} \\
 \underline{12} \quad 3 \\
 \underline{2} \\
 299 \overline{) 6}
 \end{array}$$

$$\begin{array}{r}
 23 \quad 0573 \overline{) 73} \\
 \underline{3052} \\
 1721 \overline{) 2} \\
 \underline{28} \quad 7 \\
 \underline{11} \\
 1751 \overline{) 0}
 \end{array}$$

$$\begin{array}{r}
 26 \quad 0760 \overline{) 35} \\
 \underline{580079} \\
 3801 \overline{) 8} \\
 \underline{608} \quad 2 \\
 \underline{5} \\
 4410 \overline{) 5}
 \end{array}$$

$$\begin{array}{r}
 18 \quad 80 \overline{) 6251} \\
 \underline{6317} \\
 483 \overline{) 7} \\
 \underline{24} \quad 2 \\
 \underline{08} \\
 \underline{06} \\
 509 \overline{) 3}
 \end{array}$$

$$\begin{array}{r}
 21 \quad 59 \overline{) 6159} \\
 \underline{30807} \\
 178 \overline{) 8} \\
 \underline{47} \\
 183 \overline{) 5}
 \end{array}$$

$$\begin{array}{r}
 24 \quad 0526 \overline{) 333} \\
 \underline{82828} \\
 4210 \overline{) 66} \\
 \underline{105} \quad 27 \\
 \underline{42} \quad 10 \\
 \underline{105} \quad 42 \\
 \underline{1} \\
 4359 \overline{) 51}
 \end{array}$$

$$\begin{array}{r}
 27 \quad 0314 \overline{) 16} \\
 \underline{31831} \\
 942 \overline{) 5} \\
 \underline{31} \quad 4 \\
 \underline{25} \quad 1 \\
 \underline{9} \\
 0999 \overline{) 9}
 \end{array}$$

29 As in Ex 28 we require  $4085 \times 746$  to the nearest unit

$$\begin{array}{r}
 746 \overline{) 0} \\
 \underline{4085} \\
 2984 \overline{) 7} \\
 \underline{59} \quad 7 \\
 \underline{37} \\
 3047 \overline{) 4 \text{ thousands}}
 \end{array}$$

giving answer as 3,047,000

- 30 As in Ex 28 we require  
 $45\ 28 \times 6\ 402$  to the nearest  
 unit

$$\begin{array}{r|l} 45 & 28 \\ \hline & 6\ 402 \\ 271 & 7 \\ 18 & 1 \\ & 1 \\ \hline 289 & \end{array}$$

9 hundred thousands  
 giving answer as 29,000,000

- 31 As in Ex 28 we require  
 $80\ 460 \times 5\ 073$  to the nearest  
 unit

$$\begin{array}{r|l} 80 & 460 \\ \hline & 5\ 073 \\ 402 & 3 \\ 5 & 6 \\ & 2 \\ \hline 408 & \end{array}$$

1 millions  
 giving answer as 408,000,000

- 32 As in Ex 28 we require  
 $5\ 807023 \times 43\ 07$  to the nearest  
 unit

$$\begin{array}{r|l} 5 & 807023 \\ 4 & 3\ 07 \\ \hline 232 & 3 \\ 17 & 4 \\ & 4 \\ \hline 250 & \end{array}$$

1 millions  
 giving answer as 250,000,000

33

$$\begin{array}{r|l} 39 & 3701 \\ \hline & 7\ 52 \\ 275 & 6 \\ 19 & 7 \\ & 8 \\ \hline 296 & 1 \end{array}$$

34

$$\begin{array}{r|l} 2 & 34 \\ \hline & 36 \\ 91 & 44 \\ 1 & 4\ 75 \\ \hline 914 & 4 \\ 365 & 8 \\ 64 & 0 \\ 4 & 6 \\ \hline 1348 & \end{array}$$

8 cm

35

$$\begin{array}{r|l} 45 & 25 \\ \hline & 5 \\ 226 & 25 \\ & 1\ 6093 \\ \hline 226 & 3 \\ 135 & 7 \\ 2 & 0 \\ & 1 \\ \hline 364 & \end{array}$$

1 Km

36

$$\begin{array}{r|l} 2\ 22 & 6 \\ \hline & 7\ 031 \\ 15\ 58 & 2 \\ & 6\ 7 \\ & 2 \\ \hline 15\ 65 & \end{array}$$

1 Kg

37

$$\begin{array}{r|l} 224 & 35 \\ \hline & 4\ 595 \\ 897 & 4 \\ 112 & 2 \\ 20 & 2 \\ 1 & 1 \\ \hline 1030 & \end{array}$$

9 hundred thousands

giving answer as £103,100,000

38

$$\begin{array}{r|l} 29 & 72897 \\ \hline & 4\ 247 \\ 118 & 9 \\ 5 & 9 \\ 1 & 2 \\ & 1 \\ \hline 126 & \end{array}$$

1 hundred-thousands

giving answer as £12,600,000

## EXAMPLES X d Page 202

$$\begin{array}{r} 818 \\ 1. \ 1057 \overline{)86439} \\ \underline{187} \\ 81 \\ \underline{1} \end{array}$$

$$\begin{array}{r} 1766 \\ 2 \ 4301 \overline{)7596} \\ \underline{3295} \\ 284 \\ \underline{26} \end{array}$$

$$\begin{array}{r} 03311 \\ 3 \ 8212 \overline{)27184} \\ \underline{2548} \\ 85 \end{array}$$

$$\begin{array}{r} 002988 \\ 4 \ 2184 \overline{)006527} \\ \underline{2159} \\ 193 \\ \underline{19} \end{array}$$

$$\begin{array}{r} 9617 \\ 5 \ 29332 \overline{)28210} \\ \underline{1811} \\ 51 \\ \underline{22} \\ 2 \end{array}$$

$$\begin{array}{r} 536823 \\ 6 \ 29364 \overline{)157634} \\ \underline{10814} \\ 2005 \\ \underline{243} \\ 9 \end{array}$$

$$\begin{array}{r} 047 \\ 7 \ 241 \overline{)1132} \\ \underline{168} \end{array}$$

$$\begin{array}{r} 8256 \\ 8 \ 8425 \overline{)69557} \\ \underline{2157} \\ 472 \\ \underline{51} \end{array}$$

$$\begin{array}{r} 3406 \\ 9 \ 43507 \overline{)148188} \\ \underline{17667} \\ 264 \\ \underline{3} \end{array}$$

$$\begin{array}{r} 19802 \\ 10 \ 21979 \overline{)43524} \\ \underline{215450} \\ 17639 \\ \underline{53} \end{array}$$

$$\begin{array}{r} 0024 \\ 11. \ 587 \overline{)01410} \\ \underline{236} \\ 2 \end{array}$$

$$\begin{array}{r} 0051 \\ 12 \ 804 \overline{)04162} \\ \underline{142} \\ 62 \end{array}$$

$$\begin{array}{r} 1290 \\ 13 \ 13408 \overline{)17300} \\ \underline{3892} \\ 1210 \\ \underline{4} \end{array}$$

$$\begin{array}{r} 3316 \\ 14 \ 33166 \overline{)110000} \\ \underline{10502} \\ 553 \\ \underline{221} \\ 22 \end{array}$$

$$\begin{array}{r} 0318 \\ 15 \ 3142 \overline{)1} \\ \underline{574} \\ 260 \\ \underline{9} \end{array}$$

$$\begin{array}{r} 1732 \\ 16 \ 17321 \overline{)3} \\ \underline{12679} \\ 554 \\ \underline{34} \end{array}$$

$$\begin{array}{r} 2601 \\ 17 \ 31762 \overline{)82626} \\ \underline{19102} \\ 45 \\ \underline{14} \end{array}$$

$$\begin{array}{r} 0625 \\ 18 \ 3081 \overline{)19278} \\ \underline{792} \\ 176 \\ \underline{22} \end{array}$$

$$\begin{array}{r}
 0\ 0981 \\
 19\ 9\ 135 \overline{)0\ 89700} \\
 \underline{7485} \\
 177 \\
 \underline{86}
 \end{array}$$

$$\begin{array}{r}
 24\ 9 \\
 20\ 2\ 971 \overline{)73\ 98} \\
 \underline{14\ 56} \\
 2\ 68 \\
 \underline{1}
 \end{array}$$

$$\begin{array}{r}
 0\ 0142 \\
 21\ 3\ 080 \overline{)0\ 04381} \\
 \underline{1301} \\
 69
 \end{array}$$

$$\begin{array}{r}
 901 \\
 22.\ 2\ 03 \overline{)1830\ 2} \\
 \underline{3\ 2} \\
 11
 \end{array}$$

$$\begin{array}{r}
 1132 \\
 23\ 7\ 309 \overline{)8276} \\
 \underline{967} \\
 236 \\
 \underline{17} \\
 2
 \end{array}$$

$$\begin{array}{r}
 1121 \\
 24\ 3\ 1897 \overline{)3578\ 3} \\
 \underline{388\ 6} \\
 69\ 6 \\
 \underline{5\ 9} \\
 27
 \end{array}$$

$$\begin{array}{r}
 443 \\
 25\ 7816 \overline{)346423\ 5} \\
 \underline{3378} \\
 252
 \end{array}$$

$$\begin{array}{r}
 418 \\
 26\ 2164 \overline{)90598\ 2} \\
 \underline{403} \\
 187 \\
 \underline{15}
 \end{array}$$

$$\begin{array}{r}
 8\ 120 \\
 27.\ 8\ 0637 \overline{)65\ 4812} \\
 \underline{9716} \\
 1652 \\
 \underline{39}
 \end{array}$$

$$\begin{array}{r}
 92 \\
 28\ 1\ 14 \overline{)105\ 6} \\
 \underline{2\ 8} \\
 5
 \end{array}$$

$$\begin{array}{r}
 6082\ 6 \\
 29\ 2\ 16 \overline{)13138\ 54} \\
 \underline{178\ 5} \\
 57\ 4 \\
 \underline{14\ 2} \\
 13
 \end{array}$$

$$\begin{array}{r}
 4\ 506 \\
 30\ 1\ 2481 \overline{)5\ 6248} \\
 \underline{63240} \\
 835 \\
 \underline{86}
 \end{array}$$

$$\begin{array}{r}
 20 \overline{)15\ 5s} \\
 12\ 775\text{£}
 \end{array}$$

192 Rs 15 cents = Rs 192 15

$$\begin{array}{r}
 0\ 06648 \\
 1\ 9215 \overline{)12775} \\
 \underline{1246} \\
 93 \\
 \underline{16} \\
 1
 \end{array}$$

£0.06648 = 15 96d to the nearest hundredth of 1d

$$\begin{array}{r}
 561 \\
 32\ (i)\ 5\ 791 \overline{)3252\ 7} \\
 \underline{357\ 2} \\
 97 \\
 \underline{39}
 \end{array}$$

$$\begin{array}{r}
 150 \\
 32\ (ii)\ 2\ 980 \overline{)447\ 2} \\
 \underline{149\ 2} \\
 2
 \end{array}$$

$$\begin{array}{r}
 140 \\
 32\ (iii)\ 3\ 176 \overline{)445\ 8} \\
 \underline{128\ 2} \\
 12
 \end{array}$$

$$\begin{array}{r}
 1152 \\
 33 \text{ (i) } 528 \overline{) 603266} \\
 \underline{802} \\
 274 \\
 \underline{10}
 \end{array}
 \quad
 \begin{array}{r}
 0868 \\
 \text{(ii) } 6083 \overline{) 52500} \\
 \underline{4136} \\
 486
 \end{array}$$

$$\begin{array}{r}
 0.025400 \\
 34 \text{ (i) } 3.93701 \overline{) 0.1000000} \\
 \underline{212598} \\
 15747
 \end{array}
 \quad
 \begin{array}{r}
 2011 \\
 \text{(ii) } 3.9370 \overline{) 79200} \\
 \underline{460} \\
 62 \\
 \underline{23}
 \end{array}
 \quad
 \begin{array}{r}
 1609 \\
 \text{(iii) } 3.9370 \overline{) 63360} \\
 \underline{23990} \\
 368 \\
 \underline{14}
 \end{array}$$

$$\begin{array}{r}
 14142 \\
 35 \text{ } 141421 \overline{) 200000} \\
 \underline{58579} \\
 2011 \\
 \underline{597} \\
 31
 \end{array}
 \quad
 \begin{array}{r}
 69165 \\
 36 \text{ } 35128 \overline{) 242962} \\
 \underline{32194} \\
 579 \\
 \underline{228} \\
 18
 \end{array}$$

Rs 69 165 = Rs 69 17 to the nearest cent  
 1s 3½d = £0.0656,  
 hence 6.917 × 0.656  
 = 4.5375  
 giving £4 10s 9d

$$\begin{array}{r}
 27727 \\
 37 \text{ } 62321 \overline{) 172800} \\
 \underline{49158} \\
 4533 \\
 \underline{171}
 \end{array}
 \quad
 \begin{array}{r}
 665557 \\
 38 \text{ } 15025 \overline{) 1000000} \\
 \underline{98500} \\
 8350 \\
 \underline{837} \\
 86 \\
 \underline{11}
 \end{array}
 \quad
 \begin{array}{r}
 15385 \\
 39 \text{ } 22435 \overline{) 34517} \\
 \underline{12082} \\
 865 \\
 \underline{192} \\
 13
 \end{array}$$

£665 557, giving £665 11s 2d  
 to the nearest penny

$$\begin{array}{r}
 18084 \\
 40 \text{ (i) } 419708 \overline{) 759135} \\
 \underline{339367} \\
 3553 \\
 \underline{195} \\
 28
 \end{array}
 \quad
 \begin{array}{r}
 30090 \\
 \text{(ii) } 389619 \overline{) 1172368} \\
 \underline{3511} \\
 7
 \end{array}
 \quad
 \begin{array}{r}
 2551 \\
 \text{(iii) } 56367 \overline{) 143799} \\
 \underline{31065} \\
 2861 \\
 \underline{63} \\
 7
 \end{array}$$

## EXAMPLES X e Page 205

[To save space the position of the multiplier is sometimes moved  
The number of decimal figures to be retained in each case must be  
decided as in Art 199, Ex 1 and 2]

$$\begin{array}{r}
 1 \quad 023 \overline{) 94} \\
 \underline{\phantom{0} 231} \\
 47 \phantom{0} 9 \\
 \phantom{0} 7 \phantom{0} 2 \\
 \phantom{0} \phantom{0} 2 \\
 \underline{\phantom{0} 55} \phantom{0} 3 \\
 \phantom{0} 157 \\
 \underline{\phantom{0} 55} \phantom{0} 3 \\
 \phantom{0} 27 \phantom{0} 7 \\
 \phantom{0} 3 \phantom{0} 9 \\
 \underline{\phantom{0} 086} \phantom{0} 9
 \end{array}$$

$$\begin{array}{r}
 2 \quad 1860 \overline{) 2} \\
 \underline{\phantom{0} 01457} \\
 186 \phantom{0} 0 \\
 \phantom{0} 74 \phantom{0} 4 \\
 \phantom{0} 9 \phantom{0} 3 \\
 \phantom{0} 1 \phantom{0} 3 \\
 \underline{\phantom{0} 271} \phantom{0} 0 \\
 \phantom{0} 1356 \\
 \underline{\phantom{0} 271} \phantom{0} 0 \\
 \phantom{0} 81 \phantom{0} 3 \\
 \phantom{0} 13 \phantom{0} 6 \\
 \phantom{0} 1 \phantom{0} 6 \\
 \underline{\phantom{0} 367} \phantom{0} 5
 \end{array}$$

$$\begin{array}{r}
 3 \quad 3673 \overline{) 0} \\
 \underline{\phantom{0} 0073} \\
 257 \phantom{0} 1 \\
 \phantom{0} 11 \phantom{0} 0 \\
 \underline{\phantom{0} 268} \phantom{0} 1 \\
 \phantom{0} 02367 \\
 \underline{\phantom{0} 53} \phantom{0} 6 \\
 \phantom{0} 8 \phantom{0} 0 \\
 \phantom{0} 1 \phantom{0} 6 \\
 \phantom{0} 1 \\
 \underline{\phantom{0} 063} \phantom{0} 3
 \end{array}$$

$$\begin{array}{r}
 4 \quad 097 \overline{) 13} \\
 \underline{\phantom{0} 1872} \\
 97 \phantom{0} 13 \\
 77 \phantom{0} 70 \\
 \phantom{0} 6 \phantom{0} 80 \\
 \phantom{0} 19 \\
 \underline{\phantom{0} 181} \phantom{0} 82 \\
 \phantom{0} 2423 \\
 \underline{\phantom{0} 363} \phantom{0} 64 \\
 \phantom{0} 72 \phantom{0} 73 \\
 \phantom{0} 3 \phantom{0} 64 \\
 \phantom{0} 54 \\
 \underline{\phantom{0} 440} \phantom{0} 55
 \end{array}$$

$$\begin{array}{r}
 5 \quad 2143 \overline{) 4} \\
 \underline{\phantom{0} 3721} \\
 6430 \phantom{0} 2 \\
 1500 \phantom{0} 4 \\
 42 \phantom{0} 9 \\
 \phantom{0} 2 \phantom{0} 1 \\
 \underline{\phantom{0} 8532} \phantom{0} 7975 \phantom{0} 6(934 \\
 \phantom{0} 296 \phantom{0} 8 \\
 \phantom{0} 40 \phantom{0} 8 \\
 \phantom{0} 5 \phantom{0} 7
 \end{array}$$

$$\begin{array}{r}
 6 \quad 4485 \overline{) 2} \\
 \underline{\phantom{0} 1473} \\
 4485 \phantom{0} 2 \\
 1794 \phantom{0} 1 \\
 \phantom{0} 314 \phantom{0} 0 \\
 \phantom{0} 13 \phantom{0} 4 \\
 \underline{\phantom{0} 98153} \phantom{0} 6606 \phantom{0} 7(673 \\
 \phantom{0} 717 \phantom{0} 5 \\
 \phantom{0} 30 \phantom{0} 4 \\
 \phantom{0} 1 \phantom{0} 0
 \end{array}$$

$$\begin{array}{r}
 7 \quad 2436 \overline{) 8} \\
 \underline{\phantom{0} 0394} \\
 731 \phantom{0} 0 \\
 219 \phantom{0} 2 \\
 \phantom{0} 09 \phantom{0} 7 \\
 \underline{\phantom{0} 897} \phantom{0} 959 \phantom{0} 9(107 \\
 \phantom{0} 62 \phantom{0} 9 \\
 \phantom{0} 1
 \end{array}$$

$$\begin{array}{r}
 8 \quad 078 \overline{) 59} \\
 \underline{\phantom{0} 30103} \\
 235 \phantom{0} 8 \\
 \phantom{0} 8 \\
 \underline{\phantom{0} 4628} \phantom{0} 236 \phantom{0} 6(05 \\
 \phantom{0} 5
 \end{array}$$

$$\begin{array}{r}
 9 \quad 0 \overline{) 42} \\
 \underline{\phantom{0} 9} \phantom{0} 80 \\
 37 \phantom{0} 8 \\
 \phantom{0} 3 \phantom{0} 4 \\
 \underline{\phantom{0} 41} \phantom{0} 2 \\
 \phantom{0} 5312 \\
 \underline{\phantom{0} 206} \phantom{0} 0 \\
 \phantom{0} 12 \phantom{0} 4 \\
 \phantom{0} 4 \\
 \phantom{0} 1 \\
 \underline{\phantom{0} 218} \phantom{0} 9
 \end{array}$$

$$\begin{array}{r}
 10 \quad 65 \overline{) 471} \\
 \underline{1 \ 0398} \\
 65 \overline{) 47} \\
 \underline{1 \ 96} \\
 59 \\
 \underline{05} \\
 68 \overline{) 07} \\
 \underline{10 \ 326} \\
 680 \overline{) 70} \\
 \underline{20 \ 42} \\
 1 \ 36 \\
 \underline{41} \\
 702 \overline{) 89}
 \end{array}$$

$$\begin{array}{r}
 11 \quad 3214 \overline{) 00} \\
 \underline{1 \ 01359} \\
 3214 \overline{) 0} \\
 \underline{32} \ 1 \\
 9 \ 6 \\
 \underline{1 \ 6} \\
 3 \\
 9 \ 47 \overline{) 3257} \quad 6 \ (344 \\
 \underline{416 \ 6} \\
 37 \ 8
 \end{array}$$

$$\begin{array}{r}
 12 \quad 238 \overline{) 3} \ 901 \\
 \underline{714} \\
 214 \ 2 \\
 \underline{2} \\
 4 \ 83 \overline{) 928} \ 4 \ (192 \\
 \underline{445 \ 4} \\
 10 \ 7 \\
 \underline{1 \ 0}
 \end{array}$$

$$\begin{array}{r}
 13 \quad 6 \ 081 \overline{) 75} \\
 \underline{5 \ 7002} \\
 30 \ 408 \overline{) 8} \\
 \underline{4 \ 257} \ 2 \\
 1 \ 2 \\
 34 \ 667 \overline{) 2} \ 69 \ 7324 \ (2 \ 011 \\
 \underline{398} \\
 51 \\
 16
 \end{array}$$

$$\begin{array}{r}
 14 \quad 348 \overline{) 19} \\
 \underline{502} \overline{) 6612} \ (1317 \\
 \underline{1592} \\
 860 \\
 \underline{358} \\
 7
 \end{array}$$

$$\begin{array}{r}
 15 \quad 1 \ 2327 \overline{) 2 \ 5721} \ (2 \ 086 \\
 \underline{1067} \\
 81 \\
 9
 \end{array}$$

16 *NB*—In order to obtain a result correct to 4 decimal figures (i.e., 5 significant figures) we must decimalize £2 1s 1½d in full. This gives £2 05625

$$\begin{array}{r}
 4 \ 8632 \\
 2 \ 05625 \overline{) 10 \ 00000} \\
 \underline{1 \ 77500} \\
 13000 \\
 \underline{663} \\
 46
 \end{array}$$

$$\begin{array}{r}
 17 \quad 48 \overline{) 632} \\
 \underline{400} \ 4 \\
 194528 \\
 \underline{194} \ 5 \\
 25 \ 21 \overline{) 19472 \ 2} \ 5 \ (772 \ 40 \\
 \underline{1825 \ 2} \\
 605 \\
 101
 \end{array}$$

$$\begin{array}{r}
 18. \ 2 \ 2046 \overline{) 10 \ 0000} \ (4 \ 536 \\
 \underline{1 \ 1816} \\
 793 \\
 132
 \end{array}$$



$$\begin{array}{r|l}
 19 & 1\ 414\ 21 \\
 & 1\ 414\ 21 \\
 \hline
 & 1\ 414\ 21 \\
 & 565\ 68 \\
 & 14\ 14 \\
 & 5\ 66 \\
 & 14 \\
 & 1 \\
 \hline
 & 1\ 999\ 84 \\
 & 1\ 414\ 21 \\
 \hline
 & 1\ 999\ 84 \\
 & 799\ 94 \\
 & 20\ 00 \\
 & 8\ 00 \\
 & 40 \\
 & 2 \\
 \hline
 & 2\ 828\ 20
 \end{array}$$

$$\begin{array}{r|l}
 20 & 1\ 260\ 01 \\
 & 1\ 260\ 01 \\
 \hline
 & 1\ 260\ 01 \\
 & 252\ 00 \\
 & 75\ 60 \\
 & 1 \\
 \hline
 & 1\ 587\ 62 \\
 & 1\ 260\ 01 \\
 \hline
 & 1\ 587\ 62 \\
 & 317\ 52 \\
 & 95\ 26 \\
 & 2 \\
 \hline
 & 2\ 000\ 42
 \end{array}$$

$$\begin{array}{r|l}
 21 & 0\ 00731 \\
 & 0\ 00731 \\
 \hline
 & 0\ 00005\ 1 \\
 & 2 \\
 \hline
 & 0\ 00005\ 3 \\
 & 0\ 00731 \\
 \hline
 & 0\ 00000\ 0
 \end{array}$$

$$\begin{array}{r|l}
 22 & (i) \ 0\ 003160 \\
 & 0\ 003160 \\
 \hline
 & 9\ 5 \\
 & 3 \\
 & 2 \\
 \hline
 & 0\ 000009\ 9 (=a^2)
 \end{array}$$

$$\begin{array}{r|l}
 & 0\ 003160 \\
 & 0\ 000820 \\
 \hline
 & 2\ 53 \\
 & 6 \\
 \hline
 & 0\ 000002\ 59 (=ab)
 \end{array}$$

$$\begin{array}{r|l}
 & 0\ 000820 \\
 & 0\ 000820 \\
 \hline
 & 6 \\
 & 6 \\
 \hline
 & 0\ 000000\ 6 (=b^2)
 \end{array}$$

$$\begin{array}{r}
 0\ 00065 \\
 (ii) \ 3\ 98\ 0\ 002590 \\
 \hline
 220
 \end{array}$$

$$\begin{array}{r|l}
 23 & 1\ 22735 \\
 & 11\ 1 \\
 \hline
 & 122\ 7 \\
 & 12\ 3 \\
 & 1\ 2 \\
 \hline
 & 136\ 2 \\
 & 5\ 1034 \\
 \hline
 & 681\ 0 \\
 & 13\ 6 \\
 & 4 \\
 \hline
 & 695\ 0
 \end{array}$$

$$\begin{array}{r|l}
 24 & 1249\ 1 \\
 & 00\ 55 \\
 \hline
 & 6\ 24\ 6 \\
 & 62\ 5 \\
 \hline
 & 6\ 87\ 1
 \end{array}$$

$$\begin{array}{r|l}
 & 168\ 954 \\
 & 23\ 789 \\
 \hline
 & 3379\ 1 \\
 & 506\ 9 \\
 & 118\ 2 \\
 & 13\ 4 \\
 & 1\ 4 \\
 \hline
 & 6\ 87\ 4019\ 0 (=585) \\
 & 584\ 0 \\
 \hline
 & 34\ 4
 \end{array}$$

- 25 
$$\begin{array}{r|l} 328 & 08 \\ & 32809 \\ \hline 984 & 2 \\ 65 & 6 \\ 26 & 2 \\ & 2 \\ \hline 1076 & 2 \end{array}$$
26. 
$$\begin{array}{r|l} 1093 & 6 \\ & 10936 \\ \hline 981 & 2 \\ 32 & 8 \\ 06 & 5 \\ \hline 11959 & 5 \end{array}$$
- 27 Using result of Ex 26 and dividing
- $$\begin{array}{r} 247 \\ 484 \overline{) 11959} \\ \underline{2279} \\ 343 \end{array}$$
- 28 
$$\begin{array}{r|l} 2540 & \\ & 254 \\ \hline 508 & \\ 1270 & \\ 101 & 6 \\ \hline 6451 & 6 \\ & 254 \\ \hline 12903 & 2 \\ 3225 & 8 \\ 258 & 0 \\ \hline 16387 & 0 \end{array}$$
- 29 60 m an hr = 88 ft per sec,  
whence multiplier is  $\frac{88}{60} = 1.4667$
- 30 
$$\begin{array}{r|l} 3280 & 8 \\ & 22046 \\ \hline 6561 & 6 \\ 656 & 2 \\ 13 & 1 \\ 1 & 9 \\ \hline 7232 & 8 \end{array}$$

# EXAMPLES X f. Page 211

- 1 
$$\begin{array}{r} 542 \pm 005 \\ 406 \pm 005 \\ 360 \pm 005 \\ \hline 1308 \pm 015 \end{array}$$
- 2 
$$\begin{array}{r} 2030 \pm 5 \\ 1450 \pm 5 \\ 3380 \pm 5 \\ 940 \pm 5 \\ \hline 7800 \pm 20 \end{array}$$
- 3 
$$\begin{array}{r} 607 \pm 005 \\ 836 \pm 005 \\ 740 \pm 005 \\ \hline 2183 \pm 015 \end{array}$$
- 4 
$$\begin{array}{r} 16037 \pm 001 \\ 0973 \pm 001 \\ 2001 \pm 001 \\ \hline 19011 \pm 003 \end{array}$$
- 5 
$$\begin{array}{r} 4016 \pm 0005 \\ 0101 \pm 0005 \\ 2892 \pm 0005 \\ 3000 \pm 0005 \\ \hline 10009 \pm 002 \end{array}$$
- 6 
$$\begin{array}{r} 763 \pm 005 \\ 506 \pm 005 \\ \hline 257 \pm 01 \end{array}$$
- 7 
$$\begin{array}{r} 423 \pm 05 \\ 526 \pm 005 \\ 942 \pm 0005 \\ \hline 48502 \pm 0555 \end{array}$$
- 8 
$$\begin{array}{r} 4680 \pm 0005 \\ 2073 \pm 0005 \\ \hline 2607 \pm 001 \end{array}$$
- 9 
$$\begin{array}{r} 18009 \pm 0005 \\ 11900 \pm 0005 \\ \hline 6109 \pm 001 \end{array}$$
- 10 
$$\begin{array}{r} 2308 \pm 0005 \\ 3002 \pm 0005 \\ 5310 \pm 001 \\ 5304 \pm 0005 \\ \hline 0006 \pm 0015 \end{array}$$
- 11 
$$\begin{array}{r} 7000 \pm 0005 \\ 4420 \pm 001 \\ \hline 2580 \pm 0015 \end{array}$$
- 12  $(\pm 0.05 \times 10) \text{ cm}$
- 13 Error for each brick is within  $\pm 0.5 \text{ lb}$

- 14 (i) Superior limit =  $4.75 \times 6.35$ , (ii) Superior limit =  $4.8 \times 6.4$ ,  
 inferior limit =  $4.65 \times 6.25$  inferior limit =  $4.6 \times 6.2$

- 15 (i) Limits are  $5.35 \times 4.25$  and  $5.25 \times 4.15$   
 (ii)  $6.05 \times 5.05$  and  $5.95 \times 4.95$   
 (iii)  $30 \times 22$  and  $28 \times 20$   
 (iv)  $430 \times 500$  and  $450 \times 520$

$$\begin{array}{r} 16 \quad 58 \\ \quad 37 \\ \hline 174 \\ \hline 406 \\ \hline 2146 \end{array}$$

$$\begin{array}{r} 17 \quad 732 \\ \quad 604 \\ \hline 4392 \\ \hline 2928 \\ \hline 442128 \end{array}$$

$$\begin{array}{r} 18 \quad 502 \\ \quad 408 \\ \hline 2008 \\ \hline 4016 \\ \hline 204816 \end{array}$$

$$\begin{array}{r} 19 \quad 00617 \\ \quad 335 \\ \hline 1851 \\ \hline 1851 \\ \hline 3085 \\ \hline 206695 \end{array}$$

$$\begin{array}{r} 20 \quad 426 \\ \quad 0508 \\ \hline 2130 \\ \hline 3408 \\ \hline 216408 \end{array}$$

- 21 Circumference =  $431 \times (3.1416 \pm 0.0005)$  ft  
 =  $(1354.0296 \pm 0.2155)$  ft  
 =  $1354.05115$  ft, or  $1354.00805$  ft  
 =  $1354.0$  ft correct to nearest tenth

- 22 Superior limit =  $314.165 \times 14.7075$  lbs =  $4620.6$  lbs  
 Inferior limit =  $314.155 \times 14.7065$  lbs =  $4620.1$  lbs

$$\begin{array}{r} 23 \quad 408 \\ \quad 0524 \\ \hline 204 \quad 0 \\ \quad 8 \quad 2 \\ \quad 1 \quad 6 \\ \hline 213 \quad 8 \end{array}$$

- 24 Greatest weight  
 =  $62.45 \times 0.855 \times 15$  lbs =  $800.92$  lbs  
 Least weight  
 =  $62.35 \times 0.845 \times 15$  lbs =  $790.28$  lbs

- 25 Highest cost =  $(Rs\ 3500 \times \frac{119}{100}) \times (80 \times \frac{105}{100})$   
 Lowest cost =  $(Rs\ 3500 \times \frac{99}{100}) \times (80 \times \frac{95}{100})$

- 26 Expressing the product in the form  $0.827 \times 6.435$ , we get

This apparently is correct to three significant figures

If, however, we take for the first partial product its strict value, viz

$(0.827 \pm 0.005) \times 6 = 4.962 \pm 0.03 = 4.965$  or  $4.959$ , we see that the '6' is doubtful. This means the '2' in the answer, and, of course, all figures following it are doubtful. Thus *two* figures in the answer are correct, which illustrates the principle

$$\begin{array}{r} 0.827 \\ \quad 6.435 \\ \hline 4.962 \\ \quad 3308 \\ \quad 2481 \\ \quad \quad 4135 \\ \hline 5.321745 \end{array}$$



$$\begin{array}{r}
 4\ 46 \\
 32\ (i)\ 3\ 74\ )\ 16\ 68 \\
 \underline{1\ 720} \\
 2240
 \end{array}$$

$$\begin{array}{r}
 227000 \\
 (iv)\ 3\ 72\ )\ 843700 \\
 \underline{997} \\
 2530
 \end{array}$$

$$\begin{array}{r}
 14\ 91 \\
 (ii)\ 3\ 107\ )\ 46\ 328 \\
 \underline{15\ 258} \\
 2\ 8300 \\
 \underline{337}
 \end{array}$$

$$\begin{array}{r}
 2325 \\
 (iii)\ 4\ 070\ )\ 9462\ 800 \\
 \underline{1322\ 8} \\
 101\ 80 \\
 \underline{2040}
 \end{array}$$

In every case we see the principle holds good *e.g.*, in (i) there are 3 figures in divisor. At the 3<sup>rd</sup> division the multiple of the '3' must come vertically under the '8' of the dividend, and here the quotient begins to be uncertain. Thus the *two* preceding figures of quotient are certain. So for (ii), (iii) and (iv)

$$\begin{array}{r}
 2\ 32 \\
 33\ (i)\ 3\ 1416\ )\ 7\ 2800 \\
 \underline{99680} \\
 54320
 \end{array}$$

$$\begin{array}{r}
 7920 \\
 (ii)\ 3\ 1416\ )\ 249000 \\
 \underline{290880} \\
 81360
 \end{array}$$

$$34\ \text{Superior limit} = \frac{92850}{1865} = 500\ 5\ \text{secs}$$

$$\text{Inferior limit} = \frac{92750}{1865} = 497\ 3\ \text{secs}$$

$$35\ \text{Sp gr} = \frac{1728 \times 0.377}{62.425} = \frac{651.456}{62.425} = 10.4.$$

### EXAMPLES XI a Page 217

$$1\ \text{Ratio} = \frac{\text{Rs } 2\frac{1}{2}}{\text{Rs } 2\frac{1}{2}}$$

$$2\ \text{Ratio} = \frac{31\frac{1}{2}d}{70 \times 12d}$$

$$3\ \text{Ratio} = \frac{255\frac{1}{2}s}{44\frac{3}{4}s}$$

$$4\ \text{Ratio} = \frac{13\frac{1}{2}\ \text{qrs}}{46\ \text{qrs}}$$

$$5\ \text{Ratio} = \frac{\text{Re } 1}{\text{Rs } 2\frac{1}{2}}$$

$$6\ \text{Ratio} = \frac{17\frac{1}{2}d}{31\frac{1}{4}d}$$

$$7\ \text{Ratio} = \frac{31\frac{1}{2}\ \text{tons}}{56\frac{1}{4}\ \text{tons}}$$

$$8\ \text{Ratio} = \frac{(320 + 40 + 1)\ p}{(320 + 120 + 35)\ p}$$

$$9\ \text{Percentage} = \frac{\pounds 19\ 25}{\pounds 87} \times 100$$

$$13\ \text{Percentage} = \frac{\pounds 9\ 625}{\pounds 127\ 5} \times 100$$

$$14\ \text{Percentage} = \frac{\pounds 5\ 742}{\pounds 57\ 48} \times 100$$

$$16\ \text{Value} = \text{Rs } 3\ 4\ a \times 2\frac{1}{8}$$

$$17\ \text{Value} = \text{Rs } 8\ 24 \times 3\frac{1}{8}$$

$$18\ \text{Value} = \text{Rs } 2\ 219 \times 5\ 84$$

- 19 Rs 2 130 | 4 36  
     8 520  
     639 0  
     127 8  
 ---  
 Rs 9 286 | 8
- 20 Rs 384 464  $\times$  0 008351  
     Rs 384 '46  
     ---  
     8 371  
     3 075 7  
     115 3  
     019 2  
     4  
 ---  
 Rs 3 210 | 6
- 21 Value = Rs 565  $\times$  0 04  
     = Rs 5 65  $\times$  4
- 23 Value = Rs 4 725  $\times$  4 5  
     = 2 363  $\times$  9
- 24 Value = 12 896  $\times$  6 25
- 25 Value = £0 17133  $\times$  5 5  
     = £0 85665  $\times$  1 1
- 26 As No 23
- 27 Value = £7 24  $\times$  3 375  
     = £7 24  $\times$  3  $\frac{3}{8}$
- 28 Value = £0 81512  $\times$  4 75  
     = £0 81512  $\times$  4  $\frac{3}{4}$
- 29  $\frac{1}{100} = \frac{73\ 41}{1981}$
- 30  $\frac{2}{100} = \frac{43\ 2}{98\ 473}$
- 31  $\frac{1}{100} = \frac{224 + 28 + 25}{560}$
- 32  $\frac{1}{100} = \frac{123\frac{1}{2}\text{ ft}}{151\text{ ft}}$
- 33  $\frac{1}{100} = \frac{47109}{896431}$
- 34 As No 33
- 35  $\frac{1}{1000} = \frac{61\ 14}{361420}$
- 36  $\frac{1}{100} = \frac{183750}{5250000}$
- 37 It would save (62 - 50) % of gross revenue
- 38 The three partners get  $\frac{1}{6}$ ,  $\frac{1}{4}$ ,  $\frac{1}{6}$  respectively of total profits,  
     leaving  $\frac{1}{3}$  for the funds. Of this last share, the R F gets  $\frac{2}{3}$ ,  
     and the E P F  $\frac{1}{3}$
- 39 For flax,  $\frac{1}{100} = \frac{97075 - 55442}{97075}$  For potatoes,  $\frac{1}{100} = \frac{982301 - 635321}{982301}$
- 40 For wheat,  $\frac{2}{100} = \frac{321d}{337d}$  For barley,  $\frac{1}{100} = \frac{22\frac{2}{3}}{25\frac{1}{3}} = \frac{68}{77}$

## EXAMPLES XI b Page 221

- 1 Value = 227 at Rs (10 -  $\frac{1}{4}$ )      2 Value = 309 at Rs (3 -  $\frac{1}{12}$ )
- 3 Rs 417 5 is value at Re 1, then 8 a, 2 a, 6 p
- 4 £160 25 is value at £1, then 5 s, 1 s 3 d
- 5 £437 6 is value at £1, then 10 s, 7 s, 7 d, 3  $\frac{1}{2}$  d
- 6 Rs 5 0265 is value at Re 1, then 8 a, 4 a, 4 p
- 7 Rs 16 308 is value at Re 1, then 8 a, 4 a, 2 a, 1 a 4 p
- 8 Rs 1153 5 is value at Re 1, then 8 a, 2 a
- 9 Rs 500 76875 is value at Re 1, then 5 a 4 p

10 Rs 23 775 is value at Re 1, then Rs 2, 8 a, 1 a

11 Area = 2 60125 bigs, Re 1 2 a 8 p = Rs  $1\frac{1}{8}$  Then multiply

12 Area = 345 594 ac, £2 2s 6d = £2 $\frac{1}{2}$  Then multiply

13 Weight = 1684 4446 tons, £1 12s 6d = £1 $\frac{5}{8}$  = £( $1 + \frac{1}{2} + \frac{1}{8}$ ) Then multiply

14	£2 916	666
		13
	37 916	7
2 qts = $\frac{1}{2}$ cwt	1 458	3
1 qt = $\frac{1}{4}$ cwt	729	2
4 lbs = $\frac{1}{4}$ qr	104	2
	£40 208	4

16	£3 666	66
	916	7
2 fms = $\frac{1}{4}$ mi	083	3
40 yds = $\frac{1}{11}$ of $\frac{1}{4}$ mi	£1 000	0

15	£2 283	33
		2
	4 566	7
2 qts = $\frac{1}{2}$ gal	1 141	7
1 qt = $\frac{1}{4}$ gal	570	9
1 pt = $\frac{1}{2}$ qt	285	5
	£6 564	8

17	£1 712	5
		7
	11 987	5
1 ft = $\frac{1}{3}$ yd	570	8
6 in = $\frac{1}{2}$ ft	285	4
3 in = $\frac{1}{4}$ ft	142	7
	£12 986	4

18	Rs 42870 833	3
	21435 416	6
8 a = $\frac{1}{2}$ of Re 1	14290 277	7
5 a 4 p = $\frac{1}{3}$ of Re 1	Rs 35725 694	3

19	Rs 2073 375	
	691 125	
5 a 4 p = $\frac{1}{3}$ of Re 1	691 125	
5 a 4 p = $\frac{1}{3}$ of Re 1	129 585	9
1 a = $\frac{1}{10}$ of Re 1	Rs 1511 835	9

20	£1710 725	
	855 362	5
10s = $\frac{1}{2}$ of £1	213 840	6
2s 6d = $\frac{1}{5}$ of £1	71 280	2
10d = $\frac{1}{3}$ of 2s 6d	3 564	0
$\frac{1}{2}$ d = $\frac{1}{20}$ of 10d	£1144 047	3

21	£7 714	28
	1 370	83
	7 714	3
	2 314	3
	540	0
	6	2
		2
	£10 575	0

22	£2 070	833
		1 99
	20 708	3
	18 637	5
	1 863	8
	£41 209	6

23	£1 191	666
	30 412	5
	35 750	0
	476	6
	11	9
	2	4
		6
	£36 241	5

24	Rs 27 48	
	5	90
	137 40	
	24 73	2
	Rs 162 13	2

25	Rs 5 029	7
		2 6094
	10 059	4
	3 017	8
	45	2
	2	0
	Rs 13 124	4

26 Rs 382 828125  $\times$  36 621875

Rs 3828 281	25
	3 6621875
11484 843	8
2296 968	7
229 696	8
7 656	5
382	8
306	2
26	7
1	9
Rs 14019 883	4

27  $7\frac{1}{2}d = \frac{1}{16}$  of 10s  $\left\{ \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \end{array} \right\}$   $\begin{array}{l} \text{£} \\ 7922\ 5 = \text{price at 10s} \\ \hline 1980\ 625 \\ \hline 495\ 15625 \\ \hline \text{£}7427\ 34375 \end{array}$

28  $\begin{array}{r} \text{Rs} \\ 1232 \\ 11 \\ \hline 13552 \\ 154 \\ \hline 102\ 10a\ 8p \\ \hline \text{Rs } 13808\ 10a\ 8p \end{array}$

$2a = \frac{1}{8}$  of Re 1  
 $1a\ 4p = \frac{1}{12}$  of Re 1

29 It loses 8 26666' in 6 hrs  
 „ 1 37777' in 1 hr  
 Again, 17 days 9 hrs 45 min = 417 75 hrs

1 37	7777 min
41	7 75
551 11	1
13 77	8
9 64	4
96	4
6	9
575 56	6 min

30  $(17)^3 = 4913$

4 913	
8 788	
39 304	
3 439	1
393	0
39	3
43 175	4

# EXAMPLES XII a Page 224

- They do  $(\frac{1}{6} + \frac{1}{12})$ , i.e.  $\frac{1}{4}$  in 1 day
- $(\frac{1}{4} - \frac{1}{5})$ , or  $\frac{1}{20}$  will be filled in 1 min.
- They do  $(\frac{1}{5} + \frac{1}{7\frac{1}{2}})$ , or  $\frac{1}{3}$  in 1 hour



- 4 They do  $(\frac{1}{2} + \frac{1}{3} + \frac{1}{6})$  in 1 day    5 They do  $(\frac{1}{3} + \frac{1}{6})$ , or  $\frac{5}{6}$  in 1 hour  
 6  $(\frac{1}{25} - \frac{1}{45})$ , or  $\frac{4}{225}$  is filled in 1 min  
 7  $(\frac{1}{3} + \frac{1}{4} - \frac{1}{6})$ , or  $\frac{5}{12}$  is filled in 1 hour  
 8  $(\frac{1}{m} - \frac{1}{n})$ , or  $\frac{n-m}{mn}$  is filled in 1 min  
 9 C does  $(\frac{1}{10} - \frac{1}{30} - \frac{1}{45})$ , or  $\frac{2}{45}$  in 1 day  
 10 They do  $(\frac{1}{10} + \frac{1}{12})$ , or  $\frac{11}{60}$  in 1 day  
 11 They do  $(\frac{1}{2a} + \frac{1}{a} + \frac{1}{4a})$ , or  $\frac{7}{4a}$  in 1 day  
 12 A and B do  $\frac{1}{5}$  in 15 days,    A does  $\frac{1}{20}$  in 20 days    Hence B does  $(\frac{1}{5} - \frac{1}{20})$  in 1 day  
 13 A and B do  $\frac{1}{6}$  in 2 days, and A, B, and C do  $\frac{1}{6}$  in  $6\frac{1}{2}$  days  
     A and B do  $\frac{1}{12}$  in 1 day, and A, B, and C do  $\frac{1}{12}$  in 1 day  
     C (or A) does  $(\frac{1}{12} - \frac{1}{12})$  in 1 day, and B does  $[\frac{1}{12} - (\frac{1}{12} - \frac{1}{12})]$  in 1 day  
 14 A and B do  $\frac{11}{16}$  in 22 days, or  $\frac{1}{30}$  in 1 day    A does  $\frac{4}{16 \times 20}$  in 1 day  
 15 A and B do  $\frac{2}{3}$  in 1 day, B and C do  $\frac{1}{3}$  in 1 day, C and A do  $\frac{1}{2}$  in 1 day,  
     A does  $\frac{1}{2}(\frac{1}{3} + \frac{2}{3} - \frac{1}{3})$  in 1 day, or  $\frac{1}{2}$  in 1 day, and earns  $\frac{1}{2}$  of 72d,  
     B "  $\frac{1}{2}(\frac{2}{3} + \frac{1}{3} - \frac{1}{3})$  " 1 "  $\frac{1}{2}$  " 1 " "  $\frac{1}{2}$  of 72d,  
     C "  $\frac{1}{2}(\frac{1}{3} + \frac{1}{3} - \frac{2}{3})$  " 1 "  $\frac{1}{2}$  " 1 " "  $\frac{1}{2}$  of 72d  
 16 (i)  $(\frac{1}{3} - \frac{1}{4})$  is filled in 1 hour, (ii)  $(\frac{1}{4} + \frac{1}{6} - \frac{1}{3})$  is emptied in 1 hour  
      $(\frac{1}{a} - \frac{1}{b})$  is filled in 1 hour,     $(\frac{1}{b} + \frac{1}{c} - \frac{1}{a})$  is emptied in 1 hour

## EXAMPLES XII b Page 227

$$[NB - 60 \text{ mi per hr} = \frac{60 \times 1760 \times 3}{60 \times 60} \text{ ft per sec} = 88 \text{ ft per sec}]$$

$$1 \text{ mi per hr} = \frac{88}{60} \text{ ft per sec, and } 1 \text{ ft per sec} = \frac{60}{88} \text{ mi per hr}]$$

- 9 528 yds in 6 min, or 5280 yds in 1 hour  
 10 He rides 22 ft in 1 sec    11 He rides  $\frac{88}{60} \times 9$  ft in 1 sec  
 12  $35 \times \frac{88}{60}$  ft of train pass in 1 sec    13.  $44 \times \frac{88}{60}$  miles per hour

- 14 It travels  $45 \times \frac{88}{90}$  ft in 1 sec., hence it travels  $(150+180)$  ft in  $\frac{130 \times 90}{45 \times 88}$  sec.
- 15 Let  $x$  ft. be length of induct then  $(x+252)$  ft are traversed in 21 sec But  $30 \times \frac{88}{90} \times 21$  ft are traversed in 21 secs hence  $x+252=30 \times \frac{88}{90} \times 21$
- 16 Rate of approach = 8 mi per hr and they meet in 4 hrs, 14 mi from A
- 17 Rate of approach = 10 mi per hr they meet in  $2\frac{1}{2}$  hrs,  $11\frac{1}{4}$  mi from B's starting place
- 18 Rate of approach =  $1\frac{1}{2}$  mi per hr A overtakes B in 6 hrs
- 19 Rate of approach = 8 mi per hr hence they meet at 1 45 p m
- 20 At 9 a.m they are 33 mi apart rate of approach = 11 mi per hr they meet at 12 noon, 12 miles from A
- 21 Rate of approach = 88 ft. per 1 sec, they pass in  $\frac{(188+211)}{88}$  secs
- 22 Rate of approach =  $\frac{88}{90} (25-11\frac{1}{2})$  ft per 1 sec,  
or  $\frac{88}{90} \times 13\frac{1}{2}$  ft per 1 sec.  
time req<sup>d</sup> =  $\frac{(344+196) \times 60}{88 \times 13\frac{1}{2}}$  secs
- 23 Rate of approach =  $7\frac{1}{2}$  mi per hour  
 $(170+150+1000)$  yds = 1320 yds =  $\frac{3}{4}$  mi  
time req<sup>d</sup> =  $(\frac{3}{4} \times \frac{1}{7\frac{1}{2}})$  hours
- 24 Rate of approach = 18 mi per hour, or  $18 \times \frac{88}{90} \times 25$  ft per 25 secs  
If  $x$  ft be length of passenger train, then  $x+1\frac{1}{2} \times 18 \times \frac{88}{90} \times 25$   
 $x=264$  ft, and, since 45 mi per hr = 66 ft per sec.,  
time req<sup>d</sup> =  $\frac{264 + \frac{18 \times 88 \times 25}{90}}{66}$  secs

### EXAMPLES XII c Page 229

- 1 Average height  
=  $(5 \text{ ft } 10 \text{ in } + 5 \text{ ft } 7\frac{1}{2} \text{ in } + 5 \text{ ft } 4 \text{ in } + 4 \text{ ft } 10\frac{1}{2} \text{ in } + 4 \text{ ft } 2 \text{ in}) - 5$
- 2 Average tonnage in 1904 = 126185 - 26  
Average tonnage in 1906 =  $(126185 + 10754 \times 2) - 28$
- 3 Average profits for half-year  
=  $(£416 \text{ } 8\text{s } 6\text{d}) \times 4 + £437 \text{ } 2\text{s } + £489 \text{ } 4\text{s}] - 6$

- 4 Value req<sup>d</sup> =  $[2s\ 4d \times 12 + 2s\ 6d \times 10 + 2s\ 9d \times 8] - 30$
- 5 Average from Jan to June  
 $= (£89\ 5s + £91 + £93\ 15s + £96\ 2s\ 6d + £98 + £99\ 17s\ 6d) - 6$   
 Average from July to Dec  
 $= (£99 \times 12 - \text{sum of prices from Jan to June}) - 6$
- 6  $905 \times 8 - 92 \times 7$
- 7 Distance =  $(88 + 147 + 220 + 352 + 586 + 4840)$  yds in 10 min
- 8 Average age req<sup>d</sup> =  $\{164 \times 125 + 177 \times 35\} - (125 + 35)$
- 10 Day's run =  $\frac{3514}{5\frac{1}{2}}$  mi, sea speed =  $\frac{3514}{5\frac{1}{2} \times 24} \times \frac{60}{69}$  knots  
 Rest of passage = 1000 mi, when speed =  $\frac{1000}{\frac{60}{69}} \times \frac{60}{69}$  knots
- 11 Rainfall for April, May and June, 1907 =  $(1\ 90 + 0\ 94 + 1\ 72) = 4\ 56$  in  
 Average for April, 1901-1906 =  $(1\ 90 - 0\ 77) = 1\ 13$  in  
 " May, " =  $(0\ 94 + 1\ 46) = 2\ 40$  in  
 " June, " =  $(1\ 72 + 1\ 78) = 3\ 50$  in  
 average for April, May and June, 1901-1906 =  $7\ 03$  in  
 rainfall for April, May and June, 1901-1906 =  $(7\ 03 \times 6)$  in  
 req<sup>d</sup> average for the 3 months for 7 years  
 $= \frac{7\ 03 \times 6 + 4\ 56}{7 \times 3} = 2\ 23$  in
- 12 6 mi per hr =  $\frac{6 \times 1760}{60}$ , or 176 yds per min
- |   |   |          |     |                    |
|---|---|----------|-----|--------------------|
| In the first min the spaces traversed are (i) | 0 | and (ii) | 176 | <sup>yds</sup> × 1 |
| " second                                      | " | "        | 176 | × 2                |
| " third                                       | " | "        | 176 | × 3                |
| " fourth                                      | " | "        | 176 | × 4                |
| " fifth                                       | " | "        | 176 | × 5                |
- total distance (i) = 1760 yds, and (ii) = 2640 yds  
 Average of these totals =  $\frac{1760 + 2640}{2} = 2200$  yds  
 From the formula  $s = \frac{1}{2} \times 176 \times 5^2 = 2200$  yds

### EXAMPLES XII d Page 232

- 1 (i) The large hand gains 15 min spaces in  $\frac{60}{6} \times 15$  min  
 [At 219, Ex 1]  
 (ii) " " 45 " "  $\frac{60}{6} \times 45$  min
- 2 (i) The large hand gains 45 min spaces in  $\frac{60}{6} \times 45$  min  
 (ii) " " (45 - 30) " "  $\frac{60}{6} \times (45 - 30)$  min

- 3 The large hand gains  $(30-15)$  min spaces in  $\frac{6}{5} \times (30-15)$  min  
 " "  $(30+15)$  " "  $\frac{6}{5} \times (30+15)$  "
- 4 The large hand gains  $(25-15)$  min spaces in  $\frac{6}{5} \times (25-15)$  min  
 " "  $(25+15)$  " "  $\frac{6}{5} \times (25+15)$  "
- 5 The large hand gains  $(35-12)$  min spaces in  $\frac{6}{5} \times (35-12)$  min  
 " "  $(35+12)$  " "  $\frac{6}{5} \times (35+12)$  min
- 6 In  $\frac{6}{5} \times (10-1)$  min and  $\frac{6}{5} \times (10+1)$  min after 2 o'clock
- 7  $\frac{6}{5} \times (25-3)$  min or  $\frac{6}{5} \times (25+3)$  min after 5 o'clock
- 8  $A$  gains 2 yds while  $B$  runs 9,  $A$  gains 50 yds while  $B$  runs  $9 \times 25$  yds
- 9 Suppose  $x$  lbs at  $10\frac{1}{2}$  are mixed with  $y$  lbs at 7a,  
 then  $3\frac{1}{2}x + 7y = 8(x+y)$
- 10 Suppose  $x$  gals at  $14\frac{1}{2}$  are mixed with  $y$  gals at  $17\frac{1}{2}$ ,  
 then  $14\frac{1}{2}x + 17\frac{1}{2}y = 16\frac{1}{2}(x+y)$
- 11 Let  $x$  srs at 10a be mixed with  $(18-x)$  srs at 11a,  
 then  $10x + 11(18-x) = 190$
- 12 While  $A$  walks 410 yds,  $B$  walks 420 yds  
 " "  $C$  " 413 yds  
 $B$  can give  $C$  7 yds in 420,  
 i.e.,  $B$  "  $C 7 \times \frac{420}{413}$  "  $420 \times \frac{420}{413}$ ,  
 or  $B$  "  $C$  2 " 120
- 13 Reasoning as in ex 12, we find  
 $B$  can give  $C$  16 yds in 1728,  
 i.e.,  $B$  "  $C 16 \times \frac{1728}{1712}$  "  $1728 \times \frac{1728}{1712}$ ,  
 or  $B$  "  $C$  10 " 1080
14. Reasoning as in ex 12, we find  
 $B$  can give  $C$  5 yds in 90,  
 i.e.,  $B$  "  $C 5 \times \frac{90}{88}$  "  $90 \times \frac{90}{88}$ ,  
 or  $B$  "  $C$   $8\frac{1}{2}$  " 150
- 15  $A$  scores 100 while  $B$  scores 80, (i)  
 $B$  " 100 "  $C$  " 95 (ii)  
 Multiply by 10 in (i) and by 8 in (ii)  
 Thus while  $B$  scores 800  $\begin{cases} A \text{ scores } 100 \times 10 \text{ or } 1000, \\ C \text{ " } 95 \times 8 \text{ or } 760 \end{cases}$   
 $A$  can give  $C$  240 in 1000,  
 or  $A$  "  $C$  24 " 100

- 16  $A$  scores 500, while  $B$  scores 450, and  $C$  scores 432

$B$  can give  $C$  18 in 450,  
 i.e.,  $B$  "  $C$   $18 \times \frac{450}{432}$  "  $450 \times \frac{450}{432}$ ,  
 or  $B$  "  $C$  20 " 500

- 17  $A$  fires 28 shots to  $B$ 's 12, and therefore kills 7 while  $B$  misses 6

- 18  $A$  walks 100 yds while  $B$  walks 95 yds

$B$  "  $85\frac{1}{2}$  yds while  $C$  walks 100 yds  
 $A$  "  $100 \times 85\frac{1}{2}$  "  $B$  "  $95 \times 85\frac{1}{2}$  "  $C$  "  $100 \times 95$ ,  
 i.e.,  $A$  walks  $85\frac{1}{2}$  yds while  $C$  walks 95 yds

$C$  gains  $9\frac{1}{2}$  in 95, or 19 in 190, or 1 in 10 on  $A$   
 thus  $C$  gains 176 yds in 1760 yds

- 19 Rate of approach  $= 8\frac{1}{4}$  or  $3\frac{3}{4}$  mi per hr

$$= \frac{33 \times 1760}{4 \times 60}, \text{ or } 242 \text{ yds per min}$$

they meet in  $\frac{7\frac{1}{4}}{2\frac{3}{4}}$ , or 3 min

Since  $3\frac{3}{4}$  mi per hr  $= \frac{3\frac{3}{4} \times 1760}{60}$  or 110 yds per min,

the first man in 3 min walks  $110 \times 3$  or 330 yds,  
 and the second man in 3 min walks  $(726 - 330)$  or 396 yds

- 20 In 1 sec  $A$  runs  $\frac{100}{10\frac{1}{2}}$  yds, while  $B$  runs  $\frac{100}{11\frac{1}{6}}$  yds

$A$  runs  $\frac{100}{10\frac{1}{2}} \times 10\frac{1}{2}$  yds, while  $B$  runs  $\frac{100}{11\frac{1}{6}} \times 10\frac{1}{2}$  yds,  
 or  $A$  " 100 "  $B$  "  $93\frac{3}{4}$  "  
 req<sup>d</sup> start is  $6\frac{1}{4}$  yds

- 21 While  $B$  runs 1 yd,  $A$  runs  $\frac{6\frac{2}{3}}{7}$  yds and  $C$  runs  $\frac{15}{16\frac{1}{2}}$  yds

$A$  runs  $\frac{6\frac{2}{3}}{7} \times \left(\frac{7}{6\frac{2}{3}} \times 1760\right)$  yds while  $C$  runs  $\frac{15}{16\frac{1}{2}} \times \left(\frac{7}{6\frac{2}{3}} \times 1760\right)$  yds  
 or  $A$  " 1760 "  $C$  " 1680 "  
 $C$  runs 1680 yds in 5 mins 15 secs or 315 secs  
 i.e.,  $C$  "  $1680 \times \frac{1760}{1680}$  "  $315 \times \frac{1760}{1680}$  secs  
 or  $C$  " 1760 " 330 secs or  $5\frac{1}{2}$  mins

- 22 Before they meet  $A$  runs 250 yds while  $B$  runs 200 yds,

after "  $A$  " 300 "  $B$  "  $300 \times \frac{200}{250}$  yds  
 i.e., when  $A$  reaches the starting pt  $B$  has run 240 yds  
 But when they meet,  $B$  was 250 yds from the starting point,  
 $A$  gets in when  $B$  is 10 yds away

- 23 The leak admits  $24\frac{6}{11}$  tons in 1 hr  
 when the pumps are going,  $(24\frac{6}{11} - 12)$  tons enter in 1 hr ,  
 i.e., " " "  $12\frac{6}{11} \times \frac{92}{12\frac{6}{11}}$  "  $\frac{92}{12\frac{6}{11}}$  "  
 she must sail 44 mi in  $\frac{92}{12\frac{6}{11}}$  hrs ,  
 i.e., " "  $44 \times \frac{12\frac{6}{11}}{92}$ , or 6 miles in 1 hour
24. A gained 1 yd in 40 yds, or 11 yds in 440 yds  
 length of course is 440 yds, run by A in 90 secs, and B runs  
 429 yds. in 90 secs
- 25 First rate of approach = 88 yds in 10 secs = 18 mi per hour  
 actual speed of train =  $(18 + 4)$  mi, i.e., 22 mi per hour  
 Second rate of approach = 88 yds in 9 secs = 20 mi per hour  
 second man walks at  $(22 - 20)$  mi, i.e., at 2 mi per hour
- 26 A has walked  $3\frac{1}{2}$  mi when met by B  
 A has walked for 63 mins and B has walked  $5\frac{1}{2}$  mi  
 in  $(63 + 3)$  min, i.e., 1 mile in 12 min, or 5 mi per hour
- 27 The problem is the same as if B waited 11 hrs and A waited 6 hrs,  
 or " B " 5 " A " 0 hrs  
 That is, just as if B started at 9 mi per hr, when A had walked  
 20 miles at 4 mi per hr  
 Now, rate of approach is 5 mi per hour  
 B overtakes A in 4 hours, i.e., 9 hours after A started, and  
 36 miles from London

EXAMPLES XII e Page 235

- 1 Let A be  $x$  and B  $(x+3)$  years of age ,  
 then  $\frac{2x}{3} = \frac{5(x+3)}{6} - 10$
- 2 Let A, B, C be  $x, (x+9), (x-6)$  years of age respectively ,  
 then  $\frac{3x}{4} + \frac{4(x+9)}{5} + \frac{x-6}{2} = 37$  ,  
 whence  $15x + 16(x+9) + 10(x-6) = 740$  and  $x = 16$
- 3 Let silk cost  $6x$  rupees and linen  $x$  rupees per yard ,  
 then  $6x \times 23 + x \times 50 = 211\frac{1}{2}$  , whence  $x = 1\frac{1}{2}$
- 4 Suppose there are  $x$  lbs at Re  $\frac{7}{8}$  and  $(200 - x)$  lbs at Rs  $\frac{9}{8}$  ,  
 then  $\frac{7}{8}x + \frac{9}{8}(200 - x) = 190$  ,  $7x + 9(200 - x) = 1520$  and  $x = 140$   
 K H S I E

- 5 Let rolls be  $x$  yds and  $(x+25)$  yds in length  
 $5\frac{1}{2}x + 5(x+25) = 2015$ ,  
 i.e.,  $11x + 10(x+25) = 4030$ , whence  $x = 180$
- 7 Let 1 egg cost  $x$  pence then  $30 - 16x = 12x - 5$
- 8 Let  $x$  be number of oranges then  $\frac{15x}{12} - \frac{12x}{15} = 162$   
 $75x - 48x = 9720$ , whence  $x = 360$
- 9 Let  $x$  be number required then  $\frac{6x}{5} = \frac{12x}{11} + 6$ ,  
 i.e.,  $66x = 60x + 330$ , whence  $x = 55$
- 10 Let them meet in  $x$  hours then  $4(x - 2\frac{1}{2}) + 3x = 60$ ,  
 i.e.,  $4x - 10 + 3x = 60$ , whence  $x = 10$
- 11 Let  $x$  miles be distance required He takes  $\frac{2x}{4}$  hours to walk to B and back, at 4 miles an hour Walking to B at  $3\frac{1}{2}$  miles an hour, he takes  $\frac{x}{3\frac{1}{2}}$  hours, and back to A at  $4\frac{1}{2}$  miles an hour, he takes  $\frac{x}{4\frac{1}{2}}$  hours The difference in time is  $3\frac{1}{2}$  min, i.e.,  $\frac{3\frac{1}{2}}{60}$  hrs,  
 $\frac{2x}{4} = \frac{x}{3\frac{1}{2}} + \frac{x}{4\frac{1}{2}} - \frac{3\frac{1}{2}}{60}$  or  $\frac{x}{2} = \frac{2x}{7} + \frac{2x}{9} - \frac{1}{18}$   
 $63x = 36x + 28x - 7$ , whence  $x = 7$
- 12 A gains  $1\frac{1}{2}$  miles while walking  $4\frac{1}{2}$  miles,  
 A " 5 " "  $5 \times \frac{4\frac{1}{2}}{1\frac{1}{2}}$ , i.e., 15 miles  
 Time =  $\frac{15}{4\frac{1}{2}}$  hrs =  $3\frac{1}{2}$  hrs
- 13 Let distance be  $x$  miles at  $3\frac{1}{2}$  miles per hour he takes  $\frac{x}{3\frac{1}{2}}$  hours,  
 at  $3\frac{1}{3}$  miles an hour he takes  $\frac{x}{3\frac{1}{3}}$  hours, the latter time is  
 2 mins, or  $\frac{1}{30}$  hour, longer than the former  
 $\frac{x}{3\frac{1}{2}} + \frac{1}{30} = \frac{x}{3\frac{1}{3}}$  i.e.,  $\frac{2x}{7} + \frac{1}{30} = \frac{3x}{10}$ ,  
 $60x + 7 = 63x$ , whence  $x = 2\frac{1}{3}$
- 14 Following the method of Ex 13, we obtain  
 $\frac{x}{3\frac{1}{2}} = \frac{x}{3\frac{1}{3}} + \frac{1}{30}$ ,  $\frac{2x}{7} = \frac{4x}{15} + \frac{1}{30}$ ,  
 $60x = 56x + 7$ , whence  $x = 1\frac{1}{4}$  miles

- 15 20 miles diff for the trains = 80 miles distance between the towns,  
 $\begin{array}{ccccccc} 1 & & & = 4 & & & \\ \text{, e, } 14\frac{1}{2} & & & = 4 \times 14\frac{1}{2} & & & \\ & & & & & & \end{array}$   
 giving 58 miles

- 16 If  $A$  rides for  $x$  hours before he is overtaken, then  $B$  rides for  $(x - 1\frac{1}{2})$  hours

$$8x = 10(x - 1\frac{1}{2}), \text{ whence } x = 7\frac{1}{2}, \text{ whence distance} = 60 \text{ miles}$$

For second part of question, if  $x$  = req<sup>d</sup> number of hrs after noon,

$$8x = 10(x - 1\frac{1}{2}) \pm 5, \quad [\text{see Ex. 3, p 235}]$$

whence  $x = 10$  or  $5$ , according as  $B$  is ahead of or behind  $A$

- 17 Let  $x$  be req<sup>d</sup> number of hours after 9 a.m.,  
 then  $3\frac{1}{2}x + 2\frac{1}{2}x \pm 5 = 35$  (upper sign gives first time, lower sign gives second time), whence  $x = 5$  or  $6\frac{2}{3}$

- 18 If  $x$  hours is time required, then  $12x + 9x + 18 = 60$   
 $+18$  must be changed to  $-18$

- 19  $C$  has  $\frac{3}{8}$  of 4 mi, i.e.  $\frac{1}{2}$  mi, start of  $A$

Now  $A$  in walking 5 mi gains 1 mi on  $C$  in 1 hr

$A$  " 1 mi "  $\frac{1}{5}$  mi "  $C$  "  $\frac{1}{5}$  hr

Again  $A$  has  $\frac{7}{8}$  of 5 mi, i.e.  $\frac{7}{8}$  mi, start of  $B$ ,

and  $B$  in walking 6 mi gains 1 mi on  $A$  in 1 hr

$B$  "  $\frac{7}{8}$  of 6 mi gains  $\frac{7}{8}$  mi on  $A$  in  $\frac{7}{8}$  hr

or  $B$  "  $3\frac{1}{2}$  mi "  $\frac{7}{8}$  mi "  $A$  " 35 min

- 20 Let  $x$  miles be distance up stream

My speed up stream is  $(4\frac{1}{2} - 1\frac{1}{2})$  mi, i.e., 3 mi per hour

" down "  $(4\frac{1}{2} + 1\frac{1}{2})$  mi, " 6 mi "

my time up stream is  $\frac{x}{3}$  hrs and down stream is  $\frac{x-2}{6}$  hrs

$$\frac{x}{3} + \frac{x-2}{6} = 2\frac{1}{2}, \text{ whence } x = 5$$

### Alternative Solution

Up stream the rate of rowing is  $4\frac{1}{2} - 1\frac{1}{2}$ , or 3 miles per hour

Down " "  $4\frac{1}{2} + 1\frac{1}{2}$ , or 6 " "

If I had returned to the starting point, the last 2 miles would have taken 20 minutes, and the whole time would have been  $2\frac{1}{2}$  hours, two thirds of which must have been occupied by the up stream journey, since the rates of rowing up and down are as 1 to 2

Thus the distance up stream =  $\frac{2}{3} \times \frac{5}{2} \times 3 = 5$  miles



- 21 Let him change his speed  $v$  hrs after the start  
 $9v + 15(7-v) = 75$ , whence  $v = 5$
- 22 Suppose he mixes  $x$  seers of milk with  $y$  seers of water  
 Then  $4\frac{1}{2}(x+y) = 52$ , whence  $x = 9y$  and  $\frac{x}{y} = \frac{9}{1}$
- 23 Suppose  $x$  gallons of water are added  
 then  $(18 + 14 + x) \times 16\frac{1}{2} = 18 \times 18\frac{1}{2} + 14 \times 21$ ,  
 $(32 + x) \times 33 = 18 \times 37 + 14 \times 42$ ,  
 whence  $x = 6$
- 24 Let  $A$  run 50*l* yds in 1 min  
 then  $B$  runs 45*l* yds in 1 min  
 Suppose  $A$  gets level with  $B$  in  $x$  min  
 $50x = 45(x+5)$ ,  
 $50x = 45(x+5)$ , whence  $x = 45$
- 25 Suppose he swims at  $v$  miles an hour in still water  
 In 1 hr he swims  $(x + 1\frac{1}{2})$  mi down stream,  
 „ 1 hr „  $(x - 1\frac{1}{2})$  mi up stream,  
 $x + 1\frac{1}{2} = 5(x - 1\frac{1}{2})$ , whence  $v = 2$
- 26 Let time req<sup>d</sup> be  $x$  hours after 10 30 a m  
 At this time  $A$  is  $25(x + 3\frac{1}{2})$  miles from Bristol,  
 „  $B$  „  $20(x + 2\frac{1}{4})$  „ „  
 „  $C$  „  $220 - 30x$  „ „  
 $(220 - 30x) - 25(x + 3\frac{1}{2}) = 25(x + 3\frac{1}{2}) - 20(x + 2\frac{1}{4})$ ,  
 $220 - 30x = 50(x + 3\frac{1}{2}) - 20(x + 2\frac{1}{4})$ ,  
 $220 - 30x = 50x + 175 - 20x - 45$ ,  
 whence  $x = 1\frac{1}{2}$ , giving 12 noon and 125 mi from Bristol

### MISCELLANEOUS EXAMPLES III Page 238

- 1 Wheat per cwt costs  $28s\ 5d \times \frac{1\frac{1}{2}}{3\frac{1}{2}} = 28s\ 5d \times \frac{2}{5} = 6s\ 8d$   
 Bailey „  $24s\ 5d \times \frac{1\frac{1}{2}}{4\frac{1}{2}} = 24s\ 5d \times \frac{2}{7} = 6s\ 10d$   
 Oats „  $17s\ 11d \times \frac{1\frac{1}{2}}{3\frac{1}{2}} = 17s\ 11d \times \frac{2}{5} = 6s\ 5d$
- 2 Let Rs  $x$  be the average subscription,  
 then the "other two" give Rs  $(x+2)$  and Rs  $(x+2\frac{1}{2})$  respectively  
 $11x = 2 \times 9 + 2 + 2 + x + 2\frac{1}{2}$ , whence  $x = 2\frac{1}{2}$ ,  
 the two persons give Rs 4 8a and Rs 5

3 Tax is £1 on 5 lbs

tax is £13,184,767 on  $5 \times 13184767 \times \frac{1}{2540}$  tons,

„ £13,181,767 „  $\frac{13184767}{2540}$  tons, i.e., on 29130 tons

4 35 fr per 1 Kg = £  $\frac{35}{2540}$  per 2204 lbs

=  $\frac{15 \times 40}{2540}$  pence per 2204 lbs =  $\frac{35 \times 2240}{2540 \times 2540}$  pence per 1 lb

=  $\frac{15}{699 \cdot 31}$  pence per 1 lb = 15d per 1 lb

5 He worked 44 hours per week. Had he worked for 52 weeks at 7 pice per hour, he would have earned  $(44 \times 7 \times 52)$  pice

But he lost 15 days' work, i.e.  $(8 \times 7 \times 15)$  pice

his net earnings were  $(44 \times 7 \times 52 - 8 \times 7 \times 15)$  pice

his gain on former contract of Rs  $3\frac{1}{2}$  per week

=  $(44 \times 7 \times 52 - 8 \times 7 \times 15)$  pice =  $52 \times 240$  pice

=  $8\{11 \times 7 \times 26 - 7 \times 15 - 52 \times 30\}$  pice =  $8\{7(11 \times 26 - 15) - 1560\}$  pice

=  $8\{7 \times 271 - 1560\}$  pice =  $8 \times 337$  pice = Rs 12 2a

6 Req<sup>d</sup> value =  $58135823 \times \frac{35705682}{8 \cdot 050917}$

=  $58135823 \times \frac{35705682}{136766}$

This fraction lies between 4 and 5 in value. Hence, to obtain a final result correct to two decimal figures, we multiply correct to three decimal figures, and thus secure the division being correct to two decimal places

58134	823
290670	1
52322	2
2906	8
52	3
2	3
	4
345963	

25296
136766 ) 315963
72431
1018
1313
83

7 \$4875 = £1, i.e.  $84\frac{1}{8}$  = £1, \$15 = £  $\frac{15}{4\frac{1}{8}}$

Now  $\frac{15}{4\frac{1}{8}} = \frac{40}{13} = £3.0769 = £3 1s 6d$

8 Error = £0.000495 × 317 = £0.157 = 3s 1½d

- 9 His average speed =  $\frac{25\frac{1}{2}}{2\frac{1}{4}}$  mi in 1 hr =  $\frac{25\frac{1}{2}}{2\frac{1}{4}} \times \frac{88}{60}$  ft in 1 sec  
 $= \frac{7\frac{1}{2} \times 8}{4 \times 6} \text{ ft in 1 sec} = \frac{14 \times 9}{9} \text{ ft in 1 sec} = 17 \text{ ft in 1 sec.}$   
 On returning his speed was  $\frac{1}{5}$  of 25 mi in  $2\frac{1}{4}$  hrs,  
 or 1 mi in  $\frac{2\frac{1}{4}}{\frac{1}{5} \times 25\frac{1}{2}} \times 60$  mins, or 8 mi in  $\frac{8 \times 9 \times 2 \times 60}{5 \times 4 \times 5\frac{1}{2}} \times 28$  mins,  
 i.e. 28 mi in  $\frac{108 \times 28}{17}$  mins, i.e. in 178 mins
- 10 No of gillions per head =  $\frac{1\frac{5}{8} \times 10}{4 \times 333}$  (neglecting unnecessary figs)  
 $= \frac{1\frac{20}{4} \times 1}{33} = 29.8 = 30$ , to nearest whole number  
 Revenue per head =  $\pounds \frac{1\frac{1}{4} \times 10}{4 \times 333}$  (neglecting unnecessary figures)  
 $= \pounds 0.302 = 6s$
- 11 

1 0036	59
27	33
<hr/>	
200 7318	
70 2561	3
3 0109	8
3011	0
<hr/>	
274 3000	1

$$\begin{aligned} x^3 - 109x^2 + 94x - 102 \\ &= 11^3 - 109 \times 11^2 + 94 \times 11 - 102 \\ &= 1331 - 13189 + 1034 - 102 \\ &= 121 + 14 \\ &= 135 \end{aligned}$$
- 12 18 mi =  $18 \times 1760$  yds =  $\frac{18 \times 1760 \times 64}{70}$  metres,  
 time req<sup>d</sup> in secs =  $\frac{18 \times 1760 \times 64}{70 \times 2} = \frac{9 \times 1760 \times 64}{7} = \frac{101776}{7}$   
 $= 14482\frac{2}{7}$  giving 4 hrs 1 min  $22\frac{2}{7}$  secs
- 13 B runs  $\frac{264}{300}$  yds, while C runs  $\frac{297}{300}$  yds, while A runs 1 yd  
 B „  $\frac{264 \times 300}{300 \times 297} \times 300$  yds, while C runs 300 yds  
 B „  $\frac{264}{297}$ , or  $293\frac{1}{3}$  yds, „ C „ 300 yds
- 14 34 mi =  $34 \times 1760 \times 3$  ft no of knocks in 1 hr =  $\frac{34 \times 1760 \times 3}{22}$ ,  
 whence no of knocks in  $\frac{1}{4}$  min =  $\frac{34 \times 1760 \times 3}{22 \times 60 \times 4} = 34$
- 15 Income this year = Rs  $\frac{25\frac{1}{2} \times 192}{5} = \text{Rs } 984$ ,  
 Income last year = Rs  $\frac{32\frac{1}{4} \times 192}{7} = \text{Rs } 879$ ,  
 Income next year = Rs 1089, and tax = 6534 p = Rs 34 0a. 6p
- 16 Total cost = Rs  $3\frac{1}{2} \times 10 \times 5 + \text{Rs } 2\frac{1}{2} \times 40$   
 $= \text{Rs } 675 + \text{Rs } 92.80 = \text{Rs } 767.80$   
 he saves Rs 392.80

17	1 423	056
	3	2 6231
	42 691	68
	2 846	11
	853	83
	28	46
	4	27
		14
	46 424	49

	6 799
6 4395	43 784
	5 147
	640
	61

quotient = 6.80 correct to 2 places of decimals

- 18 Let 1 man do the work in  $m$  days and 1 boy in  $b$  days

$$\frac{5}{m} = \frac{1}{14}, \text{ and } \frac{5}{m} + \frac{2}{b} = \frac{1}{12}, \quad \frac{2}{b} = \frac{1}{12} - \frac{5}{m} = \frac{1}{12} - \frac{1}{14} = \frac{1}{84},$$

whence  $m = 70$ , and  $b = 168$ , since a boy does  $\frac{1}{168}$ ,

while a man does  $\frac{1}{70}$ , his fraction of a man's work =  $\frac{1}{168} / \frac{1}{70} = \frac{5}{12}$

- 19  $(32 \times 7\frac{3}{4})$  strokes cover  $(1760 \times 3 \times 12)$  in ,

$$1 \text{ stroke covers } \frac{1760 \times 3 \times 12}{32 \times 7\frac{3}{4}} \text{ in ,}$$

$$1 \text{ " } \frac{880 \times 3 \times 12}{7\frac{3}{4}} \text{ in, i.e. } 261 \text{ in}$$

- 20  $\frac{4}{5}$  of Rs 23 4a 9p + 4757 p + Rs 17 6875

$$= \text{Rs. } 10 \text{ 5a. } 8 \text{ p} + \text{Rs } 24 \text{ 12a } 5 \text{ p} + \text{Rs } 17 \text{ 11a.}$$

$$= \text{Rs } 52 \text{ 13a } 1 \text{ p}$$

- 21 1s per 1 oz is equiv. to £1 per 20 oz, or 25 fr per  $1\frac{1}{4}$  lb,

$$\text{or } \frac{25}{1\frac{1}{4}} \text{ fr per 1 lb, or } \left(\frac{25}{1\frac{1}{4}} \times 0.002204\right) \text{ fr per gram,}$$

$$\text{or } \left(\frac{25}{1\frac{1}{4}} \times 2.204\right) \text{ fr per Kg, or } 44.08 \text{ fr per Kg}$$

- 22 The company would pay £4000 in £20000, or  $\frac{1}{5}$  in £1 ,

the bank would receive  $\frac{1}{5}$  of 5000), i.e., £1000, and A would pay the bank £4000

A would also lose  $\frac{4}{5}$  of £4000, claimed by him

he is out of pocket to the extent of  $(£4000 + \frac{4}{5} \text{ of } £4000)$ , or £7200

After the case is taken into court, the company can pay £4000 in £18000, or  $\frac{2}{9}$  in £1 ,

A has to pay the bank  $\frac{2}{9}$  of 5000)

He has already lost £2000, owing to his claim being reduced, and on this he loses  $\frac{2}{9}$  of £2000 ,

he is out of pocket to the extent of

$$£(2000 + \frac{7}{8} \text{ of } 5000 + \frac{7}{8} \text{ of } 2000)$$

i.e.,  $£7444\frac{4}{8}$ ,

he is  $£(7444\frac{4}{8} - 7200)$  more out of pocket,

i.e., „  $£244\frac{4}{8}$  „ „

23.

High Tide

<i>Boulogne</i>	<i>London Bridge</i>	<i>Cork</i>
at 10 3 (Fr)	at $\begin{cases} 12\ 33\ (F_1) \\ 12\ 23\ (Eng) \end{cases}$	at $\begin{cases} 3\ 33\ (F_1) \\ 3\ 23\ (Eng) \\ 2\ 58\ (Irish) \end{cases}$

24

31 1 gms cost  $£4\ 18s\ 10d$ ,

21 6 gms „  $£4\ 9s\ 16d \times \frac{2\ 1c}{3\ 11}$

$$\begin{array}{r|l}
 4\ 9\ 11 & 66 \\
 \hline
 & 2\ 16 \\
 9\ 883 & 3 \\
 494 & 2 \\
 296 & 5 \\
 \hline
 3\ 11\ 10\ 674 & 0 \quad (£3\ 432) \\
 \hline
 1344 & \\
 \hline
 1000 & = £3\ 8s\ 8d \\
 \hline
 67 & \\
 \hline
 5 &
 \end{array}$$

25 Let  $x$  be the number of years req<sup>d</sup>. In  $x$  yrs he will have saved  $£(3000 + 300x)$ . After  $x$  yrs  $(90 - x - 30)$  yrs remain. During that time he spends  $£300$  per annum

$$3000 + 300x = 300(90 - x - 30),$$

whence  $x = 25$ . He must work till he is 55

26 From Example 2 (iii), p 227, we see

$$\begin{aligned}
 \text{time req}^d &= \frac{\text{sum of lengths in feet}}{\text{velocity of approach in ft per sec.}} \\
 &= \frac{(210 + 318) \times 3}{(25 - 17) \times \frac{88}{60}} = \frac{528 \times 3 \times 60}{8 \times 88} = 135 \text{ secs}
 \end{aligned}$$

27

1 metre sells for 0 3 fr ,

0 914 m „ (0 3  $\times$  0 914) fr

i.e. 1 yd „  $£\frac{0\ 3 \times 0\ 914}{25\ 34}$

500 yds „  $£\frac{0\ 3 \times 0\ 914 \times 500}{25\ 34}$ ,

or 500 yds „  $£\frac{1\ 7\ 1}{25\ 34}$ , or  $£5\ 410$ , or  $£5\ 8s\ 3d$ .

Now 500 yds cost 1000d, or  $£4\ 3s\ 4d$ ,

profit is  $£1\ 4s\ 11d$

28 With no season ticket

on 3 days of the week he pays  $1s\ 10d \times 3$ , or  $5s\ 6d$ ,

on 3 other days he pays  $2s \times 3$ , or  $6s$

Since he travels for 47 weeks it costs him  $11s\ 6d \times 47$ , or  $\pounds 27\ 0s\ 6d$

With a season ticket it costs him  $\pounds 20\ 8s + 3s \times 47$ ,

„ „ „  $\pounds 20\ 8s + \pounds 7\ 1s$ ,

„ „ „  $\pounds 27\ 9s$ ,

and answer is "No, loses  $8s\ 6d$ "

29 Charge = Rs  $\left( \frac{5}{100} \text{ of } 2000 + \frac{2\frac{1}{2}}{100} \text{ of } 15350 \right)$

= Rs 100 + Rs 383 75

= Rs 483 12 a

30 No of First class passengers =  $\frac{\pounds 356926}{2d} = 356926 \times 120$

„ Second class „ =  $\frac{\pounds 472793}{1\frac{1}{4}d} = 472793 \times 240 \times \frac{4}{5}$ ,

or  $472793 \times 48 \times 4$

If Second class were abolished, the receipts would be

$(356926 \times 120 + 472793 \times 48 \times 4) \times 1\frac{1}{2}d$ ,

or  $\pounds (356926 \times 120 + 472793 \times 48 \times 4) \times \frac{3}{2 \times 240}$ ,

or  $\frac{\pounds 356926 \times 120 \times 3}{2 \times 240} + \frac{\pounds 472793 \times 48 \times 4 \times 3}{2 \times 240}$ ,

or  $\pounds 10\ 07\ 8 + \pounds 283\ 07\ 8$ ,

or  $\pounds 835046\ 2s$

But the receipts originally were  $\pounds (356926 + 472793)$ , or  $\pounds 829719$

$\pounds 5327\ 2s$  is gained

31 Total weight of five bars =  $(2075 \pm 5) \times 5\text{ gm}$

=  $10377\ 5\text{ gm}$ , or  $10372\ 5\text{ gm}$

Also weight of four bars  $(3008 \pm 5) + (2092 \pm 5) + (2014 \pm 5)$

+  $(1093 \pm 5) = 8209\text{ gm}$ , or  $8205\text{ gm}$ ,

whence limits are  $2\ 1725\text{ Kg}$ , or  $2\ 1635\text{ Kg}$

32 Each year a population of 1000 become 1009 1,

2e, „ „ „ 1 „ 1 0091

Hence, if  $x$  be population 3 years hence,

$x = 77793 \times (1\ 0091)^3 = 77793 \times 1\ 0091 \times 1\ 0091 \times 1\ 0091$

=  $78500 \times 1\ 0091 \times 1\ 0091 = 79215 \times 1\ 0091$

=  $79925 = 79900$  to nearest hundred

- 33 Suppose I travel  $x$  miles by steamer,  
 then I travel  $(132-x)$  miles by train  
 The total cost is  $x \times \frac{1}{10}$  shillings +  $(132-x) \times \frac{1}{12}$  shillings

$$\frac{x}{20} + \frac{132-x}{12} = 8, \text{ whence } x = 90$$

- 34 During the 50 miles  $A$  rows 4 stretches, i.e., 24 mi in  $\frac{24}{5\frac{1}{2}}$  hrs

" 50 "  $B$  " 4 " 24 "  $\frac{24}{6}$  hrs

$A$  then rows the last 2 mi in  $\frac{2}{5\frac{1}{2}}$  hrs

$A$  and  $B$  row 50 miles in  $\left(\frac{24}{5\frac{1}{2}} + \frac{24}{6} + \frac{2}{5\frac{1}{2}}\right)$  hrs, or  $9\frac{2}{5}$  hrs

During the 50 miles  $C$  rows 22 mi in 4 hours,

" "  $D$  " 20 mi " 4 "

also  $C$  "  $5\frac{1}{2}$  mi " 1 " ,

and  $D$  "  $2\frac{1}{2}$  mi "  $\frac{1}{2}$  "

$C$  and  $D$  row 50 miles in  $9\frac{1}{2}$  hours,

$C$  and  $D$  win

After  $D$  has reached the winning post,

$A$  rows for  $(9\frac{2}{5} - 9\frac{1}{2})$  hrs or  $\frac{1}{10}$  hrs

$A$  rows  $\frac{1}{10} \times 5\frac{1}{2}$  miles, i.e.,  $\frac{3 \times 11 \times 17 \times 2}{110 \times 2}$  yds or 264 yds

- 35  $Q$  and  $R$  ride 2 mi in 12 min  $P$  walks 2 mi in 30 min ,  
 $R$  waits 18 min During this 18 min  $Q$  walks  $(18 \times \frac{4}{60})$  mi,  
 or  $1\frac{1}{2}$  mi

Let  $x$  miles be distance between meeting places

$Q$  takes  $\frac{x}{4}$  hours to walk this

$P$  and  $R$  take  $\frac{x}{10}$  hours to ride this

But  $R$  waited 18 min, or  $\frac{18}{60}$  hrs, between  $Q$ 's departure and  $P$ 's arrival,

$$\frac{x}{4} = \frac{x}{10} + \frac{18}{60}, \text{ whence } x = 2, \text{ giving 4 mi from start}$$

Time of reaching second meeting place

= time of riding 2 mi + time of walking 2 mi

= 12 min + 30 min = 42 min (from the start)

## PART II.

### EXAMPLES XIII a Page 245

$$\begin{array}{r} 13 \quad 196,00 (140 \\ 24 \overline{) 96} \end{array} \quad \begin{array}{r} 14 \quad 954,81 (309 \\ 609 \overline{) 5481} \end{array} \quad \begin{array}{r} 16 \quad 50,1264 (708 \\ 1408 \overline{) 11264} \end{array}$$

$$\begin{array}{r} 17 \quad 81,9025 (905 \\ 1805 \overline{) 9025} \end{array} \quad \begin{array}{r} 20 \quad 1370,4804 (3702 \\ 67 \overline{) 470} \\ 7402 \overline{) 14801} \end{array}$$

$$\begin{array}{r} 21 \quad 1,006009 (1003 \\ 2003 \overline{) 6009} \end{array} \quad \begin{array}{r} 24 \quad 1157,428441 (34021 \\ 64 \overline{) 257} \\ 6802 \overline{) 14284} \\ 68041 \overline{) 68041} \end{array}$$

$$\begin{array}{r} 26 \quad 174,3456,16 (132 \\ 23 \overline{) 74} \\ 262 \overline{) 534} \quad (1) \\ \quad \quad \quad 10 \quad (2) \end{array}$$

At stage (1) we have found the first two figures in the answer

This shows that

$$171345616 = (13000)^2 + 5345616$$

Similarly at stage (2) we see that

$$174345616 = (13200)^2 + 105616$$

$$\begin{array}{r} 27 \quad 18,4642,09 (4297 \\ 184041 \\ 8587 \overline{) 60109} \\ \quad \quad \quad 60109 \end{array}$$

### EXAMPLES XIII. b Page 246

$$\begin{array}{r} 12 \quad 0,00139876 (00374 \\ 67 \overline{) 498} \\ 744 \overline{) 2976} \\ \text{K H S I} \end{array}$$

$$\begin{array}{r} 13 \quad 9,5048,89 (3083 \\ 608 \overline{) 5048} \\ 6163 \overline{) 18489} \\ \text{F} \end{array}$$



15  $0.0001595169 (0.01263$

$$\begin{array}{r} 22 \overline{) 59} \\ 246 \overline{) 1551} \\ 2523 \overline{) 7569} \end{array}$$

17  $27.04, 41.60, 16 (52.004$

$$\begin{array}{r} 102 \overline{) 204} \\ 101001 \overline{) 416016} \end{array}$$

18  $676, 20.80, 16 (260.04$

$$\begin{array}{r} 46 \overline{) 276} \\ 52004 \overline{) 208016} \end{array}$$

20  $5.354596 (2.31$

$$\begin{array}{r} 13 \overline{) 135} \\ 461 \overline{) 615} \\ 18196 \end{array}$$

Hence req<sup>d</sup> answer = 0.018496

## EXAMPLES XIII c Page 249

11  $0.1440 (0.3704$

$$\begin{array}{r} 07 \overline{) 540} \\ 749 \overline{) 7100} \\ 359 \end{array}$$

12.  $2.35, 60 (15.3192$

$$\begin{array}{r} 25 \overline{) 135} \\ 303 \overline{) 1060} \\ 3061 \overline{) 16100} \\ 30650 \overline{) 241100} \\ 8199 \end{array}$$

15  $2.90 (17.0293$

$$\begin{array}{r} 3102 \overline{) 10000} \\ 31019 \overline{) 319600} \\ 13169 \end{array}$$

$$\begin{aligned} 21 \quad \sqrt{17\frac{2}{3}} &= \sqrt{\frac{11320}{811}} = \sqrt{\frac{9 \times 1181}{811}} \\ &= \frac{3 \times 59}{29} \end{aligned}$$

28  $\sqrt{4\frac{1}{3}} = \sqrt{\frac{55}{13}} = \sqrt{\frac{715}{13}} = 1\frac{1}{3} \sqrt{715}$

29  $\sqrt{1\frac{1}{3}} = \sqrt{\frac{18}{17}} = \sqrt{\frac{1008}{17}} = 1\frac{1}{3} \sqrt{1008}$

31  $\sqrt{\frac{1}{4}} = \sqrt{25}$

32  $\sqrt{\frac{25}{0.16}} = \frac{\sqrt{250}}{1}$

33  $0.0565 (0.237$

$$\begin{array}{r} 43 \overline{) 165} \\ 467 \overline{) 3600} \\ 47.1 \overline{) 33169} \\ 47 \end{array}$$

34  $0.0360 (0.1897$

$$\begin{array}{r} 28 \overline{) 260} \\ 369 \overline{) 3600} \\ 3787 \overline{) 27000} \\ 379.1 \overline{) 139136} \\ 253 \end{array}$$

$$35 \quad \sqrt{1206} = 0,368421 (0606$$

$$\begin{array}{r} 1206 \overline{) 8421} \\ 1212 \overline{) 1185} (98 \\ \quad 94 \end{array}$$

$$36 \quad 3\sqrt[5]{27} = 3,068965 (1751$$

$$\begin{array}{r} 27 \overline{) 206} \\ 345 \overline{) 1789} \\ 3501 \overline{) 6465} \\ 3502 \overline{) 2964} (846 \\ \quad 162 \\ \quad \quad 22 \end{array}$$

## EXAMPLES XIII d Page 251

$$NB \quad \sqrt{2} = 1.4142136, \quad \sqrt{3} = 1.7320508,$$

$$\sqrt{5} = 2.2360680, \quad \sqrt{6} = 2.4494897,$$

$$\sqrt{7} = 2.6457513, \quad \sqrt{11} = 3.3166248$$

$$1 \quad \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$3 \quad \frac{2}{\sqrt{8}} = \frac{2}{\sqrt{8}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$6 \quad \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$8 \quad \frac{25}{\sqrt{11}} = \frac{100}{4\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}} = \frac{100\sqrt{11}}{4 \times 11} = \frac{331.66248}{4 \times 11}$$

$$9 \quad \frac{1}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{6}$$

$$10 \quad \frac{1}{\sqrt{24}} = \frac{1}{\sqrt{24}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$11 \quad \text{Fract}^n = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$12 \quad \text{Fract}^n = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2}$$

$$13 \quad \text{Fract}^n = \frac{4}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{4(\sqrt{5}+1)}{5-1} = \sqrt{5}+1$$

$$14 \quad \text{Fract}^n = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3-2\sqrt{3}+1}{3-1} = 2-\sqrt{3}$$

$$15 \quad \text{Fract}^n = \frac{24}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} = \frac{24(7+\sqrt{5})}{49-5} = \frac{6(7+\sqrt{5})}{11}$$

$$16 \quad \text{Fract}^n = \frac{\sqrt{5}+3}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{5+5\sqrt{5}+6}{5-4} = 11+5\sqrt{5}$$

$$17 \quad 33,17,76 (576 = 24^2$$

$$\begin{array}{r} 107 \overline{) 817} \\ 1146 \overline{) 6876} \end{array}$$

$$18 \quad 108,24,32,16 (10404 = 102^2$$

$$\begin{array}{r} 46 \overline{) 824} \\ 5201 \overline{) 83216} \end{array}$$

$$19 \quad 0,00,00,06,76,52,01 (0\ 002601) = (0\ 051)^2$$

$$\begin{array}{r} 46 \overline{) 2\ 76} \\ 5201 \overline{) 52\ 01} \end{array}$$

$$20 \quad 53\ 6860 (7\ 32$$

$$\begin{array}{r} 143 \overline{) 4\ 68} \\ 1462 \overline{) 3960} \\ 1464 \overline{) 10360 (70} \\ \quad 112 \end{array}$$

$$7\ 3270 (2\ 706$$

$$\begin{array}{r} 47 \overline{) 3\ 32} \\ 540 \overline{) 370} \end{array} \quad 4^{\text{th}} \text{ root is } 2\ 71$$

$$21 \quad 17,23,58 (41\ 51$$

$$\begin{array}{r} 81 \overline{) 1\ 23} \\ 825 \overline{) 42\ 58} \\ 8301 \overline{) 1\ 3300} \\ 8302 \overline{) 4999 (60} \\ \quad 18 \end{array}$$

$$41,51,60 (6\ 443$$

$$\begin{array}{r} 124 \overline{) 5\ 51} \\ 1284 \overline{) 55\ 60} \\ 1288 \overline{) 4\ 24} \end{array}$$

### EXAMPLES XIII e Page 253

$$1 \quad \text{Diagonal} = \sqrt{(161^2 + 240^2)} \text{ yds} = \sqrt{(25921 + 57600)} \text{ yds} = 289 \text{ yds}$$

$$2 \quad \text{Side} = \sqrt{137^2 - 88^2} \text{ ft} = \sqrt{(137+88)(137-88)} \text{ ft} = \sqrt{225 \times 49} \text{ ft} \\ = (15 \times 7) \text{ ft}$$

$$3 \quad \text{Side} = \sqrt{192000 - 156} \text{ cubits} = 438 \text{ cubits}$$

$$4 \quad \text{Side} = \sqrt{439 \text{ ac } 33 \text{ p}} = \sqrt{2125758\ 25} \text{ yds} = 1457\ 9 \text{ yds}$$

$$5 \quad \text{Side} = \sqrt{450 \times 577} \text{ m} = \sqrt{259650} \text{ m} = 509\ 5 \text{ m}$$

$$\text{Diagonal} = \sqrt{259650 \times 2} \text{ m} = \sqrt{519300} \text{ m} = 720\ 6 \text{ m}$$

$$6 \quad (i) \text{ We get } x^2 = 54 \times 1944, \text{ or } x^2 = (9 \times 6) \times (6 \times 9 \times 36), \\ x^2 = 9^2 \times 6^2 \times 6^2, \text{ and } x = 9 \times 6 \times 6$$

$$(ii) \text{ We get } x^2 = 65 \times 1625, \text{ or } x^2 = (5 \times 13) \times (13 \times 125), \\ x^2 = 13^2 \times 5^4, \text{ and } x = 13 \times 5^2$$

$$(iii) \text{ We get } x^2 = 16\ 92 \times 23\ 03 = \frac{1692 \times 2303}{100 \times 100} = \frac{9 \times 4 \times 47 \times 7 \times 7 \times 47}{100 \times 100} \\ = \frac{3^2 \times 2^2 \times 47^2 \times 7^2}{100^2}, \quad x = \frac{3 \times 2 \times 47 \times 7}{100} = 19\ 74.$$

$$7 \quad 2 \text{ sq big } 5 \text{ sq cot} = 3600 \text{ sq yds}, \quad \text{side} = 60 \text{ yds}$$

$$\text{cost} = (2\text{ a } 3\text{ p}) \times 60 \times 4 = (2\text{ a } + 3\text{ p}) \times 240 = \text{Rs } \frac{240}{8} + 60\text{ a} \\ = \text{Rs } 33\ 12\text{ a}$$

- 8 Area = 160 ac =  $\frac{640}{4}$  ac =  $\frac{1}{4}$  sq mi, side =  $\frac{1}{2}$  mile,  
whence perimeter = 2 miles, and time req<sup>d</sup> = 30 min
- 9 Let  $x$  ft and  $3x$  ft be its breadth and length respectively,  
then  $3x \times x = (17 \times 320 - 10) \frac{9}{4} = 12217.5$ ,  
whence  $x^2 = 4072.5$  and  $x = 63.8$
- 10 No of acres =  $\frac{\text{£}987}{\text{£}175 \frac{11}{16}} = \frac{90}{16}$ ,  
No of sq yds =  $\frac{90 \times 4840}{16} = \frac{90 \times 100 \times 484}{16}$ ,  
side =  $\frac{3 \times 10 \times 22}{4}$  yds, whence perimeter = 660 yds, and cost  
= £104 10s
- 11 No of slabs =  $\frac{\text{Rs } 20\frac{1}{4}}{\text{Rs } \frac{9}{14}} = 144$ , area =  $144 \times 18^2$  sq in, and side  
= 18 ft
- 12 10 ac = 100 sq chains side = 10 chains  
2 sides + 2 diagonals =  $(20 + 20\sqrt{2})$  chains or  $20(1 + \sqrt{2})$  chains  
or  $440(1 + \sqrt{2})$  yds  
Now 3 mi in 1 hr = 88 yds in 1 min  
time taken =  $\frac{440(1 + \sqrt{2})}{88}$  mins =  $5(1 + 1.414)$  mins  
= 12 mins nearly
- 13 Area =  $\frac{1}{2} \times 150 \times 88$  sq yds = 6600 sq yds,  
side of sq =  $\sqrt{6600}$  yds = 81.24 yds
- 14 (i)  $A = -r^2 = 31416 \times 100$  sq cm = 31416 sq cm  
(ii) side of sq =  $\sqrt{31416}$  cm = 177.2 cm
- 15 (i) Area =  $\frac{1}{4} \sqrt{28 \times 6 \times 10 \times 124} = \frac{1}{4} \sqrt{4 \times 7 \times 144 \times 5} = \frac{1}{4} \times 2 \times 12 \times \sqrt{35}$   
=  $6\sqrt{35} = 6 \times 5.916 = 35.49$  sq in  
(ii) Area =  $\frac{1}{4} \sqrt{30 \times 10 \times 10 \times 10} = \frac{1}{4} \sqrt{100 \times 3} = \frac{100}{4} \sqrt{3} = \frac{173.20}{4}$  sq in
- 16  $r = \sqrt{\frac{100}{31416}} = \sqrt{318309} = 5.64$  cm
- 17 We have  $8 \times d^2 \times 0.7854 = 10000$ ,  
 $d^2 = \frac{10000}{8 \times 0.7854} = \frac{1591.5457}{0.7854} = 1591.5457$  sq cm,  
whence  $d = 39.89$  cm
- 18 Velocity =  $332.4 \times \sqrt{1 + 0.00366 \times 20} = 332.4 \times \sqrt{1.0732}$   
=  $332.4 \times 1.03595 = 343.9$  m per sec.

## EXAMPLES XIV a Page 257

$$18 \text{ Cost} = (6\frac{1}{3} \times 4\frac{2}{3} \times 9) a = (19 \times 14) a = \text{Rs } 16 \text{ } 10a$$

$$19 \text{ Cost} = \left( \frac{26\frac{2}{3} \times 20\frac{1}{4}}{9} \times 18 \right) d = (20 \times 27 \times 2) d = \text{£}4 \text{ } 10s$$

$$20 \text{ Cost} = (7 \text{ } 2 \times 5 \text{ } 5 \times 0 \text{ } 75) \text{ Rs} = 29 \text{ Rs } 70 \text{ cents}$$

$$21 \text{ Cost} = (8 \text{ } 5 \times 4 \text{ } 6 \times 9) d = 351 \text{ } 9d = \text{£}1 \text{ } 9s \text{ } 4d$$

$$22 \text{ Cost} = \frac{40}{7\frac{1}{2} \times 5\frac{1}{2}} a = 1 a \qquad 23 \text{ Cost} = \frac{12 \times 16}{21\frac{1}{2} \times 20\frac{1}{2}} a \times 9 = 4 a$$

$$24 \text{ Area} = (20 \times 8 \text{ } 25) \text{ sq ch} = 165 \text{ sq ch} = 16\frac{1}{2} \text{ ac}$$

$$26 \text{ Area} = (3 \text{ } 125)^2 \text{ sq big} = 9 \text{ } 765625 \text{ sq big}$$

$$= 9 \text{ sq big } 15 \text{ sq cot } 5 \text{ sq chh}$$

$$27 \text{ Area} = \left( \frac{8}{4} \right)^2 \text{ sq ch} = (21 \text{ } 75)^2 \text{ sq ch} = 473 \text{ } 0625 \text{ sq ch}$$

$$= 47 \text{ } 30625 \text{ ac} = 47 \text{ ac } 12 \text{ gun } 4 a$$

$$28 \text{ Area} = 66 \text{ sq big} \quad \text{Length} = 19\frac{4}{5} \text{ big}$$

$$\text{req'd breadth} = \frac{66}{19\frac{4}{5}} \text{ big} = 3 \text{ big } 6 \text{ cot } 4 \text{ ft}$$

$$29 \text{ Area} = 37 \text{ } 6607 \text{ ac} = 376 \text{ } 607 \text{ sq ch} \quad \text{Breadth} = 16 \text{ } 06 \text{ ch}$$

$$\text{req'd length} = \frac{376 \text{ } 607}{16 \text{ } 06} \text{ ch} = 23 \text{ } 45 \text{ ch}$$

$$30 \text{ Area} = \left( \frac{3}{2} \right)^2 \text{ sq big} = 1\frac{9}{4} \text{ sq big},$$

$$\text{cost} = \text{Rs } 10\frac{1}{2} \times 1\frac{9}{4} = \text{Rs } 16 \text{ } 6a \text{ } 6p$$

$$31 \text{ (i) Area} = (805 \times 74) \text{ sq m} = \frac{805 \times 74}{10000} \text{ Ha} = 5 \text{ } 957 \text{ Ha}$$

$$\text{(ii) Area} = 5 \text{ } 957 \text{ Ha} = (5 \text{ } 957 \times 2 \text{ } 5) \text{ ac} = 14 \text{ } 8925 \text{ ac}$$

$$32 \text{ Cost} = \text{£}12\frac{1}{2} \times \frac{2 \text{ } 000 \times 8 \text{ } 5}{10000} = \text{£}21\frac{1}{4}$$

For second part of question we have  $2\frac{1}{2}$  ac cost  $\text{£}12 \text{ } 10s$

$$33 \text{ Area} = \frac{8 \text{ } 5}{50} \text{ big} = 16 \text{ } 5 \text{ big}, \quad \text{breadth} = \frac{1 \text{ } 0}{7 \text{ } 5} \text{ big} = 2 \text{ big } 4 \text{ cot}$$

$$34 \text{ Area} = \left( \frac{5}{8} \text{ of } 26 \text{ } 16 \times 26 \text{ } 16 \right) \text{ sq ch}$$

$$= 570 \text{ } 288 \text{ sq ch},$$

$$\text{cost} = \text{£}2 \text{ } 12s \text{ } 6d \times 57 \text{ } 0288$$

$$= \text{£}2 \text{ } 616666 \times 57 \text{ } 0288$$

$$= \text{£}149 \text{ } 225$$

57 028	8
2 616666	
114 037	6
34 217	3
570	3
342	1
34	2
3	4
	3
149 225	2

$$35 \text{ No req'd} = \frac{24\frac{1}{2} \times 15\frac{3}{4}}{\frac{3}{4} \times \frac{1}{2}} = 1029$$

$$36 \text{ Cost} = \frac{9}{20} a \times \frac{20^2 \times 144}{8 \times 4} = 810 a$$

37 Width = 16 ft 3 in = 195 in,      length =  $\frac{1638 \times 7 \times 5}{195}$  in = 294 in

38 If width is  $r$  in, then, expressing area in sq in, we have  
 $288 \times 18 \times r = 30 \times 18 \times 144$ , whence  $r = 15$

39 On trial it will be found that length of tile must be parallel to width of hall. If  $r$  be no. of tiles reqd,  
 then  $r \times 4\frac{1}{2} \times 3\frac{3}{4} = (38 \text{ ft } 1\frac{1}{2} \text{ in}) \times 12 \times (20 \text{ ft } 2\frac{1}{2} \text{ in}) \times 12$ ,  
 whence  $r = \frac{157\frac{1}{2} \times 242\frac{1}{2}}{3\frac{3}{4} \times 4\frac{1}{2}} = 122 \times 57$

$$\text{cost of tiles per doz} = \frac{\text{£}193\frac{1}{2} \times 12}{122 \times 57} = \text{£}1$$

40 9 in width divides 30 ft 9 in exactly 11 times,  
 and  $(61 \text{ ft } 6 \text{ in}) - 15 = 51\frac{1}{2}$ ,  
 $(41 \times 52)$  turfs are wanted, of which 41 must be divided

41. No reqd =  $\frac{10\frac{1}{2} \times 24 \times 111}{16 \times 16} = 519$  turfs

We have  $\frac{24 \times 12}{16} = 18$ , and  $\frac{10\frac{1}{2} \times 12}{16} = 30\frac{1}{2}$ ,

showing that 16 in divides a width of 24 ft exactly,  
 we have  $18 \times 30\frac{1}{2} = 18 \times 30 + 18 \times \frac{1}{2} = 540 + 9$ ,  
 showing 9 must be divided

42 Area =  $(34 \times 28 - 30 \times 24)$  sq yds

43 Cost =  $1\frac{1}{2}$  n.  $\times (25 \times 18 - 21 \times 14)$

44 Area of footpath =  $(83 \times 47 - 75 \times 39)$  sq ft = 976 sq ft  
 $= 976 \times 144$  sq in

Area of one tile = 36 sq in,      reqd no =  $\frac{216 \times 144}{36} = 3904$

45 Area of each bed =  $(26 \times 27)$  sq ft

46 Cost for footway =  $\frac{\text{Rs } 2\frac{5}{8} \times 67\frac{1}{2} \times 5\frac{1}{2}}{9}$ ,

cost for remainder =  $\text{Rs } 2\frac{1}{4} \times \frac{67\frac{1}{2} \times 37\frac{1}{2}}{9}$

total cost =  $\text{Rs } \frac{67\frac{1}{2}}{9} \left( \frac{21}{8} \times \frac{11}{2} + \frac{9}{1} \times \frac{149}{4} \right) = \text{Rs } \frac{15}{2} \times \frac{393}{4} = \text{Rs } 736 \frac{1}{4}$  n

47 Cost of carpet =  $\frac{28 \times 2\frac{1}{2} \times 1}{4} \text{ n} = 1232$  n

Cost of stain =  $\left( \frac{10 \times 1\frac{1}{2} \times 28 \times 2}{9} \right) \times \frac{1}{1} \text{ n} = 41$  n

total cost = 1273 n = Rs 79 9 n

$$48 \quad \text{Area of tiled portion} = (48 \times 36 - 36 \times 24) \text{ sq ft} = 864 \text{ sq ft} \\ = 864 \times 144 \text{ sq in}$$

$$\text{Area of one tile} = (9 \times 8) \text{ sq in ,}$$

$$\text{req'd no of tiles} = \frac{864 \times 144}{9 \times 8} = 1728$$

$$\text{Area of boarded portion} = (36 \times 24) \text{ sq ft} = (36 \times 24 \times 144) \text{ sq in}$$

$$\text{Area of one board} = (72 \times 6) \text{ sq in ,}$$

$$\text{req'd no of boards} = \frac{36 \times 24 \times 144}{72 \times 6} = 288$$

49 Since corridor is 2 yds wide and surrounds the court,

its length = 54 yds and its breadth = 34 yds

Since each path is 2 yds wide and is within the court,

it leaves 48 yds for *total length* and 28 yds for *total breadth* of "remainder",

$$\text{cost of grass} = (3 \times 48 \times 28)s = £201 \ 12s$$

Also cost of paths and corridor, at 15s per sq yd ,

$$= (54 \times 34 - 48 \times 28) \times 15s = £(1836 - 1344) \times \frac{1}{4} = £369$$

$$\text{total cost} = £570 \ 12s$$

#### EXAMPLES XIV b Page 263

$$1 \quad \text{No of yds} = \frac{20 \times 15}{2\frac{1}{2} \times 3}$$

$$2 \quad \text{Length} = \frac{17 \times 11\frac{1}{2}}{2\frac{1}{4} \times 3} \text{ yds}$$

$$3 \quad \text{Width} = \frac{29\frac{1}{2} \times \frac{1}{2} \times 9}{14\frac{2}{3}} \text{ ft}$$

$$4 \quad \text{Cost} = \text{Rs } \frac{9}{2} \times \frac{25 \times 18}{2\frac{1}{4} \times 3}$$

$$5 \quad \text{No of strips} = \frac{16}{2\frac{1}{4}} = 7\frac{1}{4} ,$$

8 strips are required

$$\text{Cost} = £\frac{1}{6} \times \frac{8 \times 20\frac{1}{2}}{3}$$

Also  $\frac{8}{9}$  of the last strip will be wasted

$$6 \quad \text{No of strips} = \frac{12}{2\frac{1}{2}} = 4\frac{4}{5} ,$$

5 strips are required

$$\text{Cost} = \text{Rs } 2\frac{1}{2} \times \frac{5 \times 15}{3} \quad \text{Waste} = \frac{1}{5} \text{ strip} = \frac{1}{5} \text{ of } (15 \times 2\frac{1}{2}) \text{ sq ft}$$

$$7 \quad \text{No of strips} = \frac{12\frac{3}{4}}{2\frac{1}{4}} = 5\frac{3}{4} , \text{ i.e. } 6 ,$$

$$\text{length} = 6 \times 18 \text{ ft} = 36 \text{ yds}$$

$$\text{Cost} = 5\frac{1}{2}s \times 36 = £9 \ 18s$$

$$\text{Waste} = \frac{1}{3} \text{ of } (18 \times 2\frac{1}{4}) \text{ sq ft}$$

$$8 \quad (i) \text{ Area} = 2 \times 12\frac{3}{4} \times \{16\frac{3}{4} + 15\frac{1}{4}\} \text{ sq ft} = 25\frac{1}{2} \times 32 \text{ sq ft} = 816 \text{ sq ft}$$

$$(ii) \text{ Area} = 2 \times 15\frac{1}{4} \times \{20\frac{1}{4} + 16\frac{1}{4}\} \text{ sq ft} = 31\frac{1}{2} \times 36\frac{1}{2} \text{ sq ft} \\ = 1155 \text{ sq ft}$$

$$9 \quad \text{Cost} = \text{Rs } \frac{18}{16} \times \frac{2 \times 10 \times \{12\frac{3}{4} + 11\frac{1}{4}\}}{9} = \text{Rs } \frac{18 \times 2 \times 10 \times 24}{16 \times 9} = \text{Rs } 60$$

$$10 \quad \left. \begin{aligned} \text{Cost of wall} &= \frac{2 \times 14 \times (15 + 12)}{9} \times 9 \text{ a} = \text{Rs } 47 \text{ 4 a}, \\ \text{cost of ceiling} &= \frac{15 \times 12 \times 4}{9} \text{ a} = \text{Rs } 5 \text{ 10 a}, \end{aligned} \right\} \text{ giving Rs } 52 \text{ 14 a}$$

$$11 \quad (i) \text{ Surface} = 2(6 \times 4 + 6 \times 3\frac{1}{2} + 4 \times 3\frac{1}{2}) \text{ sq ft}$$

$$(ii) \text{ Surface} = 2(3\frac{1}{2} \times 2\frac{1}{2} + 3\frac{1}{2} \times 1\frac{1}{4} + 2\frac{1}{2} \times 1\frac{1}{4}) \text{ sq ft} \\ = (4\frac{9}{8} + 1\frac{5}{4} + 3\frac{5}{8}) \text{ sq ft}$$

$$12 \quad \text{Cost} = \frac{2 \times (9 \times 4\frac{1}{2} + 9 \times 3 + 4\frac{1}{2} \times 3)}{9} \times 2 \text{ a} = 2(4\frac{1}{2} + 3 + \frac{1}{2} \times 3) \times 2 \text{ a} \\ \text{(cancelling 9)}$$

$$13 \quad \left. \begin{aligned} \text{Area of walls} &= 2(18\frac{1}{4} + 11\frac{1}{4}) \times 13\frac{1}{2} = 810 \text{ sq ft}, \\ \text{area of each piece} &= 12 \times 3 \times \frac{3}{4} = 81 \text{ sq ft}, \end{aligned} \right\} \text{ giving 10 pieces}$$

$$14 \quad \begin{aligned} \text{Area of walls} &= 2(16\frac{1}{2} + 13\frac{1}{2}) \times 11\frac{1}{4} = 675 \text{ sq ft} \\ \text{area to be papered} &= 620 \text{ sq ft and area of piece} \\ &= 12 \times 3 \times \frac{5}{4} = 63 \text{ sq ft} \end{aligned}$$

Hence no of pieces =  $\frac{620}{63} + 2 = 9\frac{5}{7} + 2$ , i.e. 12 pieces are required

$$15 \quad \begin{aligned} \text{Area to be papered} &= (4 \times 18 \times 11 - 90) = 702 \text{ sq ft} \\ \text{Area of each piece} &= 2\frac{1}{4} \times 12 \times 3 = 81 \text{ sq ft} \\ \text{No of pieces} &= \frac{702}{81} + 1 = 9\frac{2}{3}, \text{ i.e. } 10, \text{ and cost} = \text{£}3 \end{aligned}$$

$$16 \quad \text{Cost} = [2 \times (16\frac{1}{4} + 10\frac{1}{4}) \times 11 - 7 \times 3 - 2 \times 4 \times 5 - 4 \times 3\frac{1}{2}] \times \frac{1}{6} \times 9d \\ = (594 - 21 - 40 - 14)d = \text{£}2 \text{ 3s } 3d$$

$$17 \quad \begin{aligned} \text{Let } x \text{ ft be the height, then area to be papered} \\ &= 2 \times 30 \times x = 60x \text{ sq ft} \\ \text{Area of each piece} &= 2\frac{1}{2} \times 12 \times 3 = 90 \text{ sq ft, costing 5s,} \\ &35\text{s buys 630 sq ft, } 60x = 630, \text{ whence } x = 10\frac{1}{2} \end{aligned}$$

$$18 \quad \text{No reqd} = 2(5\frac{1}{4} \times 1\frac{5}{8} + 3 \times 1\frac{5}{8}) + 5\frac{1}{4} \times 3 = (2 \times \frac{1}{8} \times \frac{5}{4} + \frac{5}{4} \times 3) \text{ sq ft}$$

$$19 \quad \text{Cost} = \frac{2(3\frac{3}{4} \times 3\frac{1}{4} + 3\frac{3}{4} \times 3 + 3\frac{1}{4} \times 3)}{9} \times 4 \text{ a} = \frac{2 \times (\frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times 3 + \frac{1}{4} \times 3)}{9} \times 4 \text{ a} \\ = \frac{2 \times \frac{1}{4} \times \frac{7}{4}}{9} \text{ a} = \text{Re } 1 \text{ 14 a}$$



$$20 \text{ No of pieces} = \frac{21 \text{ } 8 \text{ } 5 \times 10 \text{ } 4 \times 2}{8 \times 6 \text{ } 4 \text{ } 5} = 126 \frac{1}{4} \text{, i.e. } 127$$

$$\text{Hence cost} = (127 \times 4 \text{ } 35) \text{ Rs} = 552 \text{ } 45 \text{ Rs}$$

$$21 \text{ Total area would cost } 2 \times (19 \frac{1}{2} + 14 \frac{3}{4}) \times 11 \frac{1}{4} \times 8 \text{ } d \\ = 2 \times 8 \times 34 \frac{1}{4} \times 11 \frac{1}{4} \text{ } d = 6165 \text{ } d$$

$$\text{Again } £21 \text{ } 6 \text{ } s \text{ } 5 \text{ } d = 5117 \text{ } d \text{,}$$

$$1 \text{ eqd no of sq ft} = \frac{6165 - 5117}{9} = 131$$

### EXAMPLES XIV c Page 265

$$1 \text{ (i) Area} = \frac{8 \text{ } 25 \times 6 \times 15 \text{ } 50}{144} = \frac{42 \text{ } 5 - 5}{6} \text{ sq ft} = 533 \text{ sq ft}$$

$$(ii) \text{ Cost} = (8 \text{ } 25 \times 6 \times 1 \frac{1}{2}) \text{ } d = 74 \text{ } 25 \text{ } d = 6 \text{ } s \text{ } 2 \frac{1}{4} \text{ } d$$

$$(iii) \text{ Cost} = 1 \text{ } d \times \frac{5 \text{ } 3 \text{ } 1}{9} = 4 \text{ } s \text{ } 11 \frac{1}{4} \text{ } d \text{ (nearly),}$$

$$6 \text{ } s \text{ } 2 \frac{1}{4} \text{ } d - 4 \text{ } s \text{ } 11 \frac{1}{4} \text{ } d, \text{ or } 1 \text{ } s \text{ } 3 \text{ } d, \text{ is saved}$$

$$2 \text{ (i) Breadth} = \frac{2 \text{ } 35 \text{ } 84}{16 \text{ } 5} = 14 \text{ } 08 \text{ m}$$

$$(ii) \text{ Breadth} = 39 \text{ } 375 \times 1408 = 46 \text{ ft } 2 \text{ in}$$

$$3 \text{ Width} = \frac{9200}{1000} \text{ m} = \frac{9 \text{ } 2 \times 39 \frac{3}{4}}{36} \text{ yds} = 10 \text{ } 1 \text{ yds}$$

$$4 \text{ Cost} = £2 \text{ } 5 \times \frac{4 \text{ } 2 \text{ } 35 \times 38 \text{ } 00}{10000} \times 2 \frac{1}{2} = £(16 \text{ } 093 \times 6 \text{ } 25) = £100 \text{ } 581 \text{ } 25$$

$$5 \text{ Area to be painted } 2 \times 4 \text{ } 5(5 \text{ } 65 + 4 \text{ } 35) - 18 = 9 \times 10 - 18 = 72 \text{ sq m}$$

$$\text{Cost} = \frac{72 \times 15 \text{ } 50}{144 \times 9} \times \frac{9}{2} \text{ } d = 387 \text{ } 5 \text{ } d = £1 \text{ } 12 \text{ } s \text{ } 3 \frac{1}{2} \text{ } d$$

$$6 \text{ } 1 \text{ sq in} = 1 \text{ sq m, } 2 \text{ } 44 \text{ sq in} = (2 \text{ } 44 \times 640) \text{ ac.} = 1561 \text{ } 6 \text{ ac}$$

$$7 \text{ } 640 \text{ ac} = (1 \text{ } 6)^2 \text{ sq cm, } 2000 \text{ Ha} = 2000 \times 2 \frac{1}{2} \times \frac{1 \text{ } 6 \times 1 \text{ } 6}{640} = 20 \text{ sq cm}$$

$$8 \text{ } 17 \frac{1}{2} \text{ ft} = 210 \text{ in, } 15 \frac{3}{4} \text{ ft} = 189 \text{ in. Of these HCF} = 21 \text{ in}$$

Since the factors of 21 are 1, 3, 7, 21, we see that the areas of the square tiles must be  $1^2, 3^2, 7^2, 21^2$  sq in respectively

Thus no of ways = 4, and the different numbers of tiles are

$$\frac{210 \times 189}{1 \times 1}, \frac{210 \times 189}{3 \times 3}, \frac{210 \times 189}{7 \times 7}, \frac{210 \times 189}{21 \times 21}$$

$$9 \text{ } 20 \frac{1}{4} \times 15 \text{ sq ft cost Rs } 16 \frac{7}{4},$$

$$(20 \frac{1}{4} + 15) \times 10 \frac{1}{2} \times 2 \text{ sq ft cost Rs } \frac{16 \frac{7}{4} \times 35 \frac{1}{4} \times 10 \frac{1}{2} \times 2}{20 \frac{1}{4} \times 15} = \text{Rs } 41 \text{ } 2 \text{ a}$$

$$10 \text{ Area} = \{40 \times 8 + (40 + 8) \times 10 \times 2 - 2 \times 7 \times 4 - 5 \frac{1}{2} \times 2\} = 1213 \text{ sq ft}$$

$$\text{and cost} = \frac{1}{2} \text{ a} \times \frac{1 \text{ } 2 \text{ } 1}{9} = \text{Rs } 37 \text{ } 14 \text{ } 6 \text{ p}$$

$$11 \text{ Area} = \frac{1}{2} \times 25 \times 3 \text{ } 21 \text{ sq ch} = 40 \text{ } 125 \text{ sq ch}$$

$$12 \text{ Height} = \frac{90 \ 61}{\frac{1}{2} \times 15 \ 45} \text{ m} = 11 \ 73 \text{ m}$$

$$13 \text{ Area} = [\frac{1}{2} \times (34 \ 72 + 27 \ 08) \times 20 \ 5] \text{ sq m} = 633 \text{ sq m}$$

$$14 \ (\frac{1}{2} \times [13 \ 46 + 11 \ 54] \times 6 \ 2) \text{ sq ch cost } \pounds 38 \ 75, \text{ or } (25 \times 31) \text{ sq ch cost } \pounds 387 \ 5,$$

$$10 \text{ sq ch cost } \frac{\pounds 387 \ 5 \times 10}{25 \times 31}, \text{ or } 1 \text{ ac costs } \pounds 5$$

$$15 \text{ (i) } A = (3 \ 6)^2 \times 0 \ 7854 = 12 \ 96 \times 0 \ 7854 = 10 \ 18 \text{ sq m}$$

$$\text{(ii) Cost} = \frac{(43)^2 \times 0 \ 7854 \times 4}{9} \text{ a} = \frac{1849 \times 3 \ 1416}{9} \text{ a} = \text{Rs } 40 \ 5 \text{ a } 5 \text{ p}$$

$$16 \text{ Area} = 0 \ 7854 \times (54^2 - 46^2) = 0 \ 7854 \times (54 + 46) \times (54 - 46) \\ = 0 \ 7854 \times 800 \text{ sq ft}$$

$$17 \text{ Area} = 0 \ 7854 \times (210^2 - 190^2) = 0 \ 7854 \times (210 + 190)(210 - 190) \text{ sq ft} \\ = \frac{0 \ 7854 \times 8000}{9} \text{ sq yds}$$

$$18 \text{ Total cost} = \text{cost of rectangle} + \text{cost of semi-circular verandah} \\ = \left[ \frac{24 \frac{1}{2} \times 13 \frac{1}{2} \times 40}{9} + \frac{(13 \frac{1}{2})^2 \times 0 \ 7854 \times 48}{2 \times 9} \right] \text{ a} \\ = (1485 + 381 \ 7044) \text{ a} = \text{Rs } 116 \ 10 \text{ a } 9 \text{ p}$$

$$19 \text{ Area} = (110 \times 55) \text{ yds} = \frac{1 \ 100 \times 5 \ 500}{40} \text{ ac}$$

$$\text{Cost} = 256d \times \frac{1 \ 100 \times 5 \ 500}{40} \times 3 \frac{1}{2} = 1120d = \pounds 4 \ 13s \ 4d$$

$$20 \text{ Side} = \sqrt{1600 \times 2 \frac{1}{4}} = \sqrt{400 \times 9} = 20 \times 3 = 60 \text{ yds}$$

$$\text{Area of path} = (60^2 - 54^2) = (60 + 54)(60 - 54) = 114 \times 6 \text{ sq yds}$$

$$\text{Cost} = \text{Rs } \frac{114 \times 6 \times 4}{10} = \text{Rs } 171$$

$$21. \ 100 \text{ sq m} = 1 \text{ sq in}, \quad 640 \times 100 \text{ ac} = 1 \text{ sq in}, \\ \text{whence } 931000 \text{ ac} = \frac{9 \ 31}{4} \text{ sq in, or } 14 \ 5 \text{ sq in}$$

$$22 \text{ Total area} = 3 \times 2 \ 3, \text{ i.e. } 6 \ 9 \text{ sq cm}$$

$$\text{Unshaded area} = (4 \times \frac{3-1}{2} \times \frac{3-1}{2}), \text{ or } 2 \ 6 \text{ sq cm}$$

$$\text{and} \quad = (2 \ 6 \times 25) \text{ sq ft},$$

$$\text{shaded area} = 4 \ 3 \text{ sq cm and } = (4 \ 3 \times 25) \text{ sq ft}$$

$$23 \text{ If sides are } x \text{ yds and } 2x \text{ yds, then diagonal} = x \times \sqrt{5} \text{ yds}$$

$$\text{But} \quad 2x^2 = 4840 \text{ and } x^2 = 484 \times 5,$$

$$5x^2 = 484 \times 25, \text{ whence } x\sqrt{5} = 22 \times 5 = 110 \text{ yds}$$

- 24 Let breadth =  $x$  yds and length  $2x$  yds ,  
 then  $2x^2 = 2244 \frac{5}{8}$  , whence  $x = 33 \frac{5}{8}$   
 Perimeter =  $6x = 6 \times 33 \frac{5}{8}$  ,  
 cost =  $(6 \times 33 \frac{5}{8} \times 13 \frac{25}{32})$  Rs =  $(201 \times 13 \frac{25}{32})$  Rs = 2663  $\frac{25}{32}$  Rs .
- 25 Let  $x$  be no of ft in the side of total area  
 Then area of path in sq ft =  $x^2 - (x - 16)^2 = 4 \times 4840 \times 9$  ,  
 $i.e. 32x - 256 = 4 \times 4840 \times 9$   
 Divide by 32 and  $x - 8 = 605 \times 9$ , giving  $x = 5453$   
 Whence  $x^2 = \frac{(5453)^2}{9 \times 4840}$  ac =  $\frac{29735209}{43560}$  ac = 682  $\frac{6}{11}$  ac
- 26 Let side of larger be  $x$  yds ,  $x^2 - 71^2 = 5 \times 4840$  ,  
 whence  $x^2 = 29241$ , and  $x = 171$
- 27 Incorrect measurement of length = 15 ft  
 Correct " " =  $\frac{2\frac{3}{4}}{3} \times 15$  ft =  $1\frac{1}{2}$  of 15 ft  
 error in length =  $\frac{1}{2}$  of 15 ft  
 Similarly, error in breadth =  $\frac{1}{2}$  of  $12\frac{1}{2}$  ft  
 Error in area = incorrect area - correct area  
 =  $(15 \times 12\frac{1}{2} - 15 \times 1\frac{1}{2} \times 12\frac{1}{2} \times \frac{1}{1\frac{1}{2}})$  sq ft =  $15 \times 12\frac{1}{2} \{1 - (\frac{1}{1\frac{1}{2}})^2\}$  sq ft  
 =  $29\frac{5}{8}$  sq ft = 30 sq ft (nearly)
- 28 Total cost = cost of land + cost of road + cost of houses  
 = £ $(53 \times 352 \times 3\frac{7}{10})$  + £ $(\frac{53 \times 352}{3} \times \frac{8}{10})$  + £87700 (1)  
 Receipts from sale = £ $[352(53 - 9) \times 5\frac{1}{10}]$  (2)  
 Subtracting (2) from (1), we have  
 Net cost = £ $[87700 + 352(\frac{53 \times 67 + 8 - 44 \times 113}{20})]$   
 = £ $[87700 + 352(\frac{3551 + 8 - 492}{20})]$   
 = £ $[87700 - 176 \times 1413]$  = £ $(87700 - 248688)$

### EXAMPLES XIV d Page 270

- 6 Let  $d$  metres be depth, then  $d \times 0.65 = 2.6$  , whence  $d = 4$ .
- 7 If  $d$  cm be depth, then  $560 \times 250 \times d = 21000 \times 1000$
- 8 Weight =  $(\frac{4.8}{1.2} \times \frac{4}{1.2} \times \frac{2.5}{1.2} \times 480 \text{ lbs}) = 150 \text{ lbs}$
- 9 Req<sup>d</sup> no =  $4^3 \times \frac{2.5}{40}$       10 Area =  $\frac{2156}{12\frac{5}{8}}$  sq ft

11 Cost =  $1\frac{1}{2} \times x \times 16 \times 7a$ , where  $x$  is depth in feet

Since Rs 21 = 336a,  $1\frac{1}{2} \times x \times 16 \times 7 = 336$

12 Let  $x$  feet be depth, then  $9x = 4\frac{1}{2} \times 2\frac{1}{4} \times 1\frac{1}{2}$

13 Let A sq yds be the area. Expressing the weight in lbs, we have

$$\frac{0.42 \times 9A \times 710}{12} = 2240$$

14. Volume of rainfall per big =  $1600 \times 9 \times \frac{2.8}{12}$  cu ft

This weighs  $\frac{1600 \times 9 \times 2.8}{12} \times \frac{1000}{16} \times \frac{7.5}{7.2 \times 40}$  mds, or  $\frac{153125}{6}$  mds

15 Volume of ice =  $\frac{4840 \times 9}{12} \times \frac{2.8}{12}$  cu ft

$$\text{total weight} = \frac{4840 \times 9 \times 2.8}{2 \times 12} \times \frac{57\frac{1}{4}}{2240} \text{ tons}$$

$$= \frac{121 \times 7 \times 2.8}{2 \times 4 \times 8} = 92.8 \text{ tons}$$

16 No. of gallons per ac = volume in cu ft  $\times 6\frac{1}{4}$

$$= 4840 \times 9 \times \frac{2.8}{12} \times \frac{2.5}{4} = \frac{605 \times 45 \times 2.5}{1} = 27225 \times \frac{100}{4} \text{ galls}$$

17 Weight = vol in cu ft  $\times 710$  lbs

$$= 224 \times 9 \times \frac{1}{8 \times 12} \times \frac{710}{112} \text{ cwt} = \frac{2130}{16} \text{ cwt}$$

18  $144 \times \frac{3}{4}$  cu in = 30 lb,

$$78 \times 42 \times \frac{7}{2} \text{ cu in} = \frac{30 \times 4}{144 \times 3} \times 78 \times 42 \times \frac{7}{2} \text{ lbs} = (5 \times 7 \times 7 \times 13) \text{ lbs}$$

$$30 \times 3 \times 6 \times \frac{13\frac{1}{2}}{12}$$

19 No req<sup>d</sup> =  $\frac{3 \times 4\frac{1}{2} \times 3}{\frac{4}{4} \times \frac{12}{12} \times \frac{3}{12}} = 120 \times 72$

20 Let A sq ft be area, expressing the volume in cu in,

$$5A \times 1728 \times \frac{9}{60 \times 2} \times \frac{1}{16} = 486, \text{ whence } A = 12$$

21 Let  $l$  ft be length req<sup>d</sup>, then

$$1 \text{ cwt} = \frac{2\frac{3}{4} \times 1\frac{1}{2} \times 1\frac{1}{2}}{11} \text{ cu ft} = \frac{l \times 120}{18 \times 144} \text{ cu ft, whence } l = 12$$

22 Vol of rainfall =  $3800 \times 1760 \times 1760 \times 3 \times 3 \times \frac{3}{12}$  cu ft

$$= \frac{1000}{4} \times 10 \times [38 \times 176 \times 176 \times 9]$$

$$= 250 \times 10 \times [\frac{38 \times 176 \times 9}{10} \times 1760 \times 3] \text{ cu ft}$$

$$= 250 \times 10 \times [2006.4 \times 1760 \times 3] \text{ cu ft}$$

Now exp<sup>n</sup> in brackets =  $2006.4 \text{ m} = (66818 \times 3) \text{ m}$

$$= (670 \times 3) \text{ m (nearly)}$$

**EXAMPLES XIV. e Page 273**

*NB* From Art 247 it follows that

**Weight = Volume  $\times$  Specific Gravity.**

- 1  $Wt = (0.3 \times 100^3 \times 7.14) \text{ gms} = 21.42 \text{ Kg}$
- 2 (i)  $Wt = (2.5)^3 \times 1.2 \text{ gms} = 15.625 \times 1.2 \text{ gms}$   
 (ii)  $Wt = 36 \times (0.8)^3 \times 1.2 \text{ gms} = 23.04 \times 1.2 \text{ gms}$
- 3  $Wt = \left(4\frac{1}{2} \times 3\frac{1}{2} \times 2\frac{1}{2} \times \frac{62\frac{1}{2} \times 2.56}{2240}\right) \text{ tons} = 2.5 \text{ tons}$
- 4  $Wt = (12 \times \frac{1}{1} \times \frac{1}{1} \times 62\frac{1}{2} \times 0.85) \text{ lbs} = (1500 \times 0.85) \text{ lbs}$
- 5  $Wt = \left(4840 \times 9 \times 2\frac{1}{3} \times \frac{62\frac{1}{2} \times 1.28}{2240}\right) \text{ tons} = 121 \times 3 \times 125 \times 0.08 \text{ tons}$   
 $= (363 \times 10) \text{ tons}$
- 6  $Wt = (44.5 \times 2.4 \times 2 \times 0.55) \text{ Kg} = (106.8 \times 1.1) \text{ Kg}$   
 $= (117.48 \times \frac{1}{9}) \text{ lbs}$
- 7 Let depth be  $x$  dm, then  $25 \times 1.6 \times x \times 13.5 = 540$ ,  
 whence  $x = 1 = 10 \text{ cms}$
- 8  $(35 \times 6 \times 25000) \text{ litres pass in 1 hour}$   
 $\left(35 \times 6 \times 25000 \times \frac{2\frac{1}{2}}{60}\right) \text{ lit pass in } 2\frac{1}{2} \text{ mins,}$   
 $\therefore (350 \times 625) \text{ litres pass in } 2\frac{1}{2} \text{ mins}$
- 9 Let depth be  $x$  dm, then  $17.5 \times 8 \times x = \frac{1}{10} \times 3500$ ,  
 whence  $x = 5 = 50 \text{ cms}$
- 10  $Wt = [(16.8 \times 1.25) \times (0.5 \times 8.8)] \text{ Kg} = [(2.1 \times 10) \times (1 \times 4.4)] \text{ Kg}$   
 $= 92.4 \text{ Kg} = (92.4 \times \frac{1}{9}) \text{ lbs} = 203.28 \text{ lbs}$
- 11  $\text{Capacity} = (9-1) \times (7-1) \times (6-1) \text{ cu in [Art 246 (ii)]}$
- 12  $\text{Volume of material} = (13 \times 11 \times 9 - 12 \times 10 \times 8) \text{ cu in [Art 246 (iii)]}$
- 13  $\text{Capacity} = 13 \times 9 \times 7 \text{ cu in}$   
 $\text{Vol of material} = (14 \times 10 \times 8 - 13 \times 9 \times 7) \text{ cu in}$
- 14  $\text{Reqd no of gallons} = 50 \times 3 \times 15 \times 6 \times 6\frac{1}{4}$ ,  
 hence  $\text{time} = \frac{50 \times 3 \times 15 \times 6 \times 6\frac{1}{4}}{125} \text{ mins} = \frac{50 \times 27}{2} \text{ mins} = 11\frac{1}{4} \text{ hours}$
- 15  $\text{No of gals in 10 mins} = 4 \times 1\frac{1}{4} \times (4 \times 1760 \times 3) \times 6\frac{1}{4} \times \frac{1}{10}$   
 $= 4400 \times 25$

- 16 Since 1 ac =  $\frac{(100)^2}{2\frac{1}{2}}$  sq m, if  $r$  dm be depth req<sup>d</sup>  
 $\left[ \frac{(100)^2}{2\frac{1}{2}} \times 10 \times 10 \times r \right]$  cu dm cover 1 ac But 1000000 litres cover it,  
 $\frac{(100)^2}{2\frac{1}{2}} \times 10 \times 10 \times r = 1000000$ ,  $r = 2.5 \text{ dm} = 25 \text{ cm}$

- 17 Let  $x$  be the fraction req<sup>d</sup>, then, expressing weight in lbs,

$$r \times 2240 = 3\frac{8}{12} \times 2\frac{6}{12} \times 2 \times 62\frac{1}{2},$$

$$\text{whence } x = \frac{2}{6} \times \frac{5}{4}$$

Again, internal surface

$$= 2(bd + ld) + bl \text{ (where } l = 2\frac{2}{3} \text{ ft, } b = 2\frac{2}{3} \text{ ft, } d = 2 \text{ ft)}$$

$$= 2(2\frac{2}{3} \times 2 + 3\frac{2}{3} \times 2) + 2\frac{2}{3} \times 3\frac{2}{3} = 35\frac{1}{3} \text{ sq ft}$$

External surface = surface enlarged by thickness of wood corresponding to internal surface + surface on which lid should rest

Thus last =  $2 \times \text{external length} \times \text{thickness} + 2 \times \text{internal breadth} \times \text{thickness}$

external surface

$$= 2(3 \times 2\frac{1}{6} + 4 \times 2\frac{1}{6}) + 3 \times 4 + 2 \times 4 \times \frac{1}{6} + 2 \times 2\frac{2}{3} \times \frac{1}{6}$$

$$= 42\frac{1}{2} + 2\frac{2}{3} = 44\frac{5}{6} \text{ sq ft}$$

$$\text{cost} = (44\frac{5}{6} + 35\frac{1}{3}) \times 9 \text{ p} = 79\frac{2}{3} \times 9 \text{ p}$$

$$= (80 - \frac{1}{3}) \times 9 \text{ p} = 80 \times 9 \text{ p} - 3 \text{ p} = \text{Rs } 3 \text{ } 11 \text{ a } 9 \text{ p}$$

- 18 Capacity =  $(2\frac{6}{12} \times 2\frac{7}{12} \times 2)$  cu ft = 13 cu ft

$$\text{No of lbs of water} = 13 \times 62\frac{1}{2} \times 19 = 247 \times 62\frac{1}{2} = 15437\frac{1}{2}$$

- 19 The water will rise as if its volume had been increased by an amount equal to the volume of the rectangular block Hence if  $h$  be the height in feet

$$2\frac{1}{2} \times 1\frac{1}{3} \times h = 1\frac{1}{2} \times 1 \times 1\frac{9}{12}, \quad h = \frac{3}{8} \text{ ft} = 4\frac{1}{2} \text{ in}$$

- 20 Vol of each brick =  $\frac{1}{4}$  cu ft Vol of water displaced by each  
 $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$  cu ft

The height  $h$  to which one brick will raise water is given by

$$5 \times 4 \times h = \frac{1}{16}, \text{ whence } h = \frac{1}{80}$$

Again, height of water at first =  $\frac{3}{4} \times \frac{9}{16} = 1\frac{1}{2}$  ft,

$$\text{height left to be covered} = 2\frac{1}{2} \text{ ft},$$

hence if  $x$  be req<sup>d</sup> no of bricks  $\frac{2}{510} = 2\frac{1}{5}$ , whence  $x = 1105$

- 21 Expressing the volume in cc and weight in gms, if  $s$  be sp gr, we have  $280 \times 195 \times 2 \times s = 851760$ , whence  $s = 7.8$

- 22 Area represented  $= (25 \times 15) \times 10^2 \times 3^2$  sq ft  $= 25 \times 15 \times 9$  sq ft  
no of blocks used  $= \frac{25 \times 15 \times 9}{\frac{1}{2} \times \frac{1}{4}} = 27000$

$$\text{Hence no of cartloads reqd} = \frac{27000 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3} \times 62\frac{1}{2} \times 0.8}{2240 \times 25} \\ = \frac{13500}{1334} = 10 \text{ roughly}$$

$$\text{Cost} = \frac{27000 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3} \times 62\frac{1}{2} \times 0.8}{2240} \times 112 \text{ pence} \\ = \frac{5625}{2} d = £11 \text{ 4s } 5d$$

- 23 The river discharges in a year

$$220000 \times 60 \times 24 \times 365 \text{ cu ft out of } \frac{31}{12} \times 4082 \times 640 \times 4840 \times 9 \text{ cu ft}$$

$$\therefore \frac{220000 \times 60 \times 24 \times 365}{\frac{31}{12} \times 4082 \times 64 \times 484 \times 9} \text{ cu ft out of 100 cu ft}$$

$$\therefore \frac{100000 \times 30 \times 365}{31 \times 4082 \times 22 \times 3} \% \text{ or } \frac{100500000}{2783924} \% \text{ or } 39 \%$$

- 24 Let  $x$  sq ft be area of reservoir Since  $\frac{400000 \times 30}{64}$  cu ft of water are wanted per day,

$$20 \times x = \frac{400000 \times 120}{6} \times 184 \text{ (July-Dec = 184 days),}$$

$$\text{whence } x = 800 \times 120 \times 184 \text{ sq ft} = 17664000 \text{ sq ft} = 406 \text{ ac}$$

Let  $y$  sq ft be area of catchment Since 10 in of rainfall are sufficient for 6 months,  $y \times \frac{1}{12}$  cu ft = vol of reservoir,

$$y \times \frac{1}{12} = 20 \times 17664000, \text{ whence } y = 423936000 \text{ sq ft or 15 sq mi}$$

- 25 Annual discharge  $= (30000000 \times 100 \times 1\frac{1}{4})$  cu m

If  $r$  be the reqd no of days, since  $x$  days  $= x \times 24 \times 60 \times 60$  mins,

$$170 \times 60 \times 60 \times 24 \times x = 30000000 \times 100 \times 1\frac{1}{4},$$

$$\text{whence } x = \frac{5000000}{10884} = 255$$

- 26  $(1400 \times 4 \times 1760 \times 3)$  cu ft of water pass in 1 hr

$$(1400 \times 4 \times 1760 \times 3 \times 6\frac{1}{4}) \text{ gals } \quad \quad \quad \text{,, } \quad \quad \quad \text{1 hr}$$

$$1400 \times 4 \times 1760 \times 3 \times \frac{25}{4} \times 24 \times 365 \text{ gals } \quad \quad \quad \text{,, } \quad \quad \quad \text{1 year,}$$

$$\text{whence } \frac{1400 \times 4 \times 1760 \times 3 \times 25 \times 24 \times 365 \times 28}{4 \times 7000 \times 2240} \text{ tons of matter pass in 1 year}$$

$$\text{This exp}^n = \frac{660 \times 365 \times 68}{7} \text{ tons} = 2374586 \text{ tons}$$

27 We have  $\frac{1}{1000}$  cu m =  $l \times (0.002)^2 \times 0.7854$ ,

$l = \frac{1}{0.0031416}$  m, and this is the length of 9 Kg of copper,

length of 7.5 Kg =  $\frac{7.5}{9} \times \frac{1}{0.0031416}$  m = 265 m

### EXAMPLES XV. a Page 279

- 16 The points, in the order given, are represented by A, B, C, D, E, F, G, H in Fig 1. By measurement the distance of any one of them from the origin is said to be 5 units.

Also if  $P(x, y)$  stands for any one of the points,

$$OP^2 = x^2 + y^2$$

The value of  $x^2 + y^2$  is found by calculation to be 25 in each case. Thus  $OP = 5$ .

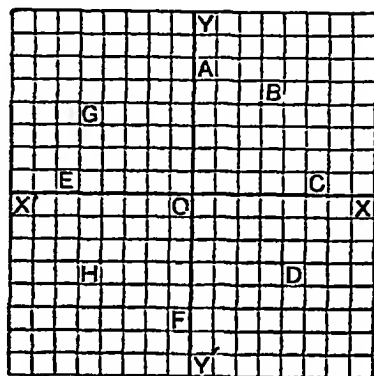


FIG 1

### EXAMPLES XV. b Page 281

- 1 The points are shown in Fig 2, the first series, in the given order, is A, B, C, D, E, and the second series P, Q, R, S, T. The coordinates of the intersection of these lines are seen to be 5 and 8, hence the point is (5, 8).

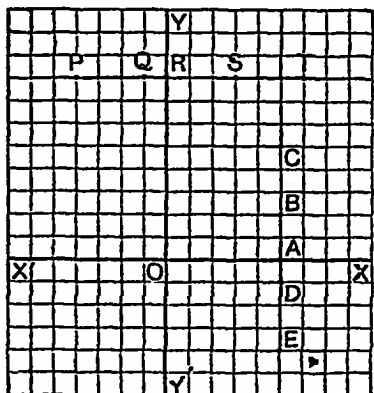


FIG 2



- 3 Substituting the given values for  $x$  in turn in the equation, the corresponding values of  $y$  are found to be 10, 12, 16, 6, 0. Hence the points required are (0, 10), (1, 12), (3, 16), (-2, 6), (-5, 0). These are the points A, B, C, D, E plotted in Fig 3, and are seen to lie on a line cutting OX at E and OY at A.

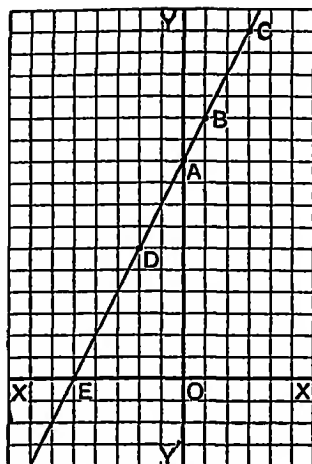


FIG 3

- 5 (i), (ii), (iii) The three lines are parallel, one passing through the origin, and the others making intercepts -4 and 6 on the  $y$  axis.
- 6 As in Ex 2, it will be seen that each graph is a line through the origin.
- 7 These points are plotted in the solution of XV a, 16. Since they are equidistant from O, the curve is a circle.

### EXAMPLES XV c Page 283

- 1 Substituting the given values for  $x$  in turn in the first equation, the corresponding values of  $y$  are found to be -8, -6, -4, -2, 0. Hence the points required are (0, -8), (1, -6), (2, -4), (3, -2), (4, 0).

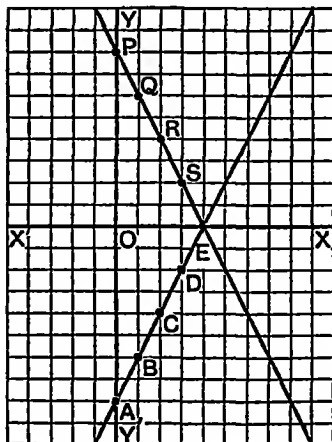


FIG 4

These are the points A, B, C, D, E plotted in Fig 4 They lie on the line cutting OX at E and OY at A

Similarly for the second equation we obtain the points (0, 8), (1, 6), (2, 4), (3, 2), (4, 0) These are the points P, Q, R, S, E They lie on a line cutting OX at E and OY at P

Thus the required coordinates of the point of intersection are 4, 0

- 2 Substituting the given values for  $x$ , the corresponding values of  $y$  in the first equation are 17, 13, 9, 5, 1, -3, in the second equation -9, -6, -3, 0, 3, 6 They will be found to lie on two straight lines intersecting at the point (2, 1)

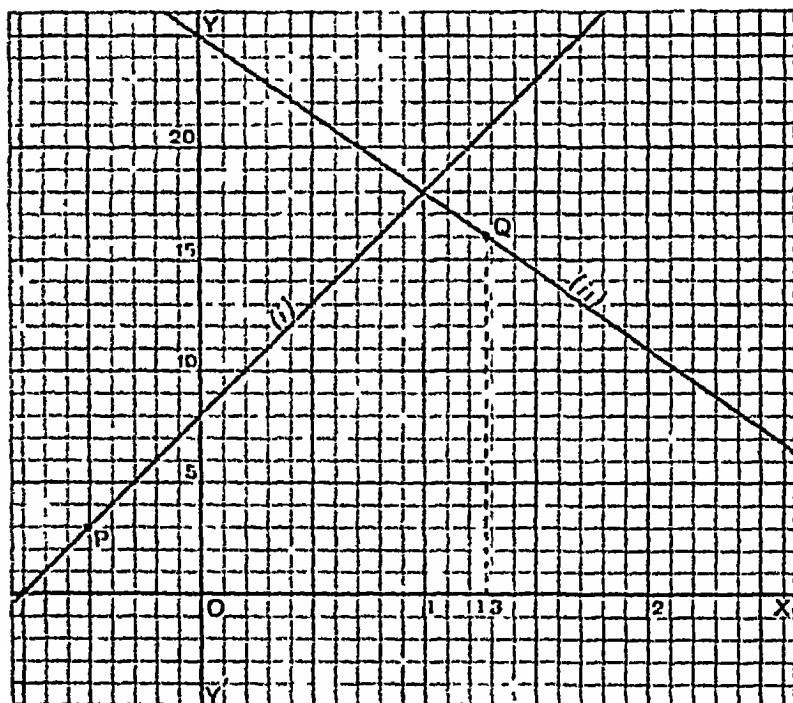


FIG 5

- 3 The lines are shown as (i), (ii) in Fig 5

On (i) at P when  $x = -\frac{1}{2}$ ,  $y$  is seen to be 3,

(ii) at Q when  $y = 16$ ,  $x$  is seen to be 13 nearly

- 4 Substituting the given values for  $x$  in turn in the equation, we obtain  $y = -4, -3, -2, -1, 0, 1$

Plotting the points, we see they lie on a straight line

When  $y=3$ ,  $x=25$ , and when  $x=-15$ ,  $y=-5$

- 5 Let us take  $\frac{1}{20}$  of an inch as unit for  $y$  and one inch as unit for  $x$ , then the graph of  $y=11x+6$  will be as in Fig 6, in which the line has been drawn by joining the points  $(0, 6)$ ,  $(2, 28)$

Now we see that  $x=1.8$  at the point P, and here  $y=26$ , nearly

Again  $y=20$  at the point Q, and  $x=OR=1.28$ , approximately  
In obtaining this last result we observe that OR is greater than 1.2 and less than 1.3, and we mentally divide the tenth in which R falls into *ten* equal parts (*i.e.* into *hundredths of the unit*) and judge as nearly as possible how many of these hundredths are to be added to 1.2

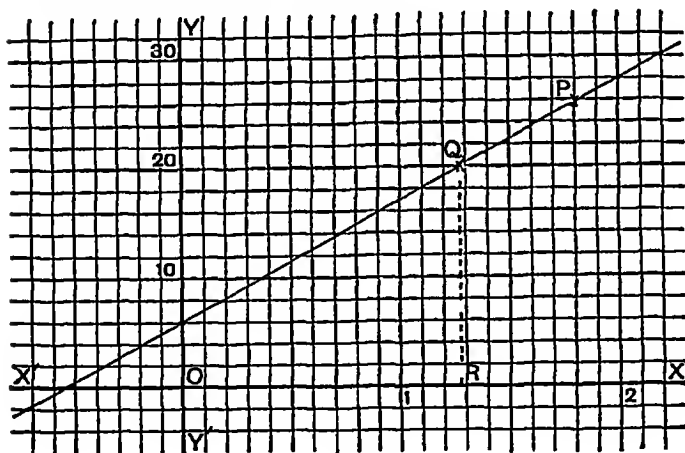


FIG 6

### EXAMPLES XV d. Page 289

- 6 Measure time horizontally and population vertically

Take 0.1 of an inch as unit in each case, also it will be convenient if we begin measuring abscissæ at 1830, and ordinates at 20

The graph is given in Fig 7, it will be seen that it passes exactly through the extreme points and lies evenly among the others

The populations in 1848 and 1875, at the points A and B respectively, will be found to be 27.8 millions and 45.3 millions

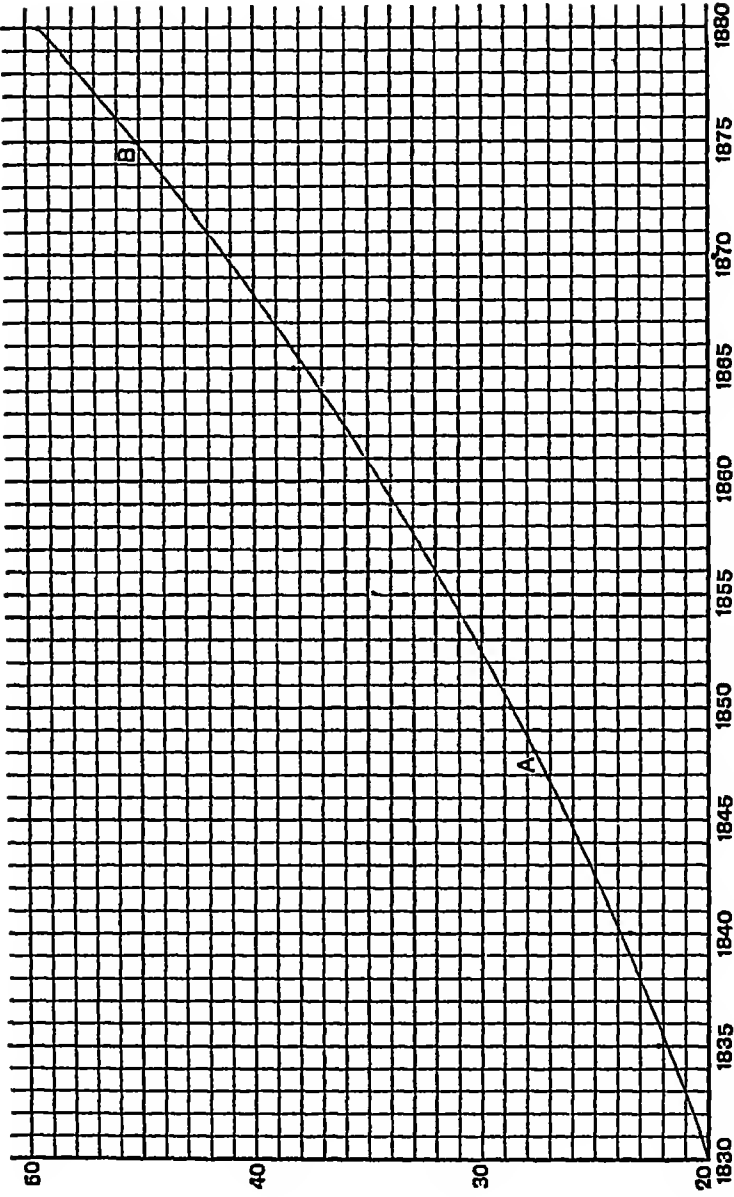


FIG 7

- 7 A convenient scale is 0.6" horizontally to each month, and 0.1" vertically to each degree. The curve is shown on half this scale in Fig 8. It should be remembered that the curve need not necessarily pass through all the points plotted, but only as near to them as possible (Art 259)

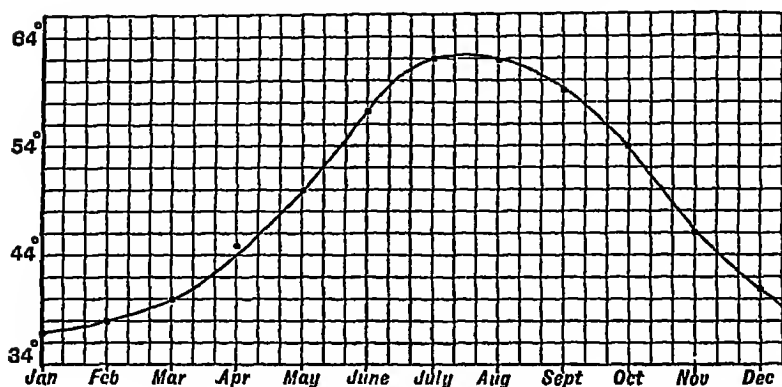


FIG 8

- 8 Take 0.1" and 0.4" to represent the units of circumference and radius respectively. The points corresponding to the given values of  $C$  and  $r$  will be found to lie on a straight line. The required values may be read off from the graph.
- 9 The required values are given by the abscissæ of  $P$  and  $Q$  (Fig 9)

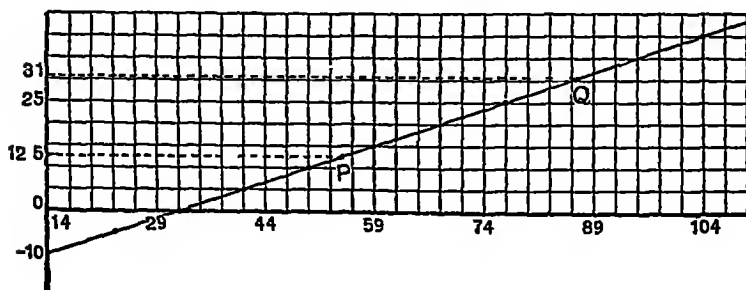


FIG 9

- 10 The points are plotted in Fig 10, and are seen to lie nearly on a straight line. By trial it is found that the line through the 1<sup>st</sup> and 4<sup>th</sup> points lies most evenly amongst the remaining points.

Assuming the equation to be  $y=ax+b$  and, substituting the coordinates of the 1<sup>st</sup> and 4<sup>th</sup> points, we find  $a=\frac{1}{11}$ ,  $b=\frac{35}{11}$

Hence the equation is  $11y=3x+35$

Putting  $y=11.5$  in this, we have  $x=30.5$ ,

. . .  $x=10$  . . .  $y=59.$

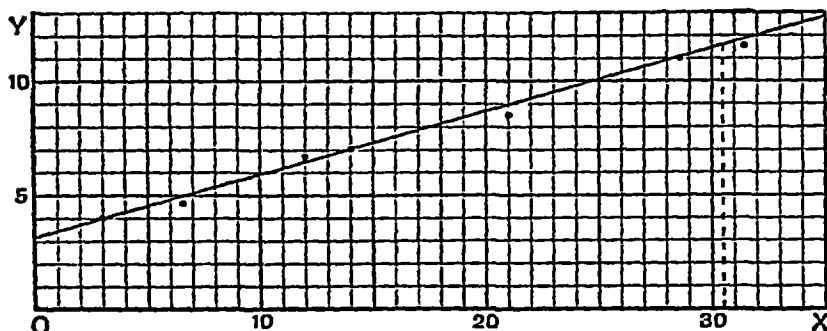


FIG 10

11. Representing the lengths from 8 in to 12 in by ordinates, and the weights from zero to 25 oz. by abscissæ, we find that the graph is a straight line cutting the axis of length at 8.1. Hence when no weight is attached the length is 8.1 in. Similarly the weight corresponding to a length of 1 foot is 24.375 oz.

12. Representing ages by abscissæ and premiums by ordinates, we find that (on the scale of £2 to an inch) the premiums corresponding to ages 34, 43 are 2.6 and 3.4. Thus the premiums required are £2 12s and £3 8s (Fig 11)

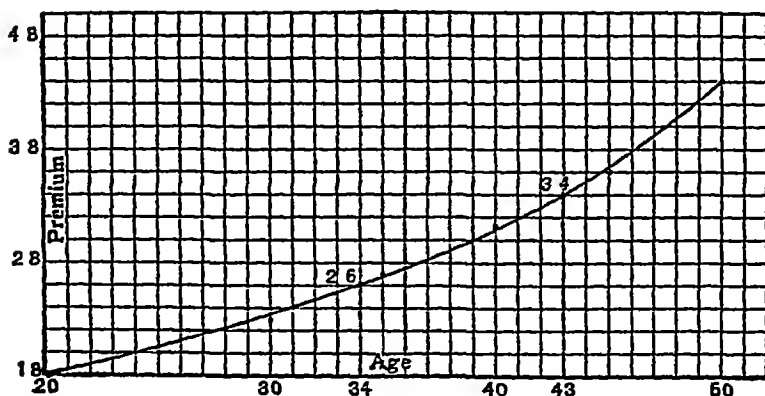


FIG 11

- 13 The graph is shown in Fig 12. The abscissæ of P and Q are 17 and 36.5 respectively. Hence the required prices are Rs 17 and Rs 36.5

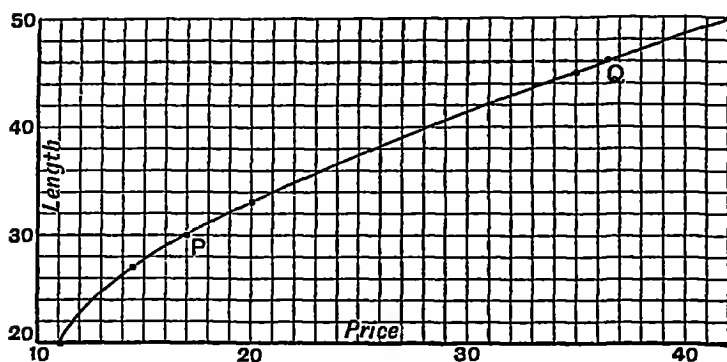


FIG 12

- 14 The graph is shown on a small scale in Fig 13. The student should draw his own on double the scales here employed.

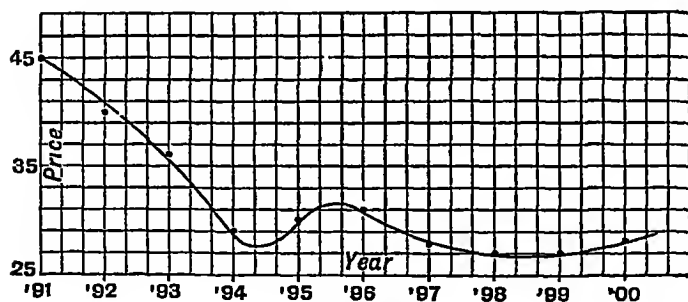


FIG 13

- 15 Take one-tenth of an inch as unit in each case

Following the method of the Example in Art 260, it will be found that a straight line passes accurately through the 2<sup>nd</sup>, 5<sup>th</sup>, and 9<sup>th</sup> points, and lies evenly among the rest.

Assume  $y = ax + b$ , and, substituting the coordinates of the first and last points, we find  $a = 0.5$ ,  $b = -3$ .

Thus the required equation is  $y = 0.5x - 3$ .

From the graph, when  $y = 15$ ,  $x = 36$ ,

$$x = 71, y = 32.5$$

From the equation, when  $x = 164$ ,  $y = 79$

16 See solution of Ex 11

17 Take a scale of 10 years to the inch horizontally and 10000 population to the inch vertically. It will also be convenient to take for the origin the point (1830, 20). As in the Example of Art 259, it will be found that the graph of P is a curve passing accurately through the 2<sup>nd</sup>, 4<sup>th</sup>, 6<sup>th</sup>, and 7<sup>th</sup> points, and lying evenly among the rest.

The graph of Q, as in the Example of Art 260, is a straight line passing accurately through the 1<sup>st</sup>, 3<sup>rd</sup>, and 7<sup>th</sup> points, and lying evenly among the others.

They intersect at the point (1883, 45), which gives the required year.

18 A scale of 0.5" horizontally to each year of Expectation, and 0.2" vertically to each year of Age should be employed, the origin being the point (34, 6). It will be found on plotting the points that between the ages 10 and 22 the graph is approximately represented by a straight line. Read off the abscissæ of those points on this line whose ordinates are 12 and 20.

### EXAMPLES XVI a Page 294

9 Since the same rate per ac. is charged,

$$\text{ratio} = \frac{\text{£}33 \text{ } 8s \text{ } 4d}{\text{£}55 \text{ } 2s \text{ } 9d} = \frac{20}{33}$$

10 Let  $d$  mi be the total distance and  $x$  and  $y$  mi per hr then rates, then  $49x = d$ , and  $84y = d$ ,  $49x = 84y$ , and  $\frac{x}{y} = \frac{84}{49} = \frac{12}{7}$

11 Ratio of speeds =  $\frac{\frac{448}{8}}{\frac{504}{12}} = \frac{4}{3}$       12 Ratio =  $\frac{\frac{146}{130}}{\frac{952}{102}} = \frac{9}{20}$

13 The first takes  $6\frac{3}{4}$  hrs, the second 6 hrs,

$$\text{as in No 10, ratio of speeds} = \frac{6}{6\frac{3}{4}} = \frac{8}{9}$$

14 The new fraction is  $\frac{1}{18}$ . But  $\frac{1}{24} = \frac{5}{120}$

This shows  $\frac{1}{18} > \frac{5}{120}$ , since  $\frac{1}{18} > \frac{1}{24}$

We now see 16 must be added to the consequent



- 15 Since  $8 > 5$ , we should have to add *more* to the numerator than to the denominator in order to get a fraction which  $= \frac{8}{5}$   
But in this case the quantities added are equal, shewing

$$\frac{8+a}{5+a} < \frac{8}{5}$$

If  $\frac{8+a}{5+a} = \frac{7}{5}$ , then  $5(8+a) = 7(5+a)$ , whence  $a = 2\frac{1}{2}$

$$17 \quad (i) \frac{548 \frac{1}{2}}{62 \frac{43}{100}}, \quad (ii) \frac{6 \frac{53}{100} \times 1728 \text{ oz}}{62 \frac{43}{100} \times 16 \text{ oz}}, \quad (iii) \frac{4 \frac{05}{100} \times 1728 \text{ oz}}{62 \frac{43}{100} \times 16 \text{ oz}}$$

- 18 Let  $V_1$  cu ft be volume of iron,  $V_2$  cu ft volume of steel, weight of  $V_1 = V_1 \times 7 \frac{2}{10} \times 62 \frac{43}{100}$  lbs, weight of  $V_2 = V_2 \times 8 \frac{0}{10} \times 62 \frac{43}{100}$  lbs

Since these are equal,  $V_1 \times 7 \frac{2}{10} \times 62 \frac{43}{100} = V_2 \times 8 \frac{0}{10} \times 62 \frac{43}{100}$

$$\frac{V_1}{V_2} = \frac{8 \frac{0}{10}}{7 \frac{2}{10}} = \frac{10}{9}$$

$$19 \quad R F = \frac{1}{100 \times 1760 \times 36}$$

$$1 \text{ in} = 100 \text{ mi}, \quad 1 \frac{1}{2} \text{ in} = 180 \text{ mi} \quad \text{Similarly } 830 \text{ mi} = 8 \frac{3}{10} \text{ in}$$

- 20  $4 \frac{5}{10} \text{ in} = 50 \text{ mi}$ , hence  $1 \text{ in} = \frac{100}{9} \text{ mi}$ , i.e. scale is 1 in to  $11 \frac{1}{9} \text{ mi}$ ,

$$R F = \frac{1}{\frac{100}{9} \times 1760 \times 36}$$

We find  $6 \frac{6}{10} \text{ in} = 73 \frac{3}{10} \text{ mi}$ , and  $3 \frac{9}{10} \text{ in} = 43 \frac{3}{10} \text{ mi}$

### EXAMPLES XVI b Page 298

- 1-6 See Art 268, Ex 1 and 2      7-9 See Art 268, Ex 4

$$10 \quad \frac{ab^3}{a^2b} = \frac{b}{ax}, \text{ whence } x=1$$

- 11-13 See Art 268, "Three quantities  $a, b, c$ , etc"

$$14 \quad \frac{a}{b} = \frac{b}{x}, \quad x = \frac{b^2}{a} \quad 15-17. \text{ See Art 268, "Three quantities, etc"}$$

$$18 \quad \frac{a^3b}{x} = \frac{x}{ab^3}, \quad x = a^2b^2$$

- 19-25 As in Art 268, Ex 3, put  $x$  for missing term

$$26 \quad \text{Ratio} = \frac{2 \frac{8}{10}}{4 \frac{6}{10}} = 0 \cdot 061$$

$$27 \quad \frac{1 \text{ lb}}{1 \text{ Kg}} = \frac{5}{11}, \quad \frac{2240 \text{ lbs}}{1 \text{ Kg}} = \frac{5 \times 2240}{11},$$

$$\text{hence } 1 \text{ ton} = \frac{5 \times 2240}{11} \text{ of } 1 \text{ Kg} = 1018 \text{ Kg}$$

$$28 \quad 2920 \text{ yds} = \frac{200}{210} \times 2920 \text{ m} = \frac{200}{3} \times 40 \text{ m} = 2667 \text{ m}$$

$$29 \quad \frac{1 \text{ lit}}{1 \text{ pt}} = \frac{7}{4}, \quad \frac{1 \text{ Kl}}{1 \text{ gall}} = \frac{7000}{8 \times 4}, \quad 1 \text{ Kl} = \frac{7000}{32} \text{ of } 1 \text{ gall} = 219 \text{ galls}$$

30 From Art 264, Ex 2, we have

$$\begin{aligned} \frac{\text{sp gr of iron}}{\text{sp gr of copper}} &= \frac{\frac{\text{wt of 1 cu ft of cast iron}}{\text{wt of 1 cu ft of water}}}{\frac{\text{wt of 1 cu ft of copper}}{\text{wt of 1 cu ft of water}}} \\ &= \frac{\text{wt of 1 cu ft of cast iron}}{\text{wt of 1 cu ft of copper}} \end{aligned}$$

$$\text{if } w \text{ lbs be weight required, } \frac{72}{86} = \frac{w}{5373},$$

$$\text{whence } w = 4498$$

### EXAMPLES XVI c Page 301

$$1 \quad (i) \text{ Rs } 18 \times \frac{11}{9}, \quad (ii)-(iv), \text{ as No } (i)$$

$$2 \quad \text{Rs } 55 \times \frac{24}{10}$$

$$3 \quad 74 \text{ mi} \times \frac{54}{12}$$

$$4 \quad 7 \text{ in} \times \frac{163}{110}$$

$$5 \quad 26 \text{ days} \times \frac{87}{88}$$

$$6 \quad 180 \text{ hr} \times \frac{10}{10}$$

$$7 \quad 21 \text{ min} \times \frac{20}{8}$$

$$8 \quad \text{Rs } 37\frac{1}{2} \times \frac{420}{200}$$

$$9 \quad 110 \text{ yds} \times \frac{3}{5\frac{1}{2}}$$

$$10 \quad 22\frac{1}{2} \text{ knots} \times \frac{5}{4\frac{1}{2}}$$

$$11 \quad 3\frac{1}{2} \text{ min} \times \frac{3}{4\frac{1}{2}}$$

$$12 \quad (i) \text{ £}56 \times \frac{21}{10}, \quad (ii) 7\frac{1}{2}d \times \frac{11}{10}$$

$$13 \quad (i) 57\frac{3}{4} \text{ mi} \times \frac{10}{11}, \quad (ii) 192 \text{ mins} \times \frac{21}{10}, \quad (iii) 28 \text{ mi} \times \frac{10}{11}$$

$$14 \quad \text{Rs } 52 \times \frac{35}{91}$$

$$15 \quad 96 \text{ Kg} \times \frac{7}{50}$$

$$16 \quad 27s \times \frac{1000}{72}$$

$$17 \quad \text{Rs } 1302 \times \frac{100}{155}$$

$$18 \quad 20 \text{ cwt} \times \frac{27}{17\frac{1}{2}}$$

$$19 \quad \text{Time without stoppages} = 270 \text{ min},$$

$$\text{average running speed} = 246 \times \frac{60}{270} \text{ mi}$$

$$\text{Time with stoppages} = 288 \text{ min},$$

$$\text{average speed} = 246 \times \frac{60}{288} \text{ mi}$$

20  $23\ 62\ \text{oz} \times \frac{1\ 00}{18}$

21  $2145\ \text{m} \times \frac{8\ 4}{13\ 2}$

22  $47\ 12\ \text{m} \times \frac{11}{15}$

23  $259 \times \frac{4}{3\frac{1}{2}}$

24.  $220 \times \frac{4\frac{1}{2}}{5\frac{1}{2}}$

25  $6\frac{1}{2}\ \text{oz} \times \frac{100}{1639}$

26  $40\ 4\ \text{mi} \times \frac{5}{8}\ \text{per hr}, \text{ i.e. } 40\ 4\ \text{mi} \times \frac{5}{8} \times \frac{48}{60}\ \text{per 48 min}$

27 Scale is  $126720\ \text{in}$  to  $1\ \text{in}$ , or  $\frac{126720}{1760 \times 36}\ \text{mi}$  to  $1\ \text{in}$ ,  
i.e.  $2\ \text{mi}$  to  $1\ \text{in}$ , giving  $38\ \text{in}$  and  $51\ \text{mi}$  for other answers

28  $118\frac{1}{2}\ \text{mi}$  to  $7\frac{1}{2}\ \text{mi} = 158\ \text{mi}$  to  $1\ \text{in}$

$$R\ F = \frac{1}{158 \times 1760 \times 3 \times 12} = \frac{1}{1001088}$$

$$69\ \text{in} = 69 \times 158\ \text{mi} = 109\ \text{mi} \text{ (nearly)}$$

29  $255\ 6\ \text{litres} \times \frac{7\ 5}{2\ 7} = 710\ \text{litres} = 710\ \text{Kg}$

30  $420\ \text{ht} = 420\ \text{Kg} = \frac{11}{5} \times 420\ \text{lbs} = 924\ \text{lbs}$  Time =  $66\ \text{min} \times \frac{112}{924}$

## EXAMPLES XVI d Page 304

1 Rs  $141\frac{1}{4} \times \frac{5\ 0}{18}$

2 £ $1404\frac{1}{12} \times \frac{4\ 8}{700}$

3 £ $49\frac{5\ 0}{80} \times \frac{5\ 6}{23}$

4.  $3276\ \text{lbs} \times \frac{2\ 7}{4\ 2}$

5  $39\ \text{mi} \times \frac{8\frac{1}{2}}{5}$

6  $65\ \text{a} \times \frac{2\frac{1}{4}}{3\frac{1}{4}}$

7  $100\ \text{min} \times \frac{3\frac{1}{4}}{3\frac{1}{8}}$

8  $27 \times \frac{32\frac{1}{2}}{22\frac{1}{2}}$

9  $33 \times \frac{150\ 675}{101\ 475} = 33 \times \frac{2\ 009}{1\ 353}$

10  $45\ \text{srs} \times \frac{29}{16\frac{5}{8}}$

11  $75\ \text{mi} \times \frac{2\frac{3}{4}}{6\frac{1}{4}}$

12 £ $10\frac{1}{2} \times \frac{3\frac{4}{5}}{36}$

13  $1\ \text{cwt} \times \frac{21\frac{5}{6}}{1\frac{1}{8}}$

14  $200\ \text{lbs} \times \frac{4\frac{1}{2}}{\frac{1}{8}}$

15 Rs  $28\frac{7}{8} \times \frac{4\ 6}{3\ 3}$

16 Re  $1 \times \frac{18\ 00}{2\ 5\ 20}$

17 £ $1 \times \frac{348\frac{1}{2}}{1520}$

18 Rs  $125 \times \frac{4\ 10}{600}$

19 £ $141\frac{1}{10} \times \frac{1\ 344}{176\frac{1}{2}}$

20 Assets =  $\frac{3}{4}$  of Rs  $1250$ , loss = Rs  $1\frac{50}{4}$

21. (i) Tax =  $\frac{0}{10}$  on every Re  $1$  of gross income,  
tax =  $\frac{0}{10}$  of gross income

(ii) Net income = gross income - tax =  $\frac{10\ 8}{100}$  of gross income

(iii) Tax =  $\frac{6}{180}$  on every Re 1 of net income,  
 tax =  $\frac{6}{180}$  of net income

22 From No 21, net income = Rs 1563  $\frac{1}{2}$   $\times$   $\frac{1}{17}$

23 From No 21, net income = Rs 6100  $\times$   $\frac{\text{Re 1} - 3 \text{ a } 9 \text{ p}}{\text{Re 1}}$   
 = Rs 6100  $\times$   $\frac{1}{10}$

24 From No 21, gross income = Rs 59  $\frac{1}{3}$   $\times$   $\frac{1}{4}$

25 From No 21, tax = Rs 2150  $\frac{1}{2}$   $\times$   $\frac{6}{180}$

26 Value = rate  $\times$   $\frac{2}{3}$  = £208  $\frac{1}{2}$   $\times$  12      27 Rates = £5676  $\frac{2}{3}$   $\times$   $\frac{31}{240}$

28 His net income would have been 187 p per Re 1 of gross income  
 instead of 189 p per Re 1 of gross income  
 Hence net income required = Rs 5264  $\times$   $\frac{1}{18}$

29 Total tax is 1s 9d in the £,      gross income = £584  $\times$   $\frac{2}{3}$

30 If total income is £1, then  $\frac{2}{3}(1 - 120) = 17 \frac{1}{10}$ , whence  $x = 580$ ;  
 Net income = total income tax

31 Let my unearned income be £1,  
 then  $\frac{1}{180}$  of 1174 +  $\frac{1}{180}$  of  $x = 70 \frac{2}{3}$ , whence  $x = 528 \frac{1}{2}$

32 47 in  $\times$   $\frac{52}{70} = \frac{21}{6}$  in, 70  $\frac{1}{2}$  m to 47 in = 15 m to 1 in

$$R F = \frac{1}{15 \times 1700 \times 1 \times 12} = \frac{1}{255000}$$

33 Rs 2450  $\times$   $\frac{1}{10} =$  Rs 245  $\frac{2}{3} =$  Rs 3524 nearly

34 2  $\frac{1}{6}$  hrs  $\times$   $\frac{1}{2} = \frac{1}{2}$  hr = 30 min

35 Rs 434  $\times$   $\frac{4}{10} =$  Rs 173  $\frac{2}{5}$       36 Rs 1500  $\times$   $\frac{48}{58} =$  Rs 1253  $\frac{1}{2}$

37 786 gms  $\times$   $\frac{1}{2}$

38 Net income =  $\frac{2}{3}$  of £1253

£1253 are saved out of every £ of net income,

$\frac{1}{3}$  of £1253 is left      Thus = £417  $\frac{1}{2}$

- 39 Since first section of 18 740 Km cost £3704, at the same rate the whole railway will cost  $\frac{18\ 740 \times 6\ 500}{18\ 740}$  of £3704,  
 or  $\frac{£27\ 13704}{18\ 74} = \frac{£25\ 31852}{9\ 37} = \frac{£468556}{9\ 37} = £5000$
- 40 1 yd =  $2\frac{3}{4}$  lbs, or  $\frac{2\frac{3}{4}}{2\frac{1}{2}}$  Kg,      1 metre =  $\frac{11}{4} \times \frac{5}{11} \times \frac{35}{32}$  Kg
- 41 We have  $11344 \times 8$  pints = 1 ac, or  $11344 \times 8 \times \frac{1}{2}$  lit =  $\frac{2}{3}$  Ha,  
 1 Ha =  $\frac{11344 \times 8 \times 4 \times 5}{1 \times 2}$  litres
- 42 1 cu in of gold weighs 0 58 oz  $\times \frac{96}{8}$ ,  
 $\frac{5 \times 87}{90 \times 0\ 58}$  cu in weigh 87 oz, i.e. 0 78 cu in weigh 87 oz

### EXAMPLES XVI e Page 314

- 7 Take one tenth of an inch as the unit in each case. Suppose  $y$  rupees are equivalent to  $x$  shillings. Then, as in Ex, Art 277,  $\frac{y}{16} = \frac{x}{21}$ , or  $y = \frac{16}{21}x$ . Hence, measuring rupees vertically and shillings horizontally, the required graph is at once obtained by joining the origin to the point whose coordinates are 21 and 16. The abscissa corresponding to ordinate 13 is 17. The ordinate corresponding to abscissa 80 is 61.  
 Thus 13 rupees = 17 shillings and 80 shillings = 61 rupees
- 8 Take scales of an inch to 1 gallon along the axis of  $x$ , and one tenth of an inch to 1 litre along the axis of  $y$ . As in Ex 7,  $y = \frac{20}{44}x$ , which is the equation of a line through the origin. Join the origin to the point (44, 20), and the graph is obtained. The ordinate corresponding to an abscissa  $2\frac{1}{2}$  is seen to be 11 4, the abscissa corresponding to an ordinate 20 9 is seen to be 46.  
 Hence  $2\frac{1}{2}$  gallons = 11 4 litres,  
 and 20 9 litres = 46 gallons
- 9 Take scales of an inch to the yard along the axis of  $x$ , and an inch to a metre along OY. Join the point whose coordinates are (6 01", 5 50") to the origin. The ordinate corresponding to an abscissa 2 22" is seen to be 2 03", and therefore  
 2 22 yards = 20 3 metres  
 Hence 22 2 yards = 20 3 metres

- 10 This is most easily done on a very large scale. Take 0.1" horizontally to each grain, and 1.0" vertically to each gram. Since 16.1 grains = 1.17 grams, we have 90.5 grains = 5.85 grams, and the graph will be the line joining the origin to the point (9.05", 5.85")

The abscissa corresponding to ordinate 3.5 is 54.1

The ordinate corresponding to abscissa 30.9 is 2.0

Hence 3.5 grams = 54.1 grains,

and 30.9 grains = 2.0 grams,

i.e. 3.09 grains = 0.2 gram

- 11 Take scales of an inch to 1 minute along the axis of  $x$ , and an inch to 1000 yards along the axis of  $y$ . Plot the points, and it will be found they are not in a straight line. Therefore the distances are not proportional to the times.

For the second part of the question, join the origin to the point (2, 700) and produce this line. Read off the ordinate corresponding to abscissa 6.1, and we get for req<sup>d</sup> distance 2300 yds.

- 12 See solution to XV d 8

- 14 Draw the graph of  $y = \frac{9}{8}x$ , and read off the values of  $y$  corresponding to  $x = 40, 64, 78$

- 15 Cf XV d 12. Plotting the points as in this example, we find they do not lie on a straight line. Hence the premium is not directly proportional to the age.

For the second part of the question, join the origin to the point (35, 28), and read off the values of  $y$  corresponding to  $x = 25, 30, 40, 45$ , on this line.

- 16 Take a scale of an inch to a year along the axis of  $x$ , and an inch to 100 increase, along the axis of  $y$ .

Regarding the "beginning of 1905" as the "end of 1904," take the point (04, 0) as origin. The increases for 1905, 1906, etc., are 200, 420, 662, 928, respectively.

Plotting the points (04, 0), (05, 200), (06, 420), (07, 662), (08, 928), we see they do not lie on a straight line. Hence the increases are not proportional to the times.

- 17 Take 0.1" vertically to represent 1 penny and 1" horizontally to represent 1 hour. Let  $y$  shillings be the wages for  $x$  hours, then the graph is that of the equation  $y = \frac{4}{8}x$ .

The required values can easily be read off.

- 18 The relation between the actual marks  $x$  and the raised marks  $y$  is clearly  $\frac{y}{x} = \frac{200}{136}$ , which represents a st line through the origin

The graph is shewn in Fig 14, the line being drawn through the origin and (68, 100)

The required marks are given by the ordinates of P and Q, and are 90 and 72 respectively

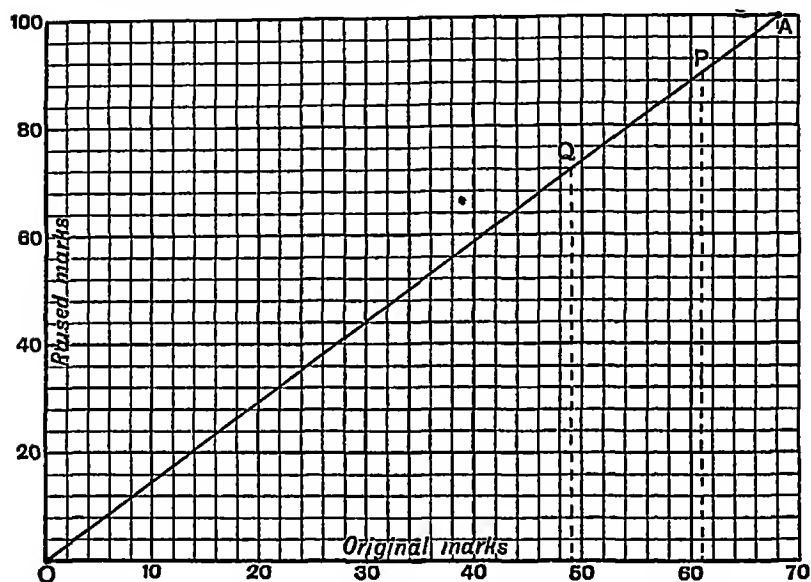


FIG 14.

### EXAMPLES XVI f Page 317

- |   |  |
|---|--|
| 1 As in Art 279, Ex 1, req <sup>d</sup> area = $96 \text{ sq ft} \times 5 \times \frac{2}{3}$                     |  |
| 2 As in Art 279, Ex 1 req <sup>d</sup> weight = $2\frac{1}{2} \text{ mds} \times 3\frac{1}{2} \times \frac{2}{6}$ |  |
| 3 $96 \text{ m1} \times \frac{3}{4} \times \frac{5}{8}$   | 4 $\text{Rs } 22\frac{1}{2} \times \frac{2}{3} \times \frac{4}{6}$                 |
| 5 $3\frac{3}{4} \text{ tons} \times \frac{1}{2} \times \frac{2}{3} \times \frac{3}{6}$                            | 6 $225 \times 2240 \text{ lbs} \times \frac{1}{2400} \times \frac{1}{7 \times 21}$ |
| 7 $20 \text{ wks} \times \frac{1050}{1600} \times \frac{6}{7}$  | 8 $\text{Rs } 1500 \times \frac{9}{6} \times \frac{24}{20}$                        |
| 9 $\text{Rs } 9 \times \frac{6}{4} \times \frac{10}{2}$   | 10 $12 \text{ men} \times \frac{18}{36} \times \frac{5}{8}$                        |
| 11 $\text{Rs } 736 \times \frac{40}{60} \times \frac{30}{6}$  | 12 $9 \text{ people} \times \frac{8}{16} \times \frac{6400}{1200}$                 |

- 13 7 men  $\times \frac{9\frac{3}{10}}{9\frac{9}{10}} \times \frac{3}{2}$       14 32 men  $\times \frac{7\frac{1}{10}}{10} \times \frac{20}{8}$
- 15 65 days  $\times \frac{4\frac{2}{3}}{11\frac{2}{3}} \times \frac{8}{5}$       16 24 days  $\times \frac{1\frac{7}{8}}{1\frac{5}{8}} \times \frac{5}{4}$
- 17 Rs. 50  $\times \frac{5\frac{1}{4}}{4\frac{1}{4}} \times \frac{34}{23}$       18 8 wks  $\times \frac{13}{17} \times \frac{257\frac{5}{6}}{121\frac{1}{3}}$
- 19 £42 75  $\times \frac{4\frac{2}{3}}{1\frac{2}{3}} \times \frac{2}{7} \times \frac{1\frac{0}{1}}{1\frac{0}{1}}$       20 120 pages  $\times \frac{2\frac{8}{11}}{4\frac{2}{11}} \times \frac{7}{11}$
- 21 (i) 1 H P  $\times \frac{3 \times 112 \times 550}{33000}$ ,      (ii) 1 H P  $\times \frac{5\frac{1}{2} \times 112 \times 100 \times 6}{33000}$ ,  
 (iii) 1 H P  $\times \frac{440 \times 10 \times 100 \times 3}{33000}$
- 22 1500 men  $\times \frac{1\frac{5}{6}}{1\frac{5}{6}} \times \frac{1\frac{0}{3}}{1\frac{0}{3}}$       23 35 days  $\times \frac{6\frac{0}{6}}{5\frac{0}{6}} \times \frac{6}{5}$
- 24 18 5s  $\times \frac{2\frac{4}{5}}{3\frac{4}{5}} \times \frac{2\frac{6}{8}}{9\frac{6}{8}}$       25 40 H.P.  $\times \frac{2\frac{2}{3} \times 4\frac{0}{6}}{2\frac{2}{3} \times 6\frac{0}{6}} \times \frac{4\frac{0}{6}}{2\frac{0}{6}}$
- 26  $3\frac{1}{2}$  hrs  $\times \frac{2}{3} \times \frac{9}{4}$       27 30 ml.  $\times \frac{7}{5} \times \frac{1\frac{0}{6}}{1\frac{0}{6}}$
- 28 40 English navvies are paid 7s  $\times 30 \times 40$ , i.e. £420 for the work,  
 56 Belgian workmen, 5s  $\times 40 \times 56$ , i.e. £560 " "  
 shewing the English navvy is more profitable.  
 The cost required = £0000  $\times \frac{5\frac{6}{10}}{4\frac{6}{10}}$

## EXAMPLES XVI g Page 321

1.-15 See Art 281

- 16 Let sides be  $5x$ ,  $7x$  and  $8x$  yds,  
 then  $5x + 7x - 12x = 270$ , whence  $x = 135$
- 17 Let the marks for the questions be  $20m$ ,  $32m$ ,  $48m$ ,  
 then  $20m + 32m + 48m = 150$ , whence  $m = 15$
- 18  $1\frac{1}{2}$ ,  $4$ ,  $2\frac{1}{2}$  are proportional to  $3$ ,  $8$ ,  $5$ , time is  $\frac{7}{16}$  of  $6\frac{1}{2}$  tons  
 To find percentages, we have  $\frac{x}{100} = \frac{3}{16}$ , etc.
- 19 Let  $\frac{x}{2}$ ,  $\frac{x}{3}$  and  $\frac{x}{9}$  metres be lengths, then  $\frac{x}{2} + \frac{x}{3} + \frac{x}{9} = 289$  and  $x = 306$
20. Let  $2\frac{1}{2}x$ ,  $3x$  and  $3\frac{1}{2}x$  feet be sides,  
 then  $2\frac{1}{2}x + 3x + 3\frac{1}{2}x = 338$ , whence  $x = 40$



- 21 The shares of the profits are  $\frac{4}{5}$  of £165½,  $\frac{5}{5}$  of £165½, and  $\frac{11}{5}$  of £165½
- 22 See No 21
- 23 Contributions are as 3 4 5,  
whence percentages are given by  $\frac{r}{100} = \frac{3}{12}$ , etc  
C gets  $\frac{r}{12}$  of Rs 15,000
- 24 They furnish  $\frac{42r, 53r, 760}{1700}$  respectively of Rs 51,000  
For second part of question  $\frac{r}{100} = \frac{425}{1700}$
- 25 The creditors receive  $\frac{250, 330, 420}{1000}$  respectively of £125
- 26 Shares are  $\frac{1r, 11r, 00}{100}$  respectively of 480d
- 27 Amount req<sup>d</sup> =  $\frac{22}{4}$  of  $300 \times 123\ 274$  grs  
=  $\frac{11}{2}$  of  $100 \times 123\ 274$  grs  
=  $\frac{11}{4}$  of  $12327\ 4$  grs = 33900 35 grs
- 28 Weight req<sup>d</sup> =  $\frac{23\ 04}{100}$  of 3750 gm
- 29 1 cu m = 1000 litres and weighs 1295 gms  
Hence weight of oxygen =  $\frac{23\ 04}{100}$  of 1295 gms = 298 3 gms
- 30 Weight req<sup>d</sup> =  $\frac{17}{100}$  of 207 4 Kg
- 31 If shares are £7r, £3r and £2x, then  $7x + 3x + 2x = 72\ 6$
- 32 Suppose C gets Rs x, then B gets Rs  $\frac{3x}{2}$  and A gets Rs  $\frac{5}{8} \times \frac{3x}{2}$ ,  
 $x + \frac{3x}{2} + \frac{15x}{16} = 20\frac{1}{4}$ , whence  $x = 6$
- 33 Since  $5\ 2 = 35\ 14$ , and  $7\ 13 = 14\ 26$ ,  
A's share B's share C's share = 35 14 26  
If shares are Rs 35r, Rs 14x and Rs 26x,  
then  $35r + 14x + 26x = 12\frac{1}{2}$ , whence  $x = \frac{1}{6}$
- 34 Since  $4s\ 5s\ 4d = 3\ 4 = 21\ 28$ ,  
and  $8s\ 9d\ 7r\ 6d = 7\ 6 = 28\ 24$ ,  
the shares may be taken as £21r, £28r and £24x,  
 $21r + 28r + 24x = 146$ ,  
and  $r = 2$

35  $3+20+3=26$   $7+56+2=65$

$$\text{Tin} = \left( \frac{3}{26} + \frac{7}{65} \right) = \frac{29}{130} \text{ of total weight}$$

$$\text{Copper} = \left( \frac{20}{26} + \frac{56}{65} \right) = \frac{213}{130} \quad ,$$

$$\text{Zinc} = \left( \frac{3}{26} + \frac{2}{65} \right) = \frac{19}{130} \quad \text{,,} \quad \text{,,}$$

36 The shares are  $\frac{20}{45} \frac{18}{45}$  respectively of £384

37 See No 36 It should be divided in proportion of  
 $50 \times 6$   $85 \times 4$ , or  $15$   $17$

38 See No 36 They should be divided in proportion of  
 $60 \times 3$   $40 \times 5$   $125 \times 2$ , or  $18$   $20$   $25$

39 A, B, and C are in the firm for 12, 8, and 6 months respectively  
 Rs 15,577 8s should be divided in proportion of  
 $40 \times 12$   $3 \times 8$   $50 \times 6$ , or  $40$   $2$   $25$

40 A, B, and C are in partnership for 3, 3, and  $2\frac{1}{2}$  years respectively  
 Rs 9660 must be divided in proportion of  
 $8 \times 3$   $10 \times 3$   $6 \times 2\frac{1}{2}$ , or  $8$   $10$   $5$

41 Suppose  $V_1, V_2$  the volumes,  $s_1$  and  $s_2$  the specific gravities,  
 then by note at end of question,  $\frac{W_1}{W_2} = \frac{V_1}{V_2} \times \frac{s_1}{s_2}$   
 $= \frac{4}{5} \times \frac{5}{6} = \frac{2}{3},$

giving 6 oz and 9 oz.

42 From note in No 41, weights are as  $3 \times 7$   $2 \times 4$   $5 \times 2$ ,  
 i.e. as  $21$   $8$   $10$ , giving 42 lbs, 16 lbs and 20 lbs.

43 The shares are as  $\frac{1}{3} \times \frac{1}{4}$   $\frac{1}{4} \times \frac{1}{2}$   $\frac{5}{12} \times 1$   
 i.e. as  $\frac{1}{12}$   $\frac{1}{8}$   $\frac{5}{12}$ , or as  $2$   $3$   $10$

44 The number of coins in each group is *jointly* proportional to the total value of the group directly, and to the value of each coin (in the group) inversely because the greater the total value of the group, the more coins we must have in it, but supposing the value of each coin is increased from a crown to a sovereign, there will be fewer coins in the group  
 no. of coins in the different groups will be as

$$\frac{14}{10} \frac{4}{7} \frac{3}{2}, \text{ or as } 7 \ 8 \ 15$$

45 The volumes are proportional to weights taken directly and to specific gravities taken inversely

$$\text{volumes are as } \frac{14}{10} \frac{15}{3} \frac{16}{4}, \text{ i.e. as } 7 \ 5 \ 4$$

Hence for second part of question we get 14 ht, 10 ht, 08 ht

## EXAMPLES XVI h Page 327

1  $24 \text{ in} \times \frac{15}{8}$       2 If  $z$  in be distance req<sup>d</sup>, then  $\frac{x}{144} = 35$

3  $40 \text{ sq cm} \times \frac{25^2}{10^2}$

4 (i)  $314 \text{ cm} \times \frac{7}{10}$ , (ii)  $314 \text{ sq cm} \times \frac{7^2}{10^2}$       5  $80 \times \frac{63^2}{25^2} \text{ sq yds}$

6  $132 \times \frac{35^2}{25^2} \text{ sq mi} = 132 \times 14^2 \text{ sq mi} = 25872 \text{ sq mi}$

7 (i)  $\frac{145}{3^2} \text{ ft}$ , (ii)  $145 \times \frac{4^2}{3^2}$  [From Art 283, let  $s = lt^2$

Now when  $t=1$ ,  $s=161$        $l=161$ , and we get  $s=161t^2$ ]

8  $328 \text{ lbs} \times \frac{1}{2^3}$  and  $328 \text{ lbs} \times \frac{3^3}{2^3}$

9 Breadth =  $6 \text{ cm} \times \frac{12}{8}$  Thickness =  $4 \text{ cm} \times \frac{12}{8}$  Wt =  $14 \text{ Kg} \times \frac{12^3}{8^3}$

10 Weight of first =  $130 \text{ lbs} \times \frac{25^3}{2^3} = 130 \times \frac{5^3}{4^3} \text{ lbs} = \frac{1625}{64} \text{ lbs}$

Weight of second =  $13 \text{ lbs} \times \frac{7^3}{2^3} = \frac{4459}{8} \text{ lbs}$

11 Let  $N$  be no of revolutions required Since the no of revolutions in a given time varies inversely as diameter of the wheel,

$$\frac{N}{130} = \frac{12}{20}, \text{ whence } N = 78$$

[The second wheel need not be taken into account]

12 Volume req<sup>d</sup> =  $1728 \times \frac{147}{84} \text{ cu in}$

13 The first is paid 28s for 48 hrs and does  $48 \times 5$ , i.e. 240 units of work

The second „ 25s „ 54 hrs „  $54 \times 4$ , i.e. 216 „ „

we have to compare the costs for 1 unit, viz  $\frac{28s}{240}$  and  $\frac{25s}{216}$

The latter is the smaller, giving 'the second man' for answer

14  $9d \times \frac{21}{3} \times \frac{1351}{100} = 10d$  and a small fraction

- 15 Let  $B$  put in Rs.  $r$

$A$  lends at the rate of [Rs  $23250 \times 4 + \text{Rs } 27000 \times 8$ ] for 1 month

$B$  lends at the rate of [Rs  $r \times 7 + \text{Rs } (r - 3000) \times 5$ ] for 1 month  
(See Ex. XVI g, No 36)

$$7r + 5(r - 3000) = 23250 \times 4 + 27000 \times 8, \text{ whence } r = 27000$$

- 16 If  $W$  be weight req<sup>d</sup>, then  $\frac{W}{0.00605} = \left(\frac{29\frac{1}{2}}{18\frac{1}{2}}\right)^2 = \frac{59^2}{37^2}$ ,

$$\text{from which } W = \frac{21.00}{1.369} = 0.01538 \text{ gm}$$

For second part of question,

difference is 0.00183 in 0.01355,

$$i.e. \frac{18.3}{135.5} \text{ in } 100, \text{ or } 13.5\%$$

- 17 If  $h$  be height req<sup>d</sup>, then  $\frac{h^3}{5^3} = \frac{1}{8}$ , whence  $h = 2\frac{1}{2}$

- 18 We have  $\frac{y}{16} = \frac{r^2}{8^2}$  or  $y = \frac{1}{4}x^2$ . Substituting the given values for  $x$  in turn in this equation, we find the corresponding values of  $y$  to be 0,  $\frac{1}{4}$ , 1,  $\frac{9}{4}$ , 4,  $\frac{25}{4}$ , 9,  $\frac{49}{4}$ .

If the given values of  $r$  were negative, the resulting values of  $y$  would be positive (see Art 54), and would be 0,  $\frac{1}{4}$ , 1,  $\frac{9}{4}$ , 4,  $\frac{25}{4}$ , 9,  $\frac{49}{4}$ . The graph is shown in Fig 15, on a reduced scale

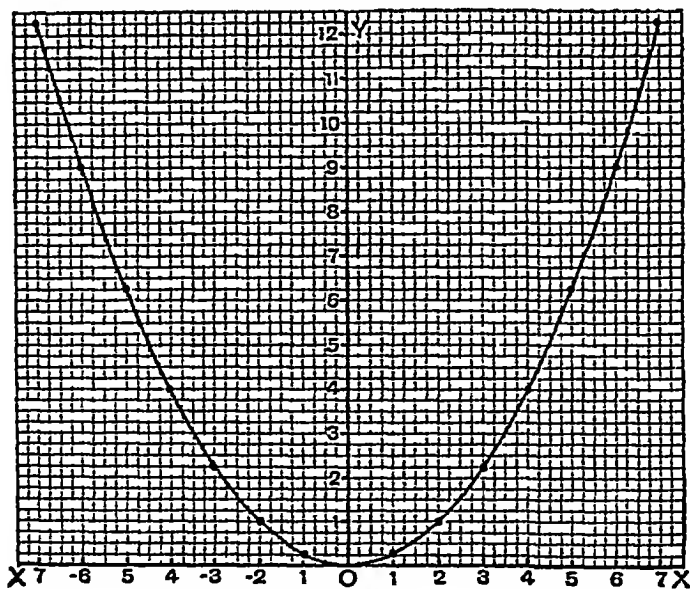


FIG 15

- 19 If we substitute the given values for  $C$  in turn in the equation, the corresponding values of  $S$  are found to be  $0.1''$ ,  $0.4''$ ,  $0.9''$ ,  $1.6''$ ,  $2.5''$ ,  $3.6''$ ,  $4.9''$ ,  $6.4''$ . Taking a scale of 2 in along the axis of  $x$  to 1 in circumference, and 1 in along the axis of  $y$  to 1 ton, and plotting the points, we obtain a graph resembling the right-hand half of that in Ex 18 (Fig 15)
- 20 On completing the table, we shall find it convenient to take a scale of  $0.1''$  to a mile horizontally and  $0.1''$  to 0.125 lb vertically. After plotting the points, we shall obtain a graph resembling the right-hand half of that in Ex 18 (Fig 15)

### MISCELLANEOUS EXAMPLES IV Page 331

#### A

- 1 (i) Multiply numerator and denominator of first fraction by 45, it becomes  $\frac{150-25+9}{120+13}$ , i.e.  $\frac{134}{133}$

Operate on second fraction in the same way with 15,

$$\text{and it becomes } \frac{18}{105-35}, \text{ i.e. } \frac{18}{133}$$

- (ii) Treating first fraction as in (i), we get

$$\exp^n = \frac{64-13}{50+79} - \frac{0.32}{18} = \frac{17}{45} - \frac{16}{90}$$

- 3  $40 \text{ mi per hr} = 40 \times \frac{5280}{3600} \text{ ft per sec}$  (See p 60 of *Key*, Note on Ex XII b)  
 $= 58\frac{2}{3} \text{ ft per sec}$

$$\begin{aligned} 33 \text{ knots} &= \frac{33 \times 6080}{60 \times 60} \text{ ft per sec} \\ &= 55\frac{1}{3} \text{ ft per sec} \end{aligned}$$

- 4  $\frac{2}{3} \text{ mi} - 4000 = \frac{2 \times 1760 \times 3 \times 12}{3 \times 4000} \text{ in} = 10.56 \text{ in}$

- 5 Total length of a side  $= 35 \times 12 \text{ in}$

$$\text{No. of tiles that can be placed} = \frac{35 \times 12}{8} = \frac{105}{2} = 52\frac{1}{2},$$

52 complete tiles must be used, leaving a 2 in space at each end. As this holds good for all sides of the courtyard, we have a square piece of paving consisting of  $52 \times 52$ , or 2704 tiles.

$$\begin{aligned} \text{Uncovered space} &= \text{area of courtyard} - \text{area of tiling} \\ &= [(35 \times 12)^2 - (52 \times 8)^2] = (420^2 - 416^2) = (420 + 416)(420 - 416) \\ &= 836 \times 4 = 3344 \text{ sq in} \end{aligned}$$

## B

7 Value = Rs 13 6875 × 20 4875

204 875	
13 6875	1 36875
204 875	
61 462	5
12 292	5
1 639	0
113	4
010	2
Rs 250 422	6

8 Exp<sup>n</sup> =  $1\frac{4}{7} - \frac{1}{7} \times \frac{1}{8} + \frac{1}{104} \times \frac{1}{8} \times \frac{1}{8} = 1\frac{4}{7} - \frac{1}{7} + \frac{1}{75}$

9 1 cwt. costs  $\text{£}1\frac{1}{8} \times \frac{1}{16} + \text{£}1\frac{1}{8} \times \frac{1}{8}$  It is sold for  $\text{£}\frac{1}{16} \times 112$ ,  
 the profit on 1 cwt. =  $\text{£}(\frac{1}{16} \times 112 - 1\frac{1}{8} \times \frac{1}{8} - 1\frac{1}{8} \times \frac{1}{16}) = \text{£}3\frac{5}{12}$ ,  
 whence profit on 2 cwt 24 lbs =  $\text{£}3\frac{5}{12} \times 2\frac{1}{4} = \text{£}7\frac{11}{8} = \text{£}7\ 11s\ 4d$

10  $\text{£}15\ 5 = (25\ 20 \times 15\ 5)$  fr, or 390 6 fr

11 Amount req<sup>d</sup> =  $\text{£}\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \text{£}0\ 755 = 15s\ 1d$

12 If  $x$  be no. of days req<sup>d</sup>, then  $\frac{2}{x} \times r = 12\frac{1}{2}$ ,  
 whence  $x = 113\frac{1}{7}$ , giving for result 114 days

## C

13 13% of 3 mds 30 srs =  $\frac{1}{10} \times \frac{1}{10}$  of 150 srs =  $\frac{1}{10} \times 150 = 15$  srs

14  $\frac{14\ 52}{83\ 1} + \frac{1\ 75}{1\ 782} = \frac{4\ 84}{2\ 77} + \frac{1\ 17}{2\ 77} = \frac{6\ 01}{2\ 77} = 0\ 288447$

15 Area =  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 12\ 8$  ac, i.e. 13 ac. (to nearest acre)

Money req<sup>d</sup> =  $\text{£}17\ 1s\ 6d \times 6\frac{1}{2} + \text{£}21\ 15s\ 6d \times 6\frac{1}{2} = \text{£}39 \times 6\frac{1}{2}$   
 $= \text{£}253\ 10s$

16. We have  $\frac{2\ ac\ 2\ 1\ 7\ 12\ po}{3\frac{1}{2}\ ac} = \frac{(2 \times 4 \times 10 + 2 \times 40 + 7\ 12)\ po}{3\frac{1}{2} \times 1 \times 40\ po}$   
 $= \frac{407\ 12}{560} = 0\ 727$

17. Ans =  $(16\frac{1}{2} \times 1016\ 05)$  Kg = 16510 8 Kg

18 A runs  $(28 \times 5)$  yds while B runs  $(16 \times 3)$  yds in 15 secs,  
 i.e. A " 140 yds " B " 138 yds,  
 " A " 700 yds " B "  $138 \times \frac{9}{4} = 690$  yds

## D

19 41 cu ft of teak weigh 2240 lbs,

1 " "  $\frac{2}{4} \times \frac{1}{1} = \frac{1}{2}$  lbs, i.e. 54 lbs

$$\begin{aligned}
 20 \quad \text{Value} &= £63\frac{2}{3} \times 0.628125 = £\frac{191}{3} \times \frac{628\frac{1}{8}}{1000} = £\frac{191}{3} \times \frac{5025}{8000} \\
 &= £\frac{19797}{32} = £39 \text{ } 19\text{s } 9\frac{3}{4}\text{d}
 \end{aligned}$$

21 Let  $A, B, C$  be the three numbers Using notation of Art 97,  
we have  $A \ B \ C = mX \ nX \ pX = (mnp \ X) \times X^2 = 29172 \times 17^2$

22 Let  $x$  annas and  $(x+5)$  annas per score be req<sup>d</sup> prices  
Then  $25x + 25(x+5) = 625$ , whence  $x = 10$ ,

23 5 cwt 3 q<sup>rs</sup> 20 lbs = 664 lbs, 2 cwt 16 lbs = 240 lbs,  
value =  $£44\frac{6}{10} \times \frac{2}{3}\frac{4}{8}\frac{0}{4} = £16$

24 If  $A$  pays  $B$  Rs 90,  $B$  pays  $C$  Rs 90 and  $C$  pays  $A$  Rs 90, then they  
are in just the same position as before, for each pays and  
receives Rs 90 After this, however,  $A$  owes  $B$  Rs 35,  $B$  owes  
 $C$  Re 0, and  $A$  owes  $C$  Rs 23 They can therefore settle  
accounts by  $A$  paying  $B$  Rs 35 and  $C$  Rs 23

## E

25 Area =  $3\frac{2}{3}\frac{1}{2}$  big, expenditure = Rs  $440 \times 3\frac{5}{12}\frac{1}{2} + \text{Rs } 3808\frac{3}{4}$   
= Rs  $(1320 + 371\frac{1}{4} + 3808\frac{3}{4}) = \text{Rs } 5500$

26 4 men and 9 boys do  $\frac{5}{6}$  of work in 1 day,  
8 " 18 "  $\frac{5}{3}$  " 1 " (1)

Again 3 " 6 "  $\frac{7}{12}$  " 1 "  
9 " 18 "  $\frac{7}{4}$  " 1 " (2)

From (1) and (2) we see that 1 man does  $(\frac{7}{4} - \frac{5}{3})$  of work in 1 day,  
i.e. 1 man does  $\frac{1}{12}$  of work in 1 day and would do the whole  
in 12 days

Again, since 3 men and 6 boys do  $\frac{7}{12}$  in 1 day,  
it follows that 6 boys do  $(\frac{7}{12} - \frac{3}{12})$  in 1 day,  
and 1 boy would do the whole in 18 days

Otherwise Let a man take  $x$  days and a boy  $y$  days,

then a man does  $\frac{1}{x}$  in 1 day and a boy  $\frac{1}{y}$  in 1 day,

$$\frac{4}{x} + \frac{9}{y} = \frac{5}{6} \quad (1)$$

$$\frac{3}{x} + \frac{6}{y} = \frac{7}{12} \quad (2)$$

Multiplying (1) by 2, multiplying (2) by 3 and subtracting,

we get  $\frac{1}{x} = \frac{1}{12}$ , and  $x = 12$

Substituting the value of  $x$  in (1), we find  $y = 18$

$$\begin{aligned} 27 \quad \text{Exp}^n &= 31\ 21325 + 3\ 20875 - 20\ 5375 - 13\ 632625 \\ &= 34\ 422 \qquad \qquad - 34\ 170125 \\ &= 0\ 251875 = 0\ 251\frac{3}{4} \end{aligned}$$

28 Let the cost of the Army be  $x$  per cent of the total

$$\begin{aligned} \text{Then } \frac{x}{100} &= \frac{91\ 710000}{183\ 592264} = \frac{91\ 7}{183\ 6} \text{ (nearly)} = 0\ 499, \\ x &= 49\ 9 = 50 \text{ (to nearest integer)} \end{aligned}$$

Similarly for the Navy,

$$\begin{aligned} \frac{x}{100} &= \frac{29,520,000}{183,592,264} = \frac{295}{1836} \text{ (nearly)} = 0\ 161, \\ x &= 16\ 1 = 16 \text{ (to nearest integer)} \end{aligned}$$

Also for the Civil Services,

$$\begin{aligned} \frac{x}{100} &= \frac{23,500,000}{183,592,264} = \frac{235}{1836} \text{ (nearly)} = 0\ 128, \\ x &= 12\ 8 = 13 \text{ (to nearest integer)} \end{aligned}$$

29 Second man runs 5214 yds in 2133 secs,

$$\begin{aligned} i.e. \quad \frac{5214 \times 66 \times 60}{2133 \times 1760} \text{ mi in 1 hr,} \\ \text{or } 5 \text{ mi in 1 hr} \end{aligned}$$

30 Let the third class fare be  $x$  shillings,

then the first-class fare is  $(x + 1\frac{1}{2})$  shillings,

$$325(x + 1\frac{1}{2}) + 1250x = 300 \times 20$$

Divide by 25, then  $13(x + 1\frac{1}{2}) + 50x = 12 \times 20$ , whence  $x = 3\frac{1}{2}$

F

$$\begin{aligned} 31 \quad (i) \quad \text{Exp}^n &= \frac{\frac{f}{4} + \frac{1}{4}}{\frac{1}{4} + 2\frac{f}{12}} \times \frac{12}{12} + \frac{11}{34} \\ &= \frac{\frac{60}{30} + \frac{6}{30}}{\frac{1}{30} + \frac{20}{30}} + \frac{11}{34} = \frac{7}{4} \end{aligned}$$

$$(ii) \quad \text{Exp}^n = \frac{\frac{11}{4} - \frac{f}{12}}{\frac{1}{4} + 2\frac{f}{12} - \frac{1}{6}} \times \frac{48}{48} = \frac{22 - 20}{1 + 0 + 8} = 1$$

32 It was seen twice in the first 13 secs, three times in the first 26 secs, and so on,

$$\text{reqd no of times} = \frac{\text{total number of secs}}{13} + 1$$



- 33 Let each man earn  $x$  annas, each boy  $y$  annas per day  
 Then " " " " "  $6y$  " per week,  

$$\left. \begin{aligned} 6x \times 8 + 6y \times 4 &= 30 \times 16, \\ \text{and } 6x \times 5 + 6y \times 3 &= 19\frac{1}{2} \times 16 \end{aligned} \right\}$$

These become on reduction,

$$\left. \begin{aligned} 12x + 6y &= 120, \\ \text{and } 10x + 6y &= 101\frac{1}{2}, \end{aligned} \right\} \text{ giving } x=8, y=4$$

- 35 From Ex 21, p 306, gross rental =  $\frac{2}{11}$  of £7101 14s 6d

- 36 1 sov  $\equiv \frac{480}{1800}$  oz  $\equiv \frac{480 \times 480}{1800}$  grs,  
 pure gold in 1 sov =  $\frac{2}{3}$  of  $\frac{480 \times 480}{1800} = \frac{70400}{3} = 113$

G

- 37 See Arts 82 and 88

- 38  $6897000 \times 56 \times 16$  oz were produced per  $187000 \times 4840$  sq yds,  
 i.e.  $\frac{6897 \times 56 \times 16}{187 \times 4840}$  oz " " 1 sq yd,  
 i.e.  $\frac{300 \times 16}{17 \times 55}$  oz, or 68 oz, were produced per 1 sq yd

- 39 Given price is £  $\frac{47}{240}$  per  $\frac{36}{50 \times 3}$  metres,  
 i.e.  $\frac{47}{240} \times 2540$  ft per  $\frac{36}{50 \times 3}$  metres,  
 i.e.  $\frac{47 \times 254 \times 10 \times 30 \times 3}{240 \times 36}$  ft per metre,  
 i.e.  $\frac{47 \times 119 \times 30 \times 30}{120 \times 36}$  ft per metre,  
 i.e.  $\frac{231}{4 \times 12}$  fr, or 5 14 ft per metre

- 40 Exp<sup>n</sup> = £  $1 \frac{3416}{107}$  + £1 6875 × 0 58 - £0 9625 × 1 439  
 = £(1 2658 + 0 9786 - 1 3850)  
 = £0 859 = 17s 2d

- 41 Total weight of 1 sov =  $\frac{1}{11}$  of weight of pure gold in it,  
 1 sov weighs  $113 \times \frac{1}{11}$  gr,  
 and  $\frac{11}{12 \times 113} \times 7000$ , or nearly 57 sovs weigh 7000 grs, or 1 lb Av
- 42 4060 yds remain, total no of men wanted during 2<sup>nd</sup> year  
 is  $1000 \times \frac{4060}{2080}$ , giving 363 extra men

H

- 43 Take scales of an inch to 5 lbs horizontally and an inch to 5 Kg vertically. As in Art 277, Example,  $y=1\frac{1}{11}$ . Join the point whose coordinates are (31, 11) to the origin. The required values may be read off from the graph

44. Total debts = £715  $\frac{1}{2}$ . Hence he owes  $\frac{315\frac{1}{2}}{745\frac{3}{4}}$  of £110 3s 3d to A,  
 $\frac{255\frac{1}{2}}{745\frac{3}{4}}$  of £110 3s 3d to B, and  $\frac{175}{745\frac{3}{4}}$  of £110 3s 3d to C

45 Let  $r$  be the no of concerts, then  $5r = 21 \times 2\frac{1}{2}$ ,  
 whence  $r = 8\frac{1}{2}$ , giving 9 as answer

46 Suppose  $x$  persons were admitted without payment  
 Then  $(15568 - x)$  paid 6d, and of these 1215 paid 1s extra,  
 $(15568 - x) \times 6 + 1215 \times 12 = 101610$  (receipts in pence),  
 and  $x = 1123$

47 71 ac 1 r 15 po =  $(71 + 0.25 + 0.09375)$

71 343	75
	2 625
142 687	5
12 806	2
1 426	9
356	7
£187 277	3

giving £187 5s 7d

48 No of men necessary to do the required work is  $7 \times 2 \times 3$ , or 42,  
 we must replace 12 of these by boys,  
 hence  $\frac{1}{2}$  of 12, or 30 boys are required

# I

49 If A sq in be area reqd, then expressing the volume in cu in,  
 $8 \times 36 \times A = 3 \times 1728$ , and  $A = 18$  sq in

50 Take a scale of 0.4" to 1 rupee horizontally, then each 0.1' represents 4a, and an estimate correct to the nearest 1a can easily be made. A scale of 0.5" to 1 rupee vertically should be taken. If  $y$  ms cost  $x$  rupees, as in Art 277, Example, since Rs 92 8a = Rs  $1\frac{8}{10}$  and 1 md = 40 ms,  $y = \frac{80}{18}x$ . This may be written  $y = 1\frac{2}{9}x$ , or  $y = \frac{10}{9}x$ . Take for origin the point (16, 37) and join it to the point (32, 71) by a straight line. From this graph the required values may be read off.

- 51 3 men or 7 women will take 64 days to do a piece of work twice as great

Now 7 men and 5 women take 7 times as long as 49 men and 35 women,

i.e. 7 men and 5 women take 7 times as long as 49 men and 15 men,

i.e. 7 men and 5 women take 7 times as long as 64 men,

i.e. 7 men and 5 women take  $64 \text{ days} \times \frac{3}{8} \times 7$ , or 21 days

NB 35 is the LCM of 5 and 7,

35 women can be replaced by an exact no of men

- 52 15 oz cost  $11\frac{1}{2}$  a, 16 oz cost  $1\frac{5}{8}$  of  $11\frac{1}{2}$  a

- 53 Let  $s$  ft be length of train,  $v$  ft per sec be its speed

From Art 214,  $s + 132 \times 3 = v \times 7\frac{1}{2}$ , ... (1)

$$s = v \times 3 \quad (2)$$

Subtracting (2) from (1),  $396 = v \times 4\frac{1}{2}$ , and  $v = 88$  ft per sec, giving 60 mi per hour

- 54 (Draw a diagram)

$$\begin{aligned} \text{Cost in annas} &= 8 \times (25\frac{1}{2} \times 19\frac{1}{4} - 21\frac{1}{2} \times 16\frac{3}{4}) + \frac{9}{5} \times 21\frac{1}{2} \times 16\frac{3}{4} \\ &= 8 \times 25\frac{1}{2} \times 19\frac{1}{4} + (\frac{9}{5} - 8) \times 21\frac{1}{2} \times 16\frac{3}{4} \\ &= 8 \times \frac{51}{2} \times \frac{469}{4} - \frac{8}{5} \times \frac{4}{2} \times \frac{17}{8} \\ &= \frac{1977}{2} - \frac{64}{5} = 3664 \\ \text{cost} &= \text{Rs } 229 \end{aligned}$$

J

- 56 (i)  $\text{Exp}^a = \frac{\frac{1}{3} \times \frac{4}{3} \times \frac{8}{5}}{\frac{4}{5} + 2\frac{2}{5}} = \frac{32}{144}$

$$(ii) \text{Exp}^a = \frac{\frac{3}{13} - \frac{1}{144}}{12\frac{1}{11} - 11\frac{1}{14} - \frac{1}{80}} = \frac{\frac{3}{13}}{\frac{6}{88}}$$

- 57 (Draw diagram) Area of path =  $(82 \times 75 - 74 \times 67)$  sq ft, which reduces to 132 sq yds 4 sq ft

- 58 A does in 8 days what B does in 15 days and what C does in 12 days,

$$B's \text{ time} = A's \text{ time} \times \frac{1}{8} = 12 \text{ wks} \times \frac{1}{8}$$

- 59 Take scales of 1" to 0.2 Kg horizontally, and 0.2" to 1 lb vertically As in Art 277, Example,  $y = \frac{2}{1} \frac{7}{8} x$ , or  $y = \frac{7}{4} x$  Take for origin the point (19, 27) Join it to the point (38, 54) From this graph the required values may be read off

$$60 \quad 27 \text{ mds } 5 \text{ srs } = 1085 \text{ sis}$$

$$\text{Cost} = 209 \text{ a} \times \frac{46\frac{1}{2}}{52\frac{1}{2}} \times \frac{700 \times 40}{1085} = 4800 \text{ a} = \text{Rs } 300$$

## K.

$$61 \quad 22 \text{ cwt } 3 \text{ qrs } 21 \text{ lbs } = 22 \text{ } 9375 \text{ cwt}$$

$$£12 \text{ } 16s \text{ } 8d = £12 \text{ } 83333$$

$$\begin{array}{r|l} 22 \text{ } 937 & 5 \\ 1 & 2 \text{ } 8333333 \\ \hline 229 \text{ } 375 & \\ 45 \text{ } 875 & \\ 18 \text{ } 350 & \\ 688 & 1 \\ 68 & 8 \\ 6 & 9 \\ & 7 \\ & 1 \\ \hline £294 \text{ } 364 & 6 \end{array}$$

*NB* It is easier to multiply by the money, on account of the '3's' recurring

giving £294 7s 3½d

$$62 \quad \text{Exp}^n = \frac{1 + \frac{2}{3} - \frac{1}{2}}{1 + \frac{1}{3} - \frac{1}{5}} \times 6\frac{6}{7} = \frac{60 + 40 - 51}{60 + 20 - 32} \times \frac{48}{7}$$

$$63 \quad \begin{array}{r} 441,00 \text{ } 01,5 \quad (210 \text{ } 00004 \\ 41 \overline{) 41} \\ 42,00 \overline{) 00 \text{ } 01,5} \end{array}$$

64 Using notation of Ex 2, page 330, we get

$$3(x+y)=27 \text{ and } 9(x-y)=27$$

These give  $x+y=9$  and  $x-y=3$

Adding, we find  $x=6$ , whence  $y=3$

Answer for (ii) =  $\frac{27}{6} = 4\frac{1}{2}$  hours

$$65 \quad 1\frac{1}{2} \text{ pints in } 1 \text{ min} = 3 \text{ gallons in } 16 \text{ min} = \frac{3}{16} \times 60 \text{ gals in } 60 \text{ min} \\ = \frac{3}{16} \times 60 \times \frac{1}{0 \text{ } 2 \text{ } 2 \text{ } 01} \text{ lit in } 1 \text{ hour,} \\ \text{ie } \frac{45}{0 \text{ } 8 \text{ } 8 \text{ } 04} \text{ lit, or } 51 \text{ lit in } 1 \text{ hr}$$

66 His time must be greater in the ratio 1760 : 1710

(See Arts 269 and 270)

$$\text{time required} = \left( 3\frac{251}{60} \times \frac{1760}{1710} \right) \text{ min} = 3\frac{27}{10} \text{ min} = 3 \text{ min } 31\frac{1}{5} \text{ secs}$$

## L

67 Let a seer cost  $z$  annas, then  $\frac{3}{2} \times 6 + z = 35$ , and  $z = 3\frac{1}{2}$

68 Required sum is £183 3s 10d  $\times \frac{14s \ 7\frac{1}{2}d}{£1}$

$$= £183 \ 692 \times 0 \ 730208$$

$$= £134 \ 2s \ 8d$$

$$\begin{array}{r|l} 183 \ 692 & \\ \hline 128 \ 584 & 4 \\ 5 \ 510 & 8 \\ 36 & 7 \\ 1 & 4 \\ \hline 134 \ 133 & 3 \end{array}$$

69  $(1600 \times 9)$  sq ft cost Rs 165 ,

$$(80 \times 1760 \times 3) \text{ sq ft cost Rs } 165 \times \frac{90 \times 1760 \times 3}{1600 \times 9}, \text{ or Rs } 4840$$

70 1320 yds or  $\frac{3}{4}$  mi =  $18\frac{3}{4}$  in , whence 1 mi = 25 in ,

1 sq mi = 625 sq in , giving second answer

71 (i) Product  $< 4 \times 0.9$ , i.e. 3.6

(ii) Quotient  $> \frac{3.6}{0.9}$ , i.e. 4.4

(iii) From Art 226, p 246, we see that first figure in  $\sqrt{0.324}$  is 0.5

72 Total expenditure = Rs 2 13a 6p  $\times 122$  = Rs 346 15a

Expenditure in Sept = Rs 3 5a  $\times 30$  = Rs 99 6a

„ Oct = Rs 2 8a  $\times 31$  = Rs 77 8a

„ Nov = Rs 1 15a  $\times 30$  = Rs 58 2a

expenditure in Dec = Rs 111 15a

This will give Rs 3 10a as answer

## M

73  $(24 \ 25 \times 16 \ 8)$  sq ch =  $(2 \ 425 \times 16 \ 8)$  ac ,

$$\text{rent} = £5 \ 775 \times 2 \ 425 \times 16 \ 8 = £(14 \ 00438 \times 16 \ 8) = £235 \ 2735$$

74 13 men and 15 boys take 16 times as long to hoe 1 piece of ground as 13  $\times$  16 men and 15  $\times$  16 boys take, or as 13  $\times$  16 men and 15  $\times$  9 men take, or as  $(208 + 135)$  men, i.e. 343 men take

$$\text{reqd no of days} = 7 \times \frac{9}{343} \times \frac{4}{16} \times \frac{16}{9} \times 16 = 10$$

Aliter If we work with a fraction of a man,

since 1 boy =  $\frac{9}{16}$  of a man,

we have 13 men and 15 boys =  $(13 + \frac{15 \times 9}{16})$  men, i.e.  $21\frac{7}{16}$  men

$$\text{no of days required} = \left( 7 \times \frac{10}{9} \times \frac{49}{16} \times \frac{9}{21\frac{7}{16}} \right) = 10$$

- 75 Let  $A$ ,  $B$  and  $C$  get Rs  $x$ , Rs  $3x$  and Rs  $4x$  respectively ,

$$x + 3x + 4x = 7852\frac{1}{2}, \text{ and } x = 981\frac{1}{8}$$

$A$  gets Rs 981  $\frac{1}{8}$  a , etc.

- 76 See Ex 2, p 227 Velocity of approach =  $(60 + 40)$  Km ,  $\therefore$  100 Km  
Sum of lengths = 175 m ,

$$\text{time reqd} = \frac{175}{100 \times 1000} \text{ hours , } \therefore 0.00175 \text{ hrs , or } 6.3 \text{ secs}$$

For second part of question, we get velocity of approach  $(60 - 40)$  Km ,  $\therefore$  20 Km, giving time reqd as 31.5 secs

- 77 Let  $x$  ft be thickness of beam

$$9x^2 \times 32 = 3\frac{1}{2} \times 112, \text{ whence } x = 1\frac{1}{8} \text{ ft}$$

- 78 Take scales of 1 in to 10 oranges horizontally, and 1 in to 20 a vertically Since Rs 3  $2\frac{1}{2}$  a = 50 a, as in Art 277, Example,

$$y = \frac{50}{20} x, \text{ or } y = \frac{5}{2} x$$

Join the point whose coordinates are (13, 10) to the origin  
From this graph we can read off the required values

### N

- 79 1 cu ft. of water  $\equiv (30.48)^3$  gms ,

$$62.426 \text{ lbs} \equiv (30.48)^3 \times 0.001 \text{ Kg} ,$$

$$1 \text{ Kg} \equiv \frac{62.426}{(30.48)^3} \text{ lbs , } \therefore \frac{62.426}{28320} \text{ lbs , } \therefore 2.2 \text{ lbs}$$

- 80 Wt =  $16 \times 5\frac{1}{2} \times 2\frac{7}{8} \times 1\frac{5}{16} \text{ oz} = \frac{78720}{256} \text{ oz} = 307.5 \text{ oz}$

As a simple test, take block to be  $(5 \times 3 \times 1\frac{1}{4})$  cu in , it then weighs 30 oz

- 81 (See Ex 2, p 330) Let  $x$  miles per hour be rate of stream and  $y$  miles the distance rowed

Then  $y = 7(8 - x)$  Also  $y = 5(8 + x)$ ,

$$7(8 - x) = 5(8 + x), \text{ whence } x = 1\frac{1}{3}$$

- 82 Let each man construct  $x$  yards per day  
Then, if  $y$  yards be length of path,

$$3x \times 2 = \frac{y}{5} - 1760, \quad (1)$$

$$\text{and } 18x = \frac{2y}{5} - 1760 \quad (2)$$

Adding (1) and (2),  $24x = \frac{3y}{5}$ , and  $6x = \frac{3y}{20}$

Substituting this value for  $6x$  in (1),

$$\text{we have } \frac{3y}{20} = \frac{y}{5} - 1760, \text{ and } y = 1760 \times 20, \text{ i.e. } 20 \text{ miles}$$

- 83 Let  $l$  ft be length of train and  $v$  ft per sec be velocity of train  
Since 2 miles an hour =  $\frac{88}{30}$  ft per sec, and 4 miles an hour =  $\frac{88}{15}$  ft per sec, velocity of approach of first person and train =  $v - \frac{88}{30}$ , of second person and train =  $v - \frac{88}{15}$

From Ex 2, p 227, we see time req<sup>d</sup> =  $\frac{\text{length}}{\text{velocity of approach}}$ ,

or length = velocity of approach  $\times$  time req<sup>d</sup>,

$$l = (v - \frac{88}{30}) \times 9, \text{ also } l = (v - \frac{88}{15}) \times 10,$$

$$10(v - \frac{88}{15}) = 9(v - \frac{88}{30}), \text{ and } v = \frac{484}{15} \text{ ft per sec,}$$

or 22 mi per hour

- 84 The equation to OP is  $y = \frac{14}{10}x$ ,  
or  $y = 1.414x$  Cf Art 275

### O

- 85  $(3+15+105)$  times, i.e. 123 times  $93 = 105 - 15 + 3$

- 86 Let  $d$  ft be the required depth Then weight of water  
=  $1000 \times 24 \times 15 \times d$  oz, and also =  $9000 \times 10 \times 16$  oz  
Since these are equal, we find  $d = 4$  ft

- 87 Time req<sup>d</sup> =  $23 \times 7 \times \frac{200}{280} \times \frac{18}{16}$  days, i.e. 138 days

- 88 Let 1 seer cost  $x$  annas At higher price 1 seer costs  $(x + 1\frac{1}{4})$  annas,

$$15\frac{1}{2} \text{ chh cost } \frac{15\frac{1}{2}}{16}(x + 1\frac{1}{4}) \text{ annas,}$$

$$\frac{15\frac{1}{2}}{16}(x + 1\frac{1}{4}) = x, \text{ and } x = 38\frac{3}{4} \text{ a}$$

- 89 For second part of question, total height of the four men  
 $= (1.72 \pm 0.05) \times 4 \text{ m} = (6.88 \pm 0.2) \text{ m}$

$$\text{Total height of the three} = (1.8 + 1.65 + 1.58 \pm 0.05 \times 3) \text{ m} \\ = (5.03 \pm 0.15) \text{ m}$$

$$\text{Difference} = (1.85 \pm 0.35) \text{ m}, \text{ giving for limits } 1.815 \text{ m}, \text{ and } 1.885 \text{ m}$$

- 90 (i) Area  $= [(11.4)^2 - (8.6)^2] \times 0.7854 \text{ sq ft} = 43.97 \text{ sq ft}$   
 $= 44 \text{ sq ft}$

$$(ii) \text{ Weight} = 6.7 \times [(7.3)^2 - (4.7)^2] \times 0.7854 \text{ gm} = 164.18 \text{ gm} \\ = 164 \text{ gm}$$

## P

- 91 Let  $x$  runs be average required

$$\text{Then } 12 \times 24 + 3x = 15 \times 30, \text{ and } x = 54$$

- 92 (i) Cost  $= \text{Rs } 5.11 \times 87 + 14 \times 6p \times 2 \times 87$

$$= (\text{Rs } 5.11 \text{ a} + \text{Rs } 1.13 \text{ a}) \times 87 = \text{Rs } 7.87 \times 87 = \text{Rs } 652.87$$

$$(ii) 8250 \text{ at Rs } 7.14 \text{ a per } 1000 = 8\frac{1}{2} \text{ at Rs } 7.14 \text{ a each} = \text{etc}$$

$$(iii) (49\frac{1}{16} \times \frac{1}{4}) \text{ m} = (50 - \frac{1}{16}) \times \frac{1}{4} \text{ m} = 25 \text{ m} - \frac{1}{64} \text{ m} \\ = 25 \text{ m} - 495 \text{ yds} = 24 \text{ m } 1265 \text{ yds}$$

- 93 Cost  $= [2 \times (5\frac{1}{2} + 4) \times 3 + 5\frac{1}{2} \times 4] \times 8 \text{ a} = 79 \times 8 \text{ a} = \text{Rs } 39.8 \text{ a}$

- 94  $(3.5 \times 7.4) \text{ sq in.} = 12535\frac{7}{100} \text{ sq in.}$

$$1 \text{ sq in} = \frac{12535\frac{7}{100}}{3.5 \times 7.4} \text{ sq m}$$

$$\text{Now } \frac{7}{100} < 0.001, \quad \frac{7}{100} - (3.5 \times 7.4) < 0.0001$$

the square root of this is 0.01

This shows  $\frac{7}{100}$  may be neglected

$$1 \text{ sq in} = \frac{12535}{3.5 \times 7.4} \text{ sq m} = 483.9 \text{ sq m}, \text{ i.e. } 484 \text{ sq m} \\ \text{scale is } 1 \text{ in to } 22 \text{ m}, \text{ since } 484 = 22^2$$

- 95 Let him row  $x$  mi per hour on still water

$$\text{Then } 21 = 3\frac{1}{2}(x - 2), \text{ and } x = 8$$

$$\text{time required} = \frac{21}{8 - 2} \text{ hrs.}, \text{ i.e. } 2 \text{ hrs}$$



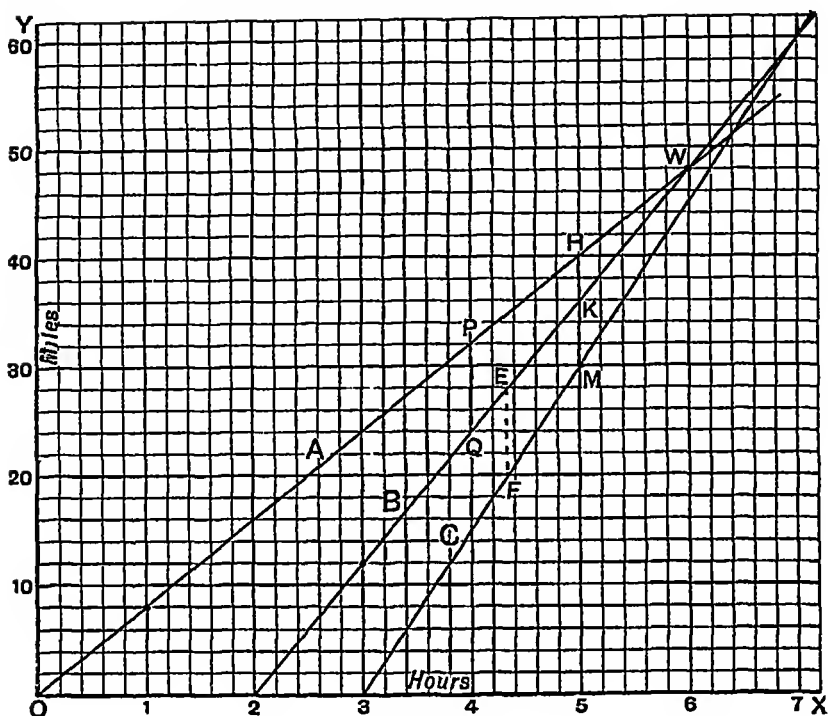


FIG 16

- 96 See Fig 16, which is drawn on half the given scale. The line is such that each ordinate is numerically eight times the corresponding abscissa. Also the distance travelled at 8 mi per hr in any number of hours is 8 times that number of hours. Hence the ordinate of any point on the line gives the distance travelled in the time indicated by the abscissa of that point.

The graph of *A*'s motion is the line OPH

Since *B* starts at 2 p.m. at 12 mi per hr and has therefore gone 12 mls at 3 p.m., his graph will be the line obtained by joining the points (2, 0) and (3, 12). Similarly *C*'s is obtained by joining (3, 0) and (4, 16).

The time and place of *B*'s overtaking *A* are given by the coords of the point common to their graphs, viz W, and are 6 p.m. and 48 mls from O.

To find at what times *A* and *B* are 8 mi apart, we must discover

for what abscissæ the vertical distance between the two graphs represents 8 m. This will be the case for the abscissæ marking 4 p.m. and 8 p.m. (The latter is not shewn in the diagram)

The rest of the solution will be obvious from the figure

### EXAMPLES XVII a Page 341

- 1 and 2 See Art 287, Ex 1
- 3 Let  $x$  be rate per cent, then  $\frac{x}{100} = \frac{3\frac{1}{2}}{25}$  4-6 As No 3
- 7-12 See Art 287, Ex 2 13-16 As No 3
- 17 Let  $x$  be the rate per cent, then  $\frac{x}{100} = \frac{38}{275}$ , and  $x = 14.0$
- 18-21 As No 17 22 Req<sup>d</sup> value =  $\frac{4}{100}$  of £573 = £22.920
- 23-24 As No 22 25 Req<sup>d</sup> value =  $\frac{4\frac{1}{2}}{100}$  of £584 = £24.99
- 26 Req<sup>d</sup> value =  $\frac{4}{100}$  of £150.675 = £6.0.6d
- 27 Req<sup>d</sup> value =  $\frac{5\frac{1}{2}}{100}$  of £804.4 = £46.253
- 28 (i) Req<sup>d</sup> value =  $\frac{7}{100}$  of £302.729 = £9.08187
- (ii) Req<sup>d</sup> value =  $\frac{5\frac{1}{2}}{100}$  of £1572.11s. 10d =  $\frac{1\frac{1}{2}}{100}$  of £1572.11s. 10d  
 =  $\frac{1\frac{1}{2}}{100}$  of £786.5s. 11d =  $\frac{1}{100}$  of £8649.5s. 1d  
 = £86.49254
- 29 Req<sup>d</sup> commission =  $\frac{3\frac{1}{2}}{100}$  of Rs 3750 = Rs 131.25
- 30 Error =  $\frac{1}{36}$  of length =  $\frac{1}{144}$  of length  
 If  $x$  be req<sup>d</sup> percentage,  $\frac{x}{100} = \frac{1}{144}$ , and  $x = 0.694$
- 31 Loss = 1141 in 7500 or  $\frac{1141}{7500}$  in 100, giving  $\frac{1141}{75}$ , or 15.2%
- 32 Increase is 268 on 4680, or  $\frac{268}{4680}$  on 100, giving 5.7%

### EXAMPLES XVII b Page 344

N.B. In some of these examples  $x$  is used for the quantity to be found

- 1 If Rs  $x$  be his income,  $\frac{77\frac{1}{2}}{100}$  of Rs  $x$  is left,  $\frac{77\frac{1}{2}}{100}x = \text{Rs } 1860$

- 2 Req<sup>d</sup> population =  $\frac{84}{100}$  of 15800
- 3 From Art 289,  $x \times \frac{100}{100}$  is the present population,  
 $x \times \frac{100}{100} = 62,130$
4. From Art 289, first rem<sup>r</sup> =  $\frac{87\frac{1}{2}}{100}$  of 145600,  
 $x = \frac{85}{100}$  of  $\frac{87\frac{1}{2}}{100}$  of 145600
- 5  $x = \frac{90}{100}$  of 7 a 6 p      6 From Art 289,  $\frac{80}{100}$  of  $x = 140$  (pence)
- 7 First remainder =  $\frac{5}{7}$  of  $x$ ,      from Art 289,  $\frac{64}{100}$  of  $\frac{52}{7} = 95880$
- 8  $\frac{95}{100}$  of  $x = 532$       9 As in 8,  $\frac{92\frac{1}{2}}{100}x = 74000$
- 10 From Art 289, a year ago the fund stood at  $\frac{11\frac{2}{3}}{100}$  of Rs  $x$ ,  
 it will now stand at  $\frac{11\frac{2}{3}}{100}$  of  $\frac{11\frac{2}{3}}{100}$  of Rs  $x$ ,  
 $\frac{11\frac{2}{3}}{100}$  of  $\frac{11\frac{2}{3}}{100}$  of Rs  $x =$  Rs 157584
- 11 Let  $a$  represent 182 460, then  $a - x = \frac{5\frac{1}{2}a}{100}$ ,  
 $x = a - \frac{5\frac{1}{2}a}{100} = \frac{94\frac{1}{2}a}{100} = \frac{94\frac{1}{2} \times 182\ 460}{100}$
- 12 Increase = Rs 177280 on Rs 4780320, i.e.  $\frac{177280}{4780320} \times 100$  on 100,  
 giving 3 71 %
- 13 Applying the principle of Art 289, we have  
 $x \times \frac{200}{100} \times \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100} = \frac{102\frac{4}{5}}{100} \times x$ , giving 2 4 %
- 14  $\frac{x}{100} = \frac{5790000}{144000000}$
- 15 Yield of olives per ac =  $\frac{15000000}{4320000}$  lbs = 35 lbs,  
 „ currants „ =  $\frac{15000000}{1080000}$  lbs = 2083 lbs,  
 „ figs „ =  $\frac{6000000}{520000}$  lbs = 1154 lbs
- If percentage of land occupied by olives is  $x$ ,  
 then  $\frac{x}{100} = \frac{4320000}{5663100}$ , and  $x = \frac{4320000 \times 100}{5663100} = 8$
- Similarly percentage of land occupied by currants  
 $= \frac{1080000 \times 100}{5663100} = 3$ ,  
 „ „ „ „ figs =  $\frac{520000 \times 100}{5663100} = 1$

16 Total imports = £523075163

As in No 15 we obtain

$$\begin{aligned} \text{percentage for British possessions} &= \frac{100530635 \times 100}{523075163} = \frac{10053}{523} = 21 \\ \text{,, US} &= \frac{18788261 \times 100}{523075163} = \frac{18788}{523} = 27, \\ \text{,, France} &= \frac{53618656 \times 100}{523075163} = \frac{5361}{523} = 10, \\ \text{,, other countries} &= \frac{221136611 \times 100}{523075163} = \frac{22114}{523} = 42. \end{aligned}$$

17 As in No 15 we obtain

$$\begin{aligned} \text{percentage for 1868} &= \frac{7054446 \times 100}{1078844} = \frac{70544}{1078} = 67, \\ \text{,, 1869} &= \frac{780106 \times 100}{1078844} = \frac{7801}{1078} = 66, \\ \text{,, 1870} &= \frac{5473460 \times 100}{847756} = \frac{5473}{847} = 65 \end{aligned}$$

$$\begin{aligned} \text{For whole period percentage} &= \frac{\text{total expenses} \times 100}{\text{total earnings}} \\ &= \frac{16274014 \times 100}{24711287} = \frac{16274}{247} = 66 \end{aligned}$$

18 As in No 15 we obtain

percentage increase for England and Wales

$$= \frac{357318 \times 100}{2300528} = \frac{3573}{2300} = 12,$$

$$\text{,, ,, Scotland} = \frac{447450 \times 100}{40284} = \frac{4474}{402} = 11$$

19 As in No 15 we obtain

$$\text{percentage for England and Wales} = \frac{1037000 \times 100}{3337800} = \frac{1037}{3337} = 15,$$

$$\text{,, Scotland} = \frac{60000 \times 100}{45800} = \frac{6000}{458} = 15,$$

$$\text{,, Ireland} = \frac{45000 \times 100}{441400} = \frac{4500}{4414} = 11$$

### EXAMPLES XVII c Page 347

1-5 See Art 291, Ex 1

6-9 See Art 291, Ex 2

10 Profit = (Rs 12a 9p - 15a) = 3a 9p on 15a,

$$\text{reqd percentage} = \frac{3a.9p}{15a} \times 100 = 25$$

11 Loss = 18s on (£3 2s + 18s), or 18s on £4,

$$\text{reqd percentage} = \frac{18s}{£4} \times 100 = 22\frac{1}{2}$$

12 CP - 4% of CP = SP, or  $\frac{96}{100}$  of CP = SP,

$$SP = \frac{96}{100} \text{ of Rs } 75 = \text{Rs } 72$$

13 SP = CP + 14% of CP, or SP =  $\frac{114}{100}$  of CP =  $\frac{114}{100}$  of 100a  
= Rs 7 2a

$$14 \quad CP - 12\frac{1}{2}\% \text{ of } CP = SP, \quad \frac{87\frac{1}{2}}{100} \text{ of } CP = Rs \ 2\frac{5}{8},$$

$$CP = Rs \ 2\frac{5}{8} \times \frac{100}{87\frac{1}{2}} = Rs \ 3$$

$$15 \quad (i) \text{ Profit} = £3 \ 7s \ 1d \text{ on } £40, \\ \text{reqd percentage} = \frac{£3 \ 7s \ 1d}{£40} \times 100 = \frac{335}{40} = 8\frac{1}{4}\% \text{ gain}$$

$$(ii) \text{ Loss} = £3 \ 10s \ 5d \text{ on } £25 \ 15s, \\ \text{reqd percentage} = \frac{£3 \ 10s \ 5d}{£25 \ 15s} \times 100 = \frac{371}{2575} = 13\frac{1}{7}\% \text{ loss}$$

$$(iii) \text{ Profit} = £9 \ 13s \ 0\frac{1}{2}d \text{ on } £77 \ 4s \ 8d, \\ \text{reqd percentage} = \frac{£9 \ 13s \ 0\frac{1}{2}d}{£77 \ 4s \ 8d} \times 100 = \frac{965}{7723} = 12\frac{1}{2}\% \text{ gain}$$

$$16 \quad 5\frac{3}{4}d \text{ per lb} = 664d \text{ per cwt}, \quad £2 \ 7s \ 11d = 575d \\ \text{gain per cent} = \frac{(644 - 575)d}{575d} \times 100 = 12$$

$$17 \quad CP \text{ per } 100 = \frac{15}{12} \times 100a, \\ SP \text{ per } 100 = \frac{112}{100} \text{ of } \frac{15}{12} \times 100a = Rs \ 8 \ 12a$$

$$18 \quad CP \text{ for } 130 \text{ lbs} = 32s \ 6d, \quad \text{gain per cent} = \frac{4s \ 10\frac{1}{2}d}{32s \ 6d} \times 100 = 15$$

$$19 \quad 110 \text{ knives cost Rs } 100, \text{ and are sold for Rs } 121, \\ \text{gain per cent} = 21$$

$$20 \quad \text{For } 25 \text{ eggs, } CP = 15a, \text{ and } SP = 18a, \\ \text{gain per cent} = \frac{3}{15} \text{ of } 100 = 20$$

$$21 \quad \text{If it cost } x \text{ annas, then } \frac{92r}{100} = 92a, \text{ and } r = 100a$$

$$22 \quad \text{If it cost } x \text{ rupees, then } \frac{120x}{100} = 15, \text{ and } r = Rs \ 12\frac{1}{2}$$

$$23 \quad \text{If } £x \text{ be } CP, \text{ then } \frac{107\frac{1}{2}x}{100} = 17\frac{1}{2}, \text{ and } x = £16$$

$$24 \quad \text{If Rs } x \text{ be } CP, \text{ then } \frac{90r}{100} = 65\frac{1}{4}, \text{ and } r = Rs \ 72\frac{1}{2}$$

$$25 \quad 150 \text{ yds at } 8a \ 9p \text{ per yd are worth Rs } 82 \ 0a \ 6p = Rs \ 82 \ 03125 \\ \text{If Rs } x \text{ be } CP, \frac{125x}{100} = 82 \ 03125, \text{ and } x = Rs \ 65 \ 625$$

$$26 \quad 9d \text{ per doz} = 15d \text{ per score if } x \text{ pence be } CP, \\ \frac{120r}{100} = 15 \text{ and } r = 12\frac{1}{2}d$$

$$27 \quad \text{Let } x \text{ pence be } CP \text{ per lb, then } \frac{112\frac{1}{2}r}{100} = 2\frac{1}{4}, \text{ and } x = 2d, \\ \text{cost per ton} = 2d \times 2240 = £18 \ 13s \ 4d$$

- 28 Let £1 be CP per gross, then  $\frac{85r}{100} = 204$ , and  $r = £24$   
 His SP must be  $\frac{120}{100}$  of £24, or £28 16s
- 29 Let  $r$  shillings be CP per half cwt, then  $\frac{93\frac{1}{2}r}{100} = 105$ , and  $r = 112s$ ,  
 CP per lb = 2s

### EXAMPLES XVII d Page 350

- 1 Req<sup>d</sup> SP =  $\frac{110}{100}$  of Rs 361      2 Req<sup>d</sup> SP =  $\frac{100}{90}$  of 19s
- 3 Req<sup>d</sup> SP =  $\frac{108}{100}$  of Re 1 9a 9p
4. Req<sup>d</sup> SP =  $\frac{100}{90}$  of  $2\frac{1}{2}d \times 112 = £1\ 5s$
- 5 Since Rs 81 =  $\frac{100}{120}$  of CP,      Rs 90 =  $\frac{90 \times 120}{81 \times 100}$  of CP =  $\frac{80}{100}$  of CP,  
 hence loss is 20 %
- 6  $105a = \frac{100}{100}$  of CP,       $93a = \frac{100}{100}$  of CP, giving 7 % loss
- 7  $43a = \frac{107\frac{1}{2}}{100}$  of CP,       $38a = \frac{107\frac{1}{2}}{100} \times \frac{38}{43}$  of CP =  $\frac{95}{100}$  of CP, and  
 loss is 5 %
- 8 Let  $r$  annas be req<sup>d</sup> price, then  $\frac{90r}{100} = 99$ , and  $r = 110a = Rs\ 6\ 14a$
- 9 If he bought at  $r$  annas per score, he paid 10 $r$  annas  
 He received  $\frac{9}{10} \times 180a$        $\frac{108}{100} \times 10r = 180 \times \frac{9}{10}$ , whence  $r = 30a$
- 10  $2\frac{1}{2}d$  per lb = 308d per cwt,       $308d = \frac{100}{100}$  of CP,  
 hence £1 11s 6d, or  $378d = \frac{100}{100} \times \frac{308}{100}$  of CP =  $\frac{108}{100}$  of CP,  
 giving 8 % profit
- 11 She sells 108 oranges for 90a, and since her loss is 10 %, then  
 CP is 100a. If she sold them at 10 for 12a, she would  
 receive 129 6a, giving a profit of 29 6 %
- [NB The number of oranges sold does not affect the gain or loss  
 per cent. 108 is a convenient number because the first SP  
 is then 90a, which at once gives 100a as the CP]
- 12  $24a = \frac{100}{100}$  of CP,       $21a = \frac{100}{100} \times \frac{21}{100}$  of CP, or  $\frac{112}{100}$  of CP,  
 giving 12 % profit
- 13 Let £1 be cost price, then  $\frac{12r}{100} = 498$ , and  $r = £415$

- 14 Let Rs  $x$  be cost price, then  $\frac{20x}{100}=7\ 5$ , and  $x=\text{Rs } 37\ 5$   
(See note to no 13)
- 15 Let £ $x$  be cost of carriage, then  $\frac{30x}{100}=10$ , and  $x=\text{£}33\frac{1}{3}$   
(See note to no 13)
- 16 Let  $x$  rupees be prime cost, then  $\frac{20x}{100}=9\frac{3}{4}$ , and  $x=\text{Rs } 46\frac{7}{8}$   
(See note to no 13)
- 17 Let  $x$  pence be C P, then  $\frac{22x}{100}=44$ , and  $x=200d$ ,  
 $\frac{8\ 8}{100}$  of C P  $=176d=14s\ 8d$  (See note to no 13)
- 18 Let  $x$  shillings be C P per doz, then  $\frac{12\frac{1}{2}x}{100}=5$ , and  $x=40s$ ,  
S P req<sup>d</sup>  $=\frac{100}{125}$  of  $40s=48s$  (See note to no 13)
- 19 See Art 293 It costs  $C$  Rs  $600 \times \frac{110}{100} \times \frac{105}{100}$ , or Rs 693
- 20 See Art 293 I must pay  $\text{£}32 \times \frac{125}{100} \times \frac{112\frac{1}{2}}{100}$ , or  $\text{£}4\ 10s$
- 21 See Art 293 He pays Rs  $2000 \times \frac{84}{100} \times \frac{105}{100}$ , or Rs 2100
- 22 Let  $x$  shillings be  $A$ 's cost, then  $x \times \frac{110}{100} \times \frac{110}{100}=121$ , or  $x=100$
- 24 He got  $40d \times \frac{100}{100} \times \frac{105}{100} \times \frac{95}{100}$ , or  $57d$
- 25 C P to dealer  $=\frac{105}{100}$  of Rs 160 = Rs 200, profit = Rs 50, i.e. 25 %
- 26 Shopkeeper pays  $\frac{110}{100}$  of first price, and sells at  $\frac{105}{100}$  of first price,  
his gain is  $\frac{11}{100}$  of C P on outlay of  $\frac{110}{100}$  of C P, or 10 %  
*Aliter* If article first cost £100, dealer paid £110 and sold at £121  
profit is £11 on £110, or 10 %
- 27 Let  $x$  be profit per cent Third S P  $=\frac{157\frac{1}{2}}{100}$  of first S P  
Also third S P  $=\frac{120}{100}$  of  $\frac{125}{100}$  of  $\frac{100+x}{100}$  of first S P,  
 $\frac{120}{100} \times \frac{125}{100} \times \frac{100+x}{100} = \frac{157\frac{1}{2}}{100}$ , i.e.  $100+x = \frac{157\frac{1}{2} \times 100 \times 100 \times 100}{100 \times 120 \times 125}$ ,  
whence  $x=5$
- 28 Let  $B$  make  $x\%$ , then  $\text{£}6 \times \frac{125}{100} \times \frac{100+x}{100} \times \frac{110}{100} = \text{£}9\ 9$ ,  
 $100+x = \frac{9 \times 100 \times 100 \times 100}{6 \times 125 \times 110}$ , and  $x=20$

$$29 \quad (i) \text{Reqd price} = 75d \times \frac{120}{100} \times \frac{110}{100} = 99d = 8s \ 3d$$

$$(ii) \text{Reqd price} = 25s \times \frac{120}{100} \times \frac{110}{100} \times \frac{112\frac{1}{2}}{100} = \frac{297}{8}s = \text{£}1 \ 17s \ 1\frac{1}{2}d$$

$$(iii) \text{ If } \text{£}x \text{ be cost reqd, then } \text{£}x \times \frac{120}{100} \times \frac{110}{100} \times \frac{112\frac{1}{2}}{100} = \text{£}14 \ 8s, \\ x = 14 \ 8s \times \frac{100}{120} \times \frac{100}{110} \times \frac{100}{112\frac{1}{2}}, \text{ or } \text{£}10$$

$$(iv) \text{ If } x \text{ be profit per cent, then } \frac{100+x}{100} \text{ of C P} = \frac{120}{100} \times \frac{110}{100} \times \frac{112\frac{1}{2}}{100}, \\ \frac{100+x}{100} = \frac{120 \times 110 \times 112\frac{1}{2}}{100 \times 100 \times 100}, \text{ and } 100+x = 148\frac{1}{2}, \text{ or } x = 48\frac{1}{2}$$

### EXAMPLES XVII e Page 353

- 1 Let  $x$  be reqd population, adding the births and subtracting the deaths,  $x + \frac{11x}{100} - \frac{4x}{100} = 85600$ , and  $x = 80000$

$$\text{births} = \frac{11x}{100} = 8800, \text{ deaths} = \frac{4x}{100} = 3200$$

- 2 Total C P =  $(8 \times 25 + 20 \times 20)s = 600s$  Total S P =  $28 \times 21s = 672s$   
gain is  $72s$  on  $600s$  outlay, or  $12s$  on  $100s$ , giving  $12\%$

- 3 Suppose 4 lbs of chicory are mixed with 17 lbs of coffee  
Total C P =  $(4 \times 3 + 17 \times 24)a = 420a$  Total S P =  $21 \times 25 = 525a$   
gain is  $105a$  on  $420a$ , or  $\frac{105 \times 100}{420}a$  on  $100a$ , giving  $25\%$

- 4 Capital reqd = Rs  $10000 \times \frac{120}{100} \times \frac{110}{100} \times \frac{112\frac{1}{2}}{100} = \text{Rs } 20736$

- 5 7 eggs are sold for  $4 \times \frac{140}{100}a$ , 7 are sold for  $4a$

- 6 Let Rs  $x$  be the whole amt reqd Then Rs  $\frac{40x}{100}$  is divided in the proportion of Rs  $12500$  Rs  $8500$

$$\text{Now Rs } 12500 + \text{Rs } 8500 = \text{Rs } 21000,$$

$$\frac{12500}{21000} \text{ of } \frac{40x}{100} = \frac{8500}{21000} \text{ of } \frac{40x}{100} + 300, \text{ and } x = \text{Rs } 3937 \ 8a$$

- 7 Let  $\text{£}x$  be rent, then  $x - 25 - \frac{1x}{100}$  of  $1150 = \frac{8}{100}$  of  $1150$ ,  
whence  $x = \text{£}134\frac{1}{2}$

- 8 Let  $x\%$  of the whole pass Now  $95\%$  of boys and  $60\%$  of girls pass,  
 $\frac{x}{100}$  of  $2500 = \frac{95}{100}$  of  $2000 + \frac{60}{100}$  of  $500$ , and  $x = 88$



# 132 MISCELLANEOUS EXAMPLES ON PERCENTAGES [CHAP

- 9 Suppose A pays £100, he receives £105,  
and D pays  $£105 \times \frac{100}{105} \times \frac{105}{100}$ ,  
i.e. D pays £157½, A would gain 57½%

- 10 Suppose each article costs 100d Then C P is 300d and S P is 400d  
Profit=100d on 300d, or 33⅓d on 100d, or 33⅓%

- 11 He actually sells at £21 - £1 6s 3d, or £19 13s 9d  
Now £21 =  $\frac{112}{100}$  of C P,  
£19 13s 9d, or £19½ =  $\frac{112}{100} \times \frac{19½}{21}$  of C P =  $\frac{105}{100}$  of C P,  
profit=5%

- 12 Let  $x\%$  be gain reqd  $\frac{1}{3}$  of Rs 36000 = Rs 12000,  
 $\frac{2}{3}$  of Rs 36000 = Rs 24000, remr = Rs 12000  
 $\frac{80}{100}$  of 12000 +  $\frac{125}{100}$  of 24000 +  $\frac{100+x}{100}$  of 12000 =  $\frac{110}{100}$  of 36000  
Cancelling we have  $9600 + 30000 + 1200(100+x) = 39600$ , and  $x=25$

- 13 Let £1 be cost of goods, then  $\frac{120x}{100} = 25$ , and  $x=20\frac{5}{8}$   
gross profit £(25 - 20⅝) = £4⅞ Now 10% of £25 = £2½,  
net profit = £(4⅞ - 2½) = £1⅞

- 14 Let  $x$  lbs of first kind be mixed with  $y$  lbs of second kind  
Expressing prices in annas we have  
 $25(x+y) = \frac{125}{100}(14x+24y)$ ,  $3x=2y$ , and  $\frac{x}{y} = \frac{2}{3}$

- 15 Let  $x$  gals of water be added to  $y$  gals of milk Expressing  
prices in pence, since 2d per pint = 16d per quart, we have  
 $16(x+y) = \frac{140}{100}$  of  $13y$ ,  $80x=11y$ , and  $\frac{x}{y} = \frac{11}{80}$

- 16 Value of total wool produced in N Z =  $\frac{180055}{100000}$  of £8593000  
Value of wool exported from N Z = £4041000  
Since amount is proportional to value, if  $x$  be reqd percentage,

$$\frac{x}{100} = \frac{4041000}{\frac{180055}{100000} \text{ of } 8593000}$$

and  $x = \frac{4041 \times 200000}{8593 \times 180055} \times 100 = \frac{4041 \times 200000}{8593 \times 180055} \times 100$   
 $= \frac{10772}{16288} \times 100$  (contracting) = 0.661 × 100 = 66%.

- 17 Let  $\pounds x$  be total value of Canada's farm products,  
then  $\frac{90x}{100} = 131$  millions and  $x = 146$  millions  
Total imports to Britain  $= \frac{100+47}{100} \times 131$  millions  
 $= \frac{147}{100} \times 131$  millions  $= 717$  millions
- 18  $\frac{20}{100}$  of CP =  $\pounds 3$ , CP =  $\pounds 15$  and SP =  $\frac{120}{100}$  of  $\pounds 15 = \pounds 18$   
If  $\pounds x$  be advertised price, then  $\frac{75x}{100} = 18$ , and  $x = \pounds 24$
- 19 Let  $x$  seers of water be added to 1 seer of milk,  
 $(x+1) \times 3a = \frac{160}{100} \times 1 \times 2\frac{1}{2}a$ , and  $x = \frac{1}{5}$
20. Let  $x$  rupees be cost price,  
then  $\frac{105x}{100} - 1 = \frac{110}{100}$  of  $\frac{95}{100}$ , and  $x = \text{Rs } 200$
21. Rs 20 corresponds to an increase of 4%,  
 $\frac{4}{100}$  of CP = Rs 20, and CP = Rs 500
- 22 Let  $x$  shillings be price per quarter at which he bought,  
then  $\frac{107\frac{1}{2}}{100}$  of  $280x + \frac{110}{100}$  of  $320x + \frac{112\frac{1}{2}}{100}$  of  $100x - 700x = \text{total gain}$   
This reduces to  $65\frac{1}{2}x$  and  $= \pounds 100$  19s 7d expressed in shillings  
 $65\frac{1}{2}x = 2019\frac{7}{2}$ , and  $x = 30\frac{5}{8}$ , i.e.  $\pounds 1$  10s 10d
- 23 Let  $x$  pence be CP per cwt, then  $\frac{160}{100}x = \frac{95}{100}$  of  $4 \times 112$   
 $x = 266d = \pounds 1$  2s 2d
24. Let  $x$  lbs of first be mixed with  $y$  lbs of second  
Then  $94(x+y) = \frac{133\frac{1}{2}}{100} \{63x + 78y\}$ , i.e.  $x = y$
- 25 Let Rs  $x$  be the price required, then  $8x + 22 \times \frac{3x}{4} = \frac{140}{100} \times 3150$ ,  
and  $x = \text{Rs } 180$
- 26 Water in first mixture  $= (2 \times 25 + 3 \times 15)$  parts out of 500, i.e. 19%  
water in second mixture  $= (\frac{120}{100}$  of  $5 + \frac{1}{4}$ ) parts out of  $5\frac{1}{4}$ , i.e. 23%,  
spirit in second mixture  $= (100 - 23)\%$ , i.e. 77%
- 27 1 Kg or  $\frac{9}{4}$  lbs cost  $3\frac{1}{2}d$ , 1 lb cost  $\frac{14}{9}d$  Since it sold for  $2d$   
profit is  $\frac{4}{9}d$  on outlay of  $\frac{14}{9}d$ , giving 28.6% as profit he  
thought he made  
Again 1 Kg or 1000 gm cost  $35d$ , 1 gm cost  $0.0035d$ , and  
453.6 gms cost  $(453.6 \times 0.0035)d$ , i.e.  $1.5876d$ ,  
gain is  $(2 - 1.5876)d$  on outlay of  $1.5876d$ , giving  $\frac{412.400}{15876}\%$ ,  
i.e. 26%

28 Cost of materials and manufacture =  $\frac{3}{100}$  of £3750

Remaining expenses = £(387 + 940 + 136 + 200 +  $\frac{1}{100}$  of 3750)  
= £1700 5

profit = £(3750 -  $\frac{3}{100}$  of 3750 - 1700 5)  
= £( $\frac{6}{100}$  of 3750 - 1700 5) = £737

In second part of question, gross sales =  $\frac{4}{100}$  of £3750, or £5250,  
cost of materials and manufacture =  $\frac{3}{100}$  of £5250

Remaining expenses = £(387 + 1880 + 136 + 200 +  $\frac{1}{100}$  of 5250)  
= £2655 5,

profit = £(5250 -  $\frac{3}{100}$  of 5250 - 2655 5)  
= £( $\frac{6}{100}$  of 5250 - 2655 5) = £757,

increase in annual profit = £(757 - 737) = £20

### EXAMPLES XVIII a Page 358

1-15 See Art 298, Ex 1 and 2

$$\begin{array}{r}
 16 \quad \begin{array}{r} \text{£} \\ 4 \ 57 \\ \hline 3 \\ 13 \ 71 \\ \hline 4\frac{1}{2} \\ 54 \ 84 \\ \hline 6 \ 855 \\ 61 \ 695 \end{array} \\
 \end{array}$$

= £61 13s 11d

$$\begin{array}{r}
 17 \quad \begin{array}{r} \text{£} \\ 51 \ 27 \\ \hline 3 \\ 153 \ 81 \\ \hline 3\frac{1}{2} \\ 461 \ 43 \\ \hline 76 \ 905 \\ 538 \ 335 \end{array} \\
 \end{array}$$

= £538 6s 8d

$$\begin{array}{r}
 18 \quad \begin{array}{r} \text{£} \\ 6 \ 41 \\ \hline 3\frac{1}{4} \\ 19 \ 23 \\ \hline 1 \ 6025 \\ 20 \ 8325 \\ \hline 2\frac{1}{8} \\ 41 \ 6650 \\ \hline 2 \ 6040 \\ 44 \ 2690 \end{array} \\
 \end{array}$$

= £44 5s 5d

$$\begin{array}{r}
 19 \quad \begin{array}{r} \text{£}37 \ 12 \\ \hline 2\frac{1}{2} \\ 74 \ 24 \\ \hline 18 \ 56 \\ 92 \ 80 \\ \hline 3\frac{3}{8} \\ 278 \ 40 \end{array} \\
 \end{array}$$

34 80 ( $\frac{1}{8}$  of 278 4)

313 20

= £313 4s

$$\begin{array}{l}
 20 \text{ Interest} = \frac{\text{£}761 \ 11s \ 1d \times 6\frac{2}{3} \times 4\frac{1}{2}}{100} \\
 = \frac{\text{£}761 \ 11s \ 1d \times 3}{10} \\
 = \text{£}228 \ 9s \ 4d
 \end{array}$$

$$\begin{array}{l}
 21 \text{ Interest} = \text{£}4 \ 08121 \times \frac{1}{2} \times 4 \\
 = \text{£}8 \ 162 = \text{£}8 \ 3s \ 3d
 \end{array}$$

$$\begin{array}{r|l}
 \text{22} & \text{Int} = \text{£}1\ 567 \\
 & 25 \times \frac{7}{4} \times \frac{7}{49} \\
 & \hline
 & 62\ 69 \\
 & 14\ 105 \quad 3 \\
 & 8 \overline{) 76\ 795} \quad 3 \\
 & \quad 9\ 599 \quad 4 \\
 \text{Prin} & 156\ 725 \\
 \hline
 \text{Amt} & \text{£}166\ 324
 \end{array}$$

23

$$\begin{aligned}
 I &= \frac{\text{£}244 \frac{1}{2} \times 3 \frac{3}{4} \times 3}{100} \\
 &= \text{£} \frac{8787}{100} = \text{£}27\ 459 \\
 &= \text{£}27\ 9s\ 2d \\
 \text{Amt} &= \text{£}271\ 10s\ 10d
 \end{aligned}$$

$$\begin{aligned}
 \text{24} \quad I &= \text{Rs } 18\ 4290625 \times 2 \frac{3}{4} \times 2 \frac{1}{4} = \text{Rs } 18\ 4290625 \times 6 = \text{Rs } 110\ 9a\ 3p \\
 \text{Amt} &= \text{Rs } 1953\ 7a\ 9p
 \end{aligned}$$

$$\begin{aligned}
 \text{25} \quad I &= \text{Rs } 11\ 47171875 \times 5 \frac{1}{2} \times 2 \frac{1}{4} = \text{Rs } 11\ 47171875 \times 12 \\
 &= \text{Rs } 137\ 660625 = \text{Rs } 137\ 10a\ 6p \quad \text{Amt} = \text{Rs } 1284\ 13a\ 3p
 \end{aligned}$$

$$\text{26} \quad I = \text{£}12\ 4 \times \frac{1}{4} \times 3 \frac{1}{2} = \text{£}10\ 6s\ 8d$$

$$\text{27} \quad I = \text{Rs } 5\ 484 \times \frac{1}{2} \times 4 \frac{1}{2} = \text{Rs } 5\ 484 \times \frac{9}{4} = \text{Rs } 12\ 34$$

$$\begin{aligned}
 \text{28} \quad I &= \text{Rs } 4\ 375 \times \frac{3}{4} \times 4 = \text{Rs } 11 \frac{3}{4} = \text{Rs } 11\ 10a\ 8p \\
 \text{Amt} &= \text{Rs } 449\ 2a\ 8p
 \end{aligned}$$

$$\text{29} \quad I = \text{Rs } 62\ 80 \times 2 \frac{1}{4} \times \frac{1}{2} = \text{Rs } 70\ 65 \quad \text{Amt} = 6350 \text{ Rs } 65 \text{ cents}$$

### EXAMPLES XVIII b Page 361

$$\begin{aligned}
 \text{1} \quad 146 \text{ days} &= \frac{2}{5} \text{ year}, \\
 I &= \text{Rs } 7\ 50 \times \frac{2}{5} \times 3 \frac{3}{4} = \text{Rs } 11\ 25, \text{ and Amt} = \text{Rs } 761\ 4a
 \end{aligned}$$

$$\begin{aligned}
 \text{2} \quad 292 \text{ days} &= \frac{4}{5} \text{ year}, \\
 I &= \text{£}18\ 95 \times \frac{4}{5} \times 1 \frac{1}{4} = \text{£}18\ 35, \text{ and Amt} = \text{£}1853\ 7s
 \end{aligned}$$

$$\begin{aligned}
 \text{3} \quad 219 \text{ days} &= \frac{3}{4} \text{ year}, \\
 I &= \text{Rs } 16\ 875 \times \frac{3}{4} \times 6 = \text{Rs } 60\ 75, \text{ and Amt} = \text{Rs } 1748\ 4a
 \end{aligned}$$

$$\begin{aligned}
 \text{4} \quad 73 \text{ days} &= \frac{1}{5} \text{ year}, \quad I = \text{£}2\ 52 \frac{1}{2} \times \frac{1}{5} \times 3 = \text{£}1\ 5125 = \text{£}1\ 10s\ 3d, \\
 \text{and Amt} &= \text{£}253\ 11s\ 11d
 \end{aligned}$$

$$\text{5} \quad I = \frac{\text{£}16\ 06 \times 400 \times 11}{365 \times 2} = \text{£}96\ 8, \quad \text{Amt} = \text{£}1702\ 16s$$

$$\begin{aligned}
 \text{6} \quad I &= \frac{\text{£}820\ 4s\ 2d \times 2 \frac{3}{5} \times 2 \frac{1}{2}}{100} = \frac{\text{£}4921\ 5s}{100} = \text{£}49\ 4s\ 3d, \\
 \text{Amt} &= \text{£}869\ 8s\ 5d
 \end{aligned}$$

$$\begin{aligned}
 \text{7} \quad \text{Time} &= 146 \text{ days} = \frac{2}{5} \text{ year}, \\
 I &= \frac{\text{Rs } 4\ 265 \times 2 \times 3}{5} = \text{Rs } 5\ 118, \text{ and Amt} = \text{Rs } 431\ 10a
 \end{aligned}$$

8 Time=196 days ,

$$I = \frac{\text{Rs } 219 \times 196 \times 10}{365 \times 3} = \text{Rs } 3 \text{ } 92, \text{ and Amt} = \text{Rs } 222 \text{ } 14 \text{ a } 9 \text{ p}$$

9 Time=219 days= $\frac{3}{4}$  year ,  $I = \frac{\text{Rs } 37575 \times 3 \times 35}{5 \times 8} = \frac{\text{Rs } 3945375}{40}$   
 =Rs 98634375, and Amt =Rs 385 9a 9p

10 Time=146 days= $\frac{2}{3}$  year ,

$$I = \frac{\text{£}7808 \times 2 \times 55}{5 \times 16} = \text{£}10736, \text{ and Amt} = \text{£}791 \text{ } 10 \text{ s } 9 \text{ d}$$

11  $I = \text{Rs } 316 \times \frac{33}{365} \times 5 = \frac{\text{Rs } 316 \times 33}{73} = \text{Rs } 1428 = \text{Re } 1 \text{ } 6 \text{ a } 9 \text{ p}$

12  $I = \text{Rs } 547 \times \frac{54}{365} \times \frac{1}{2} = \text{Rs } 547 \times \frac{27}{365} = \text{Rs } 20231 = \text{Rs } 2 \text{ } 0 \text{ a } 3 \text{ p}$

13  $I = \text{Rs } 70 \times \frac{86}{365} \times 4 = \frac{\text{Rs } 14 \times 344}{73} = \text{Rs } 659726 = \text{Rs } 65 \text{ } 15 \text{ a } 6 \text{ p}$

14  $I = \text{Rs } 708 \times \frac{113}{365} \times \frac{7}{2} = \frac{\text{Rs } 560028}{73} = \text{Rs } 76716 = \text{Rs } 7 \text{ } 10 \text{ a } 9 \text{ p}$

15  $I = \text{Rs } 96590625 \times \frac{98}{365} \times \frac{4}{100} = \text{Rs } 96590625 \times \frac{784}{73} = \frac{\text{Rs } 7572705}{73}$   
 =Rs 103735 =Rs 10 6a

16 Time=65 days ,  $I = \text{£}413 \times \frac{65}{365} \times 4 = \frac{\text{£}21476}{73} = \text{£}29419$  ,  
 and Amt =£415 18s 10d

17 Time=80 days ,  $I = \text{£}510 \times \frac{80}{365} \times 3 = \frac{\text{£}2448}{73} = \text{£}33534$  ,  
 and Amt =£513 7s 1d

18 Time=101 days ,  $I = \text{£}2450 \times \frac{101}{365} \times \frac{7}{2} = \frac{\text{£}173215}{73} = \text{£}237281$  ,  
 and Amt =£2473 14s 7d

19 Time=212 days ,  $I = \text{£}136875 \times \frac{212}{365} \times \frac{19}{4} = \frac{\text{£}27375 \times 1007}{73}$   
 = $\frac{\text{£}27566625}{73} = \text{£}377624$  , and Amt =£1406 10s 3d

20 Time=108 days ,

$$I = \text{£}4025375 \times \frac{108}{365} \times \frac{41}{100} = \text{£}4025375 \times \frac{108}{365} \times \frac{33}{8}$$

$$= \text{£}4025375 \times \frac{891}{73} = \frac{\text{£}3586609}{73} = \text{£}49131$$

and Amt =£407 9s

21 Time = 146 days =  $\frac{2}{5}$  year,  $I = \text{Rs } 1250 \times \frac{2}{5} \times \frac{1}{4} = \text{Rs } 1875$ ,  
and Amt = Rs 1268 12 a

22 From July 5 to Nov 18 is 136 days,  
 $I = \text{£}5 \times \frac{136}{365} \times \frac{5}{2} = \frac{\text{£}340}{73} = \text{£}4\ 6575$ ,  
on Nov 18 he receives £204 13s 2d

From Nov 18 to Feb 20 is 94 days,  
 $I = \text{£}3 \times \frac{94}{365} \times \frac{5}{2} = \frac{\text{£}141}{73} = \text{£}1\ 9315$ ,  
on Feb 20 he receives £301 18s 8d

23 From May 1 to Dec 31 is 244 days, Aug 1 to Dec 31, 152 days,  
 $I = \text{Rs } 350 \times \frac{244}{365} \times \frac{1}{4} + \text{Rs } 270 \times \frac{152}{365} \times \frac{1}{4}$   
 $= \left( \frac{\text{Rs } 1708}{73} + \frac{\text{Rs } 8208}{73} \right) \times \frac{15}{4} = \frac{\text{Rs } 25288}{73} \times \frac{15}{4}$   
 $= \frac{\text{Rs } 9483}{73} = \text{Rs } 12\ 9904$

I owe Rs 632 15 a 9 p

### EXAMPLES XVIII c Page 363

[In these examples the symbols P, I, r, n are used as in Art 298]

1-8 See Art 302, Ex 1      9-12 See Art 302, Ex 2

13  $P + \frac{P \times \frac{1}{4} \times 2\frac{1}{2}}{100} = 5430\frac{3}{8}$ , or  $P = \frac{43443}{8} \times \frac{1600}{1609} = \text{£}5400$

14  $3\frac{1}{2} \times 2\frac{1}{2} = 8\ 75$ ,  $P = \frac{\text{given Amt}}{\text{Amt of £1}} = \frac{527\ 4375}{1\ 0875} = \text{£}485$

15  $\frac{2}{5} \times 6 = 2\ 4$ ,  $P = \frac{\text{given Amt}}{\text{Amt of £1}} = \frac{2048}{1\ 024} = \text{£}200$

16  $P + P \times \frac{2\frac{1}{2} \times 3}{100} = 254\ \frac{14}{100}$ ,  $P = 254\ \frac{7}{100} \times \frac{200}{215} = \text{£}236\frac{1}{5}$

17 Int on £P = (Int on £1)  $\times$  P,  $P = \frac{152}{0\ 1125} = \text{Rs } 1351\ 1\ a\ 9$

18 As in 17,  $P = \frac{249}{0\ 105} = \text{£}237$  (to nearest £)

$$19 \text{ As in 17, } P = \frac{171\frac{5}{8}}{\frac{8}{12} \times \frac{5}{2}} = \frac{287 \times 100 \times 3}{16 \times 5} = \text{£}1076 \text{ (to nearest £)}$$

$$20 \quad P + P \times \frac{1\frac{3}{4} \times 3}{100} = 327 \text{ 117,} \quad P = 327 \text{ 117} \times \frac{400}{421} = \text{£}310.16s$$

$$21 \quad P + P \times \frac{5 \times 2\frac{1}{2}}{100} = 2000, \quad P = 2000 \times \frac{200}{225} = \text{£}1778 \text{ (to nearest £)}$$

22-26 See Art 303, Ex 1

27-30 Find interest by subtracting Principal from Amount, then see Art 303, Ex 1

31 If  $n$  be req<sup>d</sup> fraction of a year,

$$n = \frac{\text{given Int}}{\text{Int for 1 year}} = \frac{3\frac{1}{2}}{250 \times \frac{5}{100}} = 0.28 \text{ yr} = 103 \text{ days}$$

$$32 \text{ As in 31, } n = \frac{237}{1572.85 \times \frac{5\frac{1}{2}}{100}} \text{ yrs} = \frac{4740}{1572.85 \times 11} \times 365 \text{ days}$$

$$= \frac{4740 \times 73}{3474.27} = \frac{346.02}{3.4727} = 100 \text{ days}$$

$$33 \text{ As in 31, } n = \frac{4675}{382 \times \frac{4}{100}} \text{ yrs} = \frac{467.5}{1528} \times 365 \text{ days}$$

$$= \frac{170.63}{1.528} \text{ days} = 112 \text{ days}$$

34-37 See Art 304      38-40 Find interest and see Art 304

$$41 \quad r = \frac{\text{given Int}}{\text{Int at 1 per cent}} = \frac{12\frac{3}{10}}{650 \times \frac{6}{12} \times \frac{1}{100}} = 4\frac{1}{2}$$

$$42 \text{ As in No 41, } r = \frac{11\frac{2}{3}}{437\frac{1}{2} \times \frac{2}{3} \times \frac{1}{100}} = 4$$

43 If Principal is £100, then Interest is £200, and, as in 41,

$$r = \frac{200}{100 \times 25 \times \frac{1}{100}} = 8$$

$$44 \text{ As in 41, } r = \frac{50}{564 \times 2\frac{1}{2} \times \frac{1}{100}} = \frac{500}{141} = 3.55$$

$$45 \text{ As in 41, } r = \frac{100}{3550 \times \frac{5}{12} \times \frac{1}{100}} = \frac{480}{71} = 6 \text{ } 76$$

$$46 \text{ } P + P \times \frac{4\frac{1}{2} \times 5}{100} = 2519 \text{ } 9 \quad P = \frac{200}{245} \text{ of } 2519 \text{ } 9 = \text{£}2057$$

$$47 \text{ (1) Annual interest is } \text{£}\frac{1}{8} \times 52, \text{ i.e. } \text{£}6 \text{ } 5$$

$$P \times \frac{2\frac{3}{4} \times 1}{100} = 6 \text{ } 5, \text{ or } P = 650 \times \frac{4}{11} = \text{£}236 \text{ } 7s$$

$$(11) \text{ Annual interest} = \text{£}\frac{3 \text{ } 6 \text{ } 5}{240}$$

$$P \times \frac{2\frac{1}{2} \times 1}{100} = \frac{365}{240}, \text{ or } P = \frac{3650}{66} = \text{£}55 \text{ } 6s$$

$$48 \text{ As in Art 303, Ex 1, } n = \frac{1}{30 \text{ } 5 \times \frac{5}{100}} \text{ yrs} \\ = \frac{100}{30 \text{ } 5 \times 5} \times 365 \text{ days} = 240 \text{ days}$$

$$49 \text{ As in Art 303, Ex 1, } n = \frac{12 \text{ } 904}{1000 \times \frac{3}{100}} \text{ yrs} = 0 \text{ } 43 \text{ yrs} = 157 \text{ days}$$

This gives Jan 9th

$$50. \text{ From Jan 1 to Aug 8} = 219 \text{ days} = \frac{3}{8} \text{ yr} \quad \text{From Art 303,}$$

$$P = \frac{36}{\frac{3}{8} \times \frac{4}{100}} = \text{£}1500$$

### EXAMPLES XVIII d Page 368

$$1 \text{ Disc} = \text{Rs } 20 \text{ } 20 \times \frac{1}{4} \times 4 = \text{Rs } 20 \text{ } 3a \text{ } 3p$$

$$2 \text{ Disc} = \text{Rs } 63 \text{ } 31 \times \frac{7}{12} \times \frac{7}{12} = \frac{\text{Rs } 4431 \text{ } 7}{48} = \text{Rs } 92 \text{ } 5a \text{ } 3p$$

$$3 \text{ Disc} = \text{Rs } 6 \text{ } 94203125 \times \frac{1 \text{ } 1}{12} \times \frac{9}{12} = \text{Rs } 0 \text{ } 8677 \times 33 = \text{Rs } 28 \text{ } 634 \\ = \text{Rs } 28 \text{ } 10a \text{ } 3p$$

$$4 \text{ Disc} = \text{Rs } 5 \text{ } 31375 \times \frac{2}{6} \times 3 = \text{Rs } 6 \text{ } 6a \quad (146 \text{ days} = \frac{2}{6} \text{ year})$$

$$5 \text{ Disc} = \text{Rs } 3 \text{ } 67921875 \times \frac{3}{6} \times \frac{7}{12} = \text{Rs } 5 \text{ } 518 = \text{Rs } 5 \text{ } 8a \text{ } 3p \\ (219 \text{ days} = \frac{3}{6} \text{ yr})$$

$$6 \text{ Disc} = \text{Rs } 8 \text{ } 90 \times \frac{1 \text{ } 0}{12} \times 7 = \text{Rs } 51 \text{ } 9166 = \text{Rs } 51 \text{ } 14 \text{ } 1 \text{ } 9p$$



- 7 Disc = Rs 7 2990625  $\times \frac{2}{3} \times \frac{1}{2}$  = Rs 12 165 = Rs 12 2a 9p
- 8 Disc = Rs 5 31265625  $\times \frac{1}{12} \times 3$  = Rs 14 609 = Rs 14 9a 9p
- 9 Disc = £7 62  $\times \frac{176}{365} \times 4$  =  $\frac{£1072 896}{73}$  = £14 697 = £14 13s 11d
- 10 Disc  $\times$  £2 0535  $\times \frac{87}{365} \times 3$  =  $\frac{£0 4107 \times 261}{73}$  =  $\frac{£107 1927}{73}$   
= £1 468 = £1 9s 4d
- 11 10% =  $\frac{1}{10}$  Neglecting 3s 8d for percentage purposes, we have  
sum req<sup>d</sup> = £25 13s 8d -  $\frac{1}{10}$  of £25 10s = £23 2s 8d
- 12 As in 11, sum req<sup>d</sup> = £14 3s 2d -  $\frac{1}{10}$  of £14 = £13 9s 2d
- 13 As in 11, sum req<sup>d</sup> = £41 11s -  $\frac{1}{10}$  of £41 10s = £39 9s 6d
- 14 As in 11, sum req<sup>d</sup> = £38 4s 8d -  $\frac{1}{40}$  of £38 = £37 5s 8d
- 15 5% =  $\frac{1}{20}$  If Rs  $x$  be amt req<sup>d</sup>,  $\frac{197}{20} = 26 359$  and  $x$  = Rs 27 75
- 16 10% =  $\frac{1}{10}$  If £ $x$  be amt req<sup>d</sup>,  $\frac{9x}{10} = 16 65$  and  $x$  = £18 5

### EXAMPLES XVIII e Page 370

- 1 Time = 73 days =  $\frac{1}{5}$  year, Disc = Rs 6 03  $\times \frac{1}{5} \times \frac{5}{2}$  = Rs 3 0a 3p
- 2 Time = 219 days =  $\frac{3}{5}$  year, Disc = Rs 9 5275  $\times \frac{1}{5} \times 5$   
= Rs 28 9a 3p
- 3 Time = 99 days, Disc = Rs 3 81  $\times \frac{99}{365} \times 4$  =  $\frac{Rs 0 762 \times 396}{73}$   
=  $\frac{Rs 301 752}{73}$  = Rs 4 1335 = Rs 4 2a 3p
- 4 Time = 83 days, Disc = Rs 4 86  $\times \frac{83}{365} \times 3$  =  $\frac{Rs 0 972 \times 249}{73}$   
=  $\frac{Rs 242 028}{73}$  = Rs 3 315 = Rs 3 5a
- 5 Time = 61 days, Disc = Rs 27 6325  $\times \frac{61}{365} \times \frac{5}{2}$  =  $\frac{Rs 1685 5825}{146}$   
= Rs 11 545 = Rs 11 8a 9p

$$6 \text{ Time} = 154 \text{ days,} \quad \text{Disc} = \text{Rs } 317 \times \frac{154}{365} \times \frac{5}{2} = \frac{\text{Rs } 244.09}{73}$$

$$= \text{Rs } 3.437 = \text{Rs } 3.5a \text{ 6p,} \quad \text{Cash reqd} = \text{Rs } 317 - \text{Rs } 3.5a \text{ 6p}$$

$$7 \text{ Time} = 292 \text{ days} = \frac{4}{5} \text{ yr,} \quad \text{Disc} = \text{£}19342 \times \frac{4}{5} \times \frac{2}{4} = \text{£}348156$$

$$= \text{£}34.16s \text{ 4d,} \quad \text{Cash reqd} = \text{£}1934.4s - \text{£}34.16s \text{ 4d}$$

$$8 \text{ Time} = 83 \text{ days,} \quad \text{Disc} = \text{£}367555 \times \frac{83}{365} \times \frac{7}{2} = \frac{\text{£}73511 \times 581}{146}$$

$$= \frac{\text{£}42709801}{146} = \text{£}29253 = \text{£}29.5s \text{ 1d,}$$

$$\text{Cash reqd} = \text{£}3675.11s - \text{£}29.5s \text{ 1d}$$

$$9 \text{ Time} = 121 \text{ days,} \quad \text{Disc} = \text{£}745917 \times \frac{121}{365} \times \frac{7}{2} = \frac{\text{£}0745917 \times 847}{73}$$

$$= \frac{\text{£}6317917}{73} = \text{£}8655 = \text{£}8.13s \text{ 1d,}$$

$$\text{Cash reqd} = \text{£}745.18s \text{ 4d} - \text{£}8.13s \text{ 1d}$$

$$10 \text{ Time} = 11 \text{ days,} \quad \text{Disc} = \text{£}1031642 \times \frac{11}{365} \times 6$$

$$= \frac{\text{£}1031642 \times 492}{73} = \frac{\text{£}5075689}{73} = \text{£}6953 = \text{£}6.19s \text{ 1d,}$$

$$\text{Cash reqd} = \text{£}1031.12s \text{ 10d} - \text{£}6.19s \text{ 1d}$$

$$11 \text{ From Art 298, } I = \frac{P \times r \times n}{100}, \text{ where } I \text{ is in this case the discount,}$$

$$r = \frac{100 \times I}{P \times n} = \frac{100 \times 505}{505 \times \frac{1}{4}} = 4\%$$

$$12 \text{ As in No 11, } r = \frac{100 \times 225}{270 \times \frac{1}{4}} = 2\frac{1}{2}\%$$

$$13 \text{ From formula in No 11, } r = \frac{100 \times I}{P \times n} = \frac{100 \times 61\frac{6}{8}}{175 \times 5} = \frac{1}{2} \text{ yr} = 9 \text{ months}$$

$$14. \text{ As in No 13, } n = \frac{100 \times 128}{960 \times 4} = \frac{1}{3} \text{ yr} = 4 \text{ months}$$

$$15 \text{ Off a bill of £100 he takes £4, he charges £4 for the loan of £96 for 10 months, i.e. £4 \times \frac{10}{12} for the loan of £96 for 1 year,}$$

$$\text{hence rate reqd} = 5\%$$

$$16 \text{ As in 15, he charges £2\frac{1}{2} for the loan of £97\frac{1}{2} for 3 months, i.e. £10 for the loan of £97\frac{1}{2} for 12 months,}$$

$$\text{hence rate reqd} = 10.26\%$$

## EXAMPLES XVIII f Page 372

*N B* In this set of examples,

$A_n$  denotes Amount in  $n$  years,

$P_n$  „ the Principal at the beginning of the  $n^{\text{th}}$  year,

$I_n$  „ the  $n^{\text{th}}$  year's interest

1	Rs $P_1$ 200 $I_1$ 10 $P_2$ 210 $I_2$ 10 5 $A_2$ 220 5	2	Rs $P_1$ 2500 $I_1$ 100 $P_2$ 2600 $I_2$ 104 $A_2$ 2704	3	Rs $P_1$ 3750 $I_1$ 112 5 $P_2$ 3862 5 $I_2$ 115 875 $A_2$ 3978 375
4	Rs $P_1$ 3500 $I_1$ 175 $P_2$ 3675 $I_2$ 183 75 $P_3$ 3858 75 $I_3$ 192 9375 $A_3$ 4051 6875	5	Rs $P_1$ 750 $I_1$ 15 $P_2$ 765 $I_2$ 15 3 $P_3$ 780 3 $I_3$ 15 606 $P_4$ 795 906 $I_4$ 15 918 $A_4$ 811 824	6	Rs $P_1$ 3300 $I_1$ 165 $P_2$ 3465 $I_2$ 173 25 $P_3$ 3638 25 $I_3$ 181 912   5 $P_4$ 3820 162   5 $I_4$ 191 008   12 $A_4$ 4011 171
7	£ $P_1$ 225 $I_1$ 9 $P_2$ 234 $I_2$ 9 36 $P_3$ 243 36 $P_1$ 225 CI 18 36	8	£ $P_1$ 425 $I_1$ 17 $P_2$ 442 $I_2$ 17 63 $P_3$ 459 68 $I_3$ 18 387   2 $P_4$ 478 067   2 $P_1$ 425 CI 53 067	9	£ $P_1$ 725 $I_1$ 21 75 $P_2$ 746 75 $I_2$ 22 402   5 $P_3$ 769 152   5 $I_3$ 23 074   57 $P_4$ 792 227   07 $I_4$ 23 766   81 $P_5$ 815 993   8 $P_1$ 725 CI 90 994
10	£ $P_1$ 3546 $I_1$ 177 3 $P_2$ 3723 3 $I_2$ 186 165   $P_3$ 3909 465   $I_3$ 195 473   25 $P_4$ 4104 938   25 $P_1$ 3546 CI 558 938	11	£ $P_1$ 345 75 $I_1$ 13 83 $P_2$ 359 58 $I_2$ 14 383   2 $A_2$ 373 963	12	£ $P_1$ 472 9 $I_1$ 14 187 $P_2$ 487 087 $I_2$ 14 612   61 $P_3$ 501 699   61 $I_3$ 15 050   97 $A_3$ 516 751

		£
13	P <sub>1</sub>	1243 713
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 62 185 \\ 65 \end{array}$
	P <sub>2</sub>	1305 895
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 65 291 \\ 93 \end{array}$
	A <sub>1</sub>	1371 194

		Rs
14	P <sub>1</sub>	473 85
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 18 954 \\ 0 \end{array}$
	P <sub>2</sub>	492 801
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 19 712 \\ 16 \end{array}$
	P <sub>3</sub>	512 516
	I <sub>3</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 20 500 \\ 64 \end{array}$
	A <sub>3</sub>	533 017

		Rs
15	P <sub>1</sub>	5000
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 125 \\ 125 \end{array}$
	P <sub>2</sub>	5125
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 128 125 \\ 125 \end{array}$
	P <sub>3</sub>	5253 125
	I <sub>3</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 131 328 \\ 1 \end{array}$
	A <sub>3</sub>	5384 453

		Rs
16	P <sub>1</sub>	4500
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 180 \\ 11 25 \end{array}$
	P <sub>2</sub>	4691 25
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 187 65 \\ 11 728 \\ 12 \end{array}$
	A <sub>2</sub>	4890 628

		Rs
17	P <sub>1</sub>	4750
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 142 5 \\ 23 75 \end{array}$
	P <sub>2</sub>	4916 25
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 147 487 \\ 24 581 \\ 5 \end{array}$
	P <sub>3</sub>	5088 318
	I <sub>3</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 152 649 \\ 25 411 \\ 59 \end{array}$
	A <sub>3</sub>	5266 410

		Rs
18	P <sub>1</sub>	6750
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 270 \\ 33 75 \end{array}$
	P <sub>2</sub>	7053 75
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 282 15 \\ 35 268 \\ 75 \end{array}$
	P <sub>3</sub>	7371 168
	I <sub>3</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 291 816 \\ 36 855 \\ 81 \end{array}$
	A <sub>3</sub>	7702 871

		£
19	P <sub>1</sub>	5016 575
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 200 663 \\ 25 082 \\ 87 \end{array}$
	P <sub>2</sub>	5242 320
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 209 692 \\ 26 211 \\ 83 \end{array}$
	A <sub>2</sub>	5478 225

		£
20	P <sub>1</sub>	1601 233
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 48 036 \\ 4 003 \\ 09 \end{array}$
	P <sub>2</sub>	1653 273
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 49 598 \\ 4 133 \\ 18 \end{array}$
	P <sub>3</sub>	1707 004
	I <sub>3</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 51 210 \\ 4 267 \\ 51 \end{array}$
	A <sub>3</sub>	1762 482

		£
21	P <sub>1</sub>	2000
	I <sub>1</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 100 \\ 105 \end{array}$
	P <sub>2</sub>	2100
	I <sub>2</sub>	$\left\{ \begin{array}{l} 1000 \\ 1000 \end{array} \right. \begin{array}{l} 2205 \\ 55 125 \end{array}$
	A <sub>2</sub>	2260 125
	P <sub>1</sub>	2000
	CI	260 125

		£	
22	$P_1$	5620	
	$I_1$	224 8	
	$P_2$	5844 8	
	$I_2$	233 792	
	$P_{2\frac{1}{2}}$	6078 592	
	$I_{2\frac{1}{2}}$	121 571	8
	$A_{2\frac{1}{2}}$	6200 163	8
	$P_1$	5620	
	CI	580 164	

		£	
23	$P_1$	3600	
	$I_1$	90	
	$P_2$	3690	
	$I_2$	92 25	
	$P_3$	3782 25	
	$I_3$	94 556	25
	$P_{3\frac{1}{2}}$	3876 806	25
	$I_{3\frac{1}{2}}$	48 460	1
	$A_{3\frac{1}{2}}$	3925 266	3
	$P_1$	3600	
	CI	325 266	

		£	
24	$P_1$	8457 725	
	$I_1$	$\left\{ \begin{smallmatrix} 100 \\ 400 \end{smallmatrix} \right.$	$\begin{smallmatrix} 84 577 \\ 21 144 \end{smallmatrix} \begin{smallmatrix} 25 \\ 31 \end{smallmatrix}$
	$P_2$	8563 446	56
	$I_2$	$\left\{ \begin{smallmatrix} 100 \\ 400 \end{smallmatrix} \right.$	$\begin{smallmatrix} 85 634 \\ 21 408 \end{smallmatrix} \begin{smallmatrix} 46 \\ 61 \end{smallmatrix}$
	$P_{2\frac{1}{2}}$	8670 489	63
	$I_{2\frac{1}{2}}$	$\left\{ \begin{smallmatrix} 100 \\ 1800 \end{smallmatrix} \right.$	$\begin{smallmatrix} 21 676 \\ 5 419 \end{smallmatrix} \begin{smallmatrix} 22 \\ 05 \end{smallmatrix}$
	$A_{2\frac{1}{2}}$	8697 584	9
	$P_1$	8457 725	
	CI	239 860	

		Rs	
25	$P_1$	504 687	5
	$I_1$	$\left\{ \begin{smallmatrix} 100 \\ 200 \end{smallmatrix} \right.$	$\begin{smallmatrix} 20 187 \\ 2 523 \end{smallmatrix} \begin{smallmatrix} 5 \\ 44 \end{smallmatrix}$
	$P_2$	527 398	44
	$I_2$	$\left\{ \begin{smallmatrix} 100 \\ 200 \end{smallmatrix} \right.$	$\begin{smallmatrix} 21 095 \\ 2 636 \end{smallmatrix} \begin{smallmatrix} 94 \\ 90 \end{smallmatrix}$
	$P_{2\frac{1}{2}}$	551 131	37
	$I_{2\frac{1}{2}}$	$\left\{ \begin{smallmatrix} 100 \\ 200 \end{smallmatrix} \right.$	$\begin{smallmatrix} 5 511 \\ 2 755 \end{smallmatrix} \begin{smallmatrix} 31 \\ 66 \end{smallmatrix}$
	$A_{2\frac{1}{2}}$	559 398	34
	$P_1$	504 687	5
	CI	54 711	

*NB* In Nos 26-31,  $P_1, P_{1\frac{1}{2}}, P_2, P_{2\frac{1}{2}}$ , etc, denote the Principals for successive half years

		£	
26	$P_1$	320 75	
1 <sup>st</sup> half yr's	$I \left( \frac{2}{100} \right)$	6 415	
	$P_{1\frac{1}{2}}$	327 165	
2 <sup>nd</sup> half-yr's	$I \left( \frac{2}{100} \right)$	6 543	3
	$P_2$	333 708	3
3 <sup>rd</sup> half yr's	$I \left( \frac{2}{100} \right)$	6 674	16
	$A_{1\frac{1}{2}}$	340 382	

		£	
27	$P_1$	520 8	
1 <sup>st</sup> half-yr's	$I \left( \frac{1}{100} \right)$	13 02	
	$P_{1\frac{1}{2}}$	533 82	
2 <sup>nd</sup> half-yr's	$I \left( \frac{1}{100} \right)$	13 345	5
	$P_2$	547 165	5
3 <sup>rd</sup> half-yr's	$I \left( \frac{1}{100} \right)$	13 679	14
	$P_{2\frac{1}{2}}$	560 844	64
4 <sup>th</sup> half-yr's	$I \left( \frac{1}{100} \right)$	14 021	11
	$A_2$	574 866	

28	$P_1$	$R_s$	16000
1 <sup>st</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	320	40
	$P_{1\frac{1}{2}}$	16360	
2 <sup>nd</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	327 2	40 9
	$P_2$	16728 1	
3 <sup>rd</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	334 562 1	41 820 2
	$A_{1\frac{1}{2}}$	17101 182	

30	$P_1$	$R_s$	250
1 <sup>st</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	2 5	0 625
	$P_{1\frac{1}{4}}$	253 125	
2 <sup>nd</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	2 531 25	0 632 81
	$P_{1\frac{1}{2}}$	256 289	06
3 <sup>rd</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	2 562 89	0 640 72
	$P_{1\frac{3}{4}}$	259 492	67
4 <sup>th</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	2 594 92	0 649 73
	$A_1$	262 736	

31 (ii)	$P_1$	£	100
1 <sup>st</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	1	0 25
	$P_{1\frac{1}{4}}$	101 25	
2 <sup>nd</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	1 012 5	0 253 12
	$P_{1\frac{1}{2}}$	102 515	62
3 <sup>rd</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	1 025 15	0 256 29
	$P_{1\frac{3}{4}}$	103 797	06
4 <sup>th</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{400} \end{array} \right.$	1 037 97	0 259 49
	$A_1$	105 095	

effective annual rate  
= 5.095 %

29	$P_1$	$R_s$	100000
1 <sup>st</sup> half yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	1000	250
	$P_{1\frac{1}{2}}$	101250	
2 <sup>nd</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	1012 5	253 125
	$P_2$	102515 625	
3 <sup>rd</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	1025 156	256 289 06
	$P_{2\frac{1}{2}}$	103797 070	31
4 <sup>th</sup> half yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	1037 970	259 492 67
	$P_3$	105094 533	68
5 <sup>th</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{100} \end{array} \right.$	1050 945	262 736 33
	$A_{2\frac{1}{2}}$	106408 215	

31 (i)	$P_1$	£	100
1 <sup>st</sup> half yr's I	$\left\{ \begin{array}{l} \frac{1}{200} \end{array} \right.$	5	
	$P_{1\frac{1}{2}}$	105	
2 <sup>nd</sup> half-yr's I	$\left\{ \begin{array}{l} \frac{1}{200} \end{array} \right.$	5 25	
	$P_2$	110 25	
effective annual rate = 10.250 %			

31 (iii)	$P_1$	£	100
1 <sup>st</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{800} \end{array} \right.$	0 5	0 125
	$P_{1\frac{1}{4}}$	100 625	
2 <sup>nd</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{800} \end{array} \right.$	0 503 12	0 125 78
	$P_{1\frac{1}{2}}$	101 253	90
3 <sup>rd</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{800} \end{array} \right.$	0 506 27	0 126 57
	$P_{1\frac{3}{4}}$	101 886	74
4 <sup>th</sup> quarter's I	$\left\{ \begin{array}{l} \frac{1}{800} \end{array} \right.$	0 509 43	0 127 36
	$A_1$	102 524	

effective annual rate  
= 2.524 %

## EXAMPLES XVIII g Page 376

*N.B.*  $A_n$ ,  $P_n$ ,  $I_n$  have the same meanings as in Ex XVIII f

$$1 \quad A_n = Rs \ 387 \times (1.03)^2 = Rs \ 387 \times 1.0609 = Rs \ 110.5683$$

$$\begin{aligned} 2 \quad A_1 &= Rs \ 873.5 \times (1.04)^3 \\ &= Rs \ 873.5 \times 1.124864 \\ &= Rs \ 982.569 \end{aligned}$$

873 5	
1 124864	
873 5	
87 35	
17 470	
3 494	0
698	8
52	4
3	5
982 568	7

$$\begin{aligned} 3 \quad A_1 &= £953.775 \times (1.05)^3 \\ &= £953.775 \times 1.157625 \\ &= £1104.113 \end{aligned}$$

953 775	
1 157625	
953 775	
95 377	5
47 688	8
6 676	4
572	3
18	1
4	8
1104 112	9

$$4 \quad P = \frac{\text{given Amt}}{\text{Amt of } £1} = \frac{£296}{(1.03)^2} = \frac{£296}{1.0609} = £279$$

$$5 \quad P = \frac{\text{given Amt}}{\text{Amt of } £1} = \frac{£340.95}{(1.03)^3} = \frac{£340.95}{1.092727} = £312$$

$$6 \quad P = \frac{\text{given Amt}}{\text{Amt of } £1} = \frac{£3244.9}{(1.04)^2} = \frac{£3244.9}{1.0816} = £3000$$

$$7 \quad P = \frac{\text{given Amt}}{\text{Amt of } £1} = \frac{£5343.112}{(1.04)^3} = \frac{£5343.112}{1.124864} = £4750$$

$$8 \quad P = \frac{\text{given Amt}}{\text{Amt of } £1} = \frac{£3820.162}{(1.05)^3} = \frac{£3820.162}{1.157625} = £3300$$

$$9 \quad P = \frac{\text{given Amt}}{\text{Amt of } £1} = \frac{£275.95}{(1.025)^4} = \frac{£275.95}{(1.0506)^2} = \frac{£275.95}{1.1038} = £250$$

$$10 \quad P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 300}{(1.01)^3} = \frac{\text{Rs } 300}{1.030301} = \text{Rs } 266.699$$

266 699
1 124864 ) 300
750272
75354
7862
1113
101

$$11 \quad P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 850}{(1.05)^2} = \frac{\text{Rs } 850}{1.1025} = \text{Rs } 770.975$$

$$12 \quad P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 2000}{(1.03)^2} = \frac{\text{Rs } 2000}{1.0609} = \text{Rs } 1830.283$$

1830 283
1 092727 ) 2000
9072730
30914
3096
911
37
5

$$13 \quad P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 6027.875}{(1.0375)^2} = \frac{\text{Rs } 6027.875}{1.07640625} = \text{Rs } 5600$$

$$14. \quad P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 3925.265625}{(1.025)^3 \times 1.0125} = \frac{\text{Rs } 3925.265625}{1.076890625 \times 1.0125} = \frac{\text{Rs } 3925.266}{1.090351758} = \text{Rs } 3600$$

$$15 \quad A = \text{Rs } 250 \times 1.04 \times 1.08 = \text{Rs } 280.8$$

$$16 \quad \text{From Art 313, the Amt in 2 years} = P \times (1.05)^2, \\ \text{hence Amt in } 2\frac{1}{2} \text{ yrs} = P \times (1.05)^2 \times 1.025$$

For second part of question

$$P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 22050}{(1.05)^2 \times (1.025)} = \frac{\text{Rs } 22050}{1.1025 \times 1.025} = \frac{\text{Rs } 22050}{1.13006} = \text{Rs } 19512$$

19512
1 13006 ) 22050
107494
5789
139
26
3

$$17 \quad P = \frac{\text{given Amt}}{\text{Amt of Re 1}} = \frac{\text{Rs } 2811.90625}{1.03 \times 1.04 \times 1.05} = \frac{\text{Rs } 2811.90625}{1.12476} = \text{Rs } 2500$$



$$18 \quad A_2 = P \times (1.035)^2 = 1.07123 \text{ (approx.)}$$

$$19 \quad CI = A_3 - P = P \times (1.05)^3 - P = P \{ (1.05)^3 - 1 \} = P \times 0.15763 \text{ (approx.)},$$

we have £50 =  $P \times 0.15763$ , and  $P = £50 \div 0.15763 = £317$

$$20 \quad \text{Pop}^n \text{ for } 1891 = \frac{37632}{1.12} = 33600, \quad \text{pop}^n \text{ for } 1881 = \frac{33600}{1.12} = 30000$$

$$21 \quad \text{Req}^d \text{ population} = 347865 \times 1.02 \times 1.025 \times 1.03 \text{ to nearest hundred}$$

$$= 3478.65 \times 1.0769 \text{ to nearest unit}$$

$$= 3746$$

Hence answer = 374600

$$22 \quad CI \text{ on } £1 = £(1.04)^3 - £1 = £0.124864$$

$$SI \text{ on } £1 = 0.04 \times 3 = £0.12, \quad \text{diff for } £1 = £0.004864$$

$$\text{Hence diff for } £3125 = £3125 \times 0.004864 = £15.2$$

$$23 \quad CI \text{ on } £1 = £(1.04)^4 - £1 = £0.169859$$

$$SI \text{ on } £1 = 0.04 \times 4 = £0.16,$$

$$\text{diff for } £1 = £0.009859, \text{ and diff for } £1000 = £9.859$$

$$24 \quad CI \text{ on } £1 = £(1.05)^3 - £1 = £0.157625$$

$$SI \text{ on } £1 = £0.05 \times 3 = £0.15, \quad \text{diff for } £1 = £0.007625$$

$$\text{Hence diff for } £2400 = £2400 \times 0.007625 = £18.3$$

$$25 \quad CI \text{ on } £1 = £(1.0375)^3 - £1 = £0.07640625$$

$$SI \text{ on } £1 = £0.0375 \times 2 = £0.075, \quad \text{diff for } £1 = £0.00140625$$

$$\text{Hence diff for } £2960 = £2960 \times 0.00140625 = £4.163$$

$$26 \quad CI \text{ on } £1 = £(1.02)^4 - £1 = £0.08243216$$

$$SI \text{ on } £1 = 0.02 \times 4 = £0.08, \quad \text{diff for } £1 = £0.00243216$$

$$\text{Hence diff for } £7545.5 = £7545.5 \times 0.00243216$$

$$= £18.352$$

$$27 \quad \text{From Note, Ex 2, p 376,}$$

$$P = \frac{\text{diff on } £P}{\text{diff on } £1} = \frac{£0.6}{0.0025} = £240$$

$$28 \quad \text{As in 27, } P = \frac{£15.2}{0.04864} = £3125 \quad 29 \quad \text{As in 27, } P = \frac{Rs 3.375}{0.0009} = Rs 3750$$

$$30 \quad \text{As in 27, } P = \frac{Rs 38.125}{0.007625} = Rs 5000$$

$$31 \quad \text{The CI is reckoned on Rs } (26333.3125 - 13000),$$

$$\text{i.e. Rs } 13333.3125 \quad \text{Amt} = Rs 13333.3125 \times (1.035)^3$$

$$= Rs 13333.3125 \times 1.071225 = Rs 14282.977$$

	Rs
32 $P_2$	4000
$I_2$	<u>200</u>
$P_3$	4200
$I_3$	<u>210</u>
$P_4$	4410
$I_4$	<u>220 5</u>
$P_5$	4630 5
$I_5$	<u>231 525</u>
$P_6$	4862 025

$$\text{total savings} = \begin{cases} P_2 & 1000 \\ P_3 & 4200 \\ P_4 & 4410 \\ P_5 & 4630 5 \\ P_6 & 4862 025 \end{cases}$$

$$\underline{\hspace{10em}} 22102 525$$

i.e. Rs 22103

	Rs
33 $P_1$	200
$I_1$	<u>10</u>
$A_1$	210
Subtract	<u>50</u>
$P_2$	160
$I_2$	<u>8</u>
$A_2$	168
Subtract	<u>50</u>
$P_3$	118
$I_3$	<u>5 9</u>
$A_3$	123 9
Subtract	<u>50</u>
	<u>73 9</u>
$I_4$	<u>3 695</u>
$A_4$	77 595
Subtract	<u>50</u>
	<u>27 595</u>

i.e. Rs 27 9a 6p

34 Its value at the end of each year  
 $= \left(1 - \frac{12\frac{1}{2}}{100}\right)$  of its value at the  
beginning of the year

This fraction reduces to  $\frac{7}{8}$

$$\text{Value reqd} = \text{Rs } 24000 \times \left(\frac{7}{8}\right)^4$$

$$= \text{Rs } 24000 \times \frac{2401}{64} = \text{Rs } 14068$$

### EXAMPLES XVIII h Page 381

2 See Ex, Art 315

3 Since £(100 - 12½), or £87½, is cash payment for a sum of £100, after discount at 12½ per cent has been deducted, we have  $\frac{7}{87\frac{1}{2}} = \frac{7}{100}$ , which becomes  $\gamma = \frac{35}{40}$ . This is the graph formed by joining the origin to the point (40, 35). A convenient scale is 01" to 1 shilling along each axis. The required values can easily be read off.

4 From Art 316 we see the graph will be linear

Measure time horizontally (1 inch to 10 years), and Amount vertically (1 inch to £40) beginning at £260

The first graph is the line joining the points (6, 260) and (15, 350)  
The second graph is the line joining the points (5, 330) and (20, 420) In each of these lines the ordinate of any point gives the Amount for the number of years given by the corresponding abscissa

Again, these graphs intersect at a point where  $x=25$ ,  $y=450$   
Thus each Principal with its Interest amounts to £450 in 25 years

When  $x=0$  there is no Interest, thus the Principals will be obtained by reading off the values of the intercepts made by the two graphs on the  $y$ -axis These are £200 and £300 respectively

NOTE To obtain the result  $y=200$  it will be necessary to continue the  $y$ -axis downwards sufficiently far to shew this ordinate

### EXAMPLES XIX a Page 384

1-6 See Art 324, Ex 1

7-12 See Art 324, Ex 2

13 Rs 100 stock is worth Rs 95 cash,

Rs 2880 stock sells for Rs  $\frac{95}{100} \times 2880$  cash

14 Rs 3500 stock costs Rs 3220 cash,

Rs 100 stock costs Rs  $\frac{3220}{3500} \times 100$  cash

15 £285 stock costs £228 cash,

£100 stock costs £ $\frac{228}{285} \times 100$  cash

16 £100 Consols is worth £83½ cash,

£855 Consols is worth £ $\frac{83\frac{1}{2}}{100} \times 855$  cash

17 £102½ cash buys £100 stock,

£1000 cash buys £ $\frac{100}{102\frac{1}{2}} \times 1000$  cash

18 £560 stock costs £480 cash,

£100 stock costs £ $\frac{480}{560} \times 100$  cash

19 Rs 100 stock sells for Rs 94½ cash,

Rs 2912 stock sells for Rs  $\frac{94\frac{1}{2}}{100} \times 2912$ , or Rs  $(94\frac{1}{2} \times 29\frac{1}{2})$  cash

- 20 Rs 100 stock costs Rs  $125\frac{1}{2}$  cash,  
Rs 7782 stock costs Rs  $\frac{125\frac{1}{2}}{100} \times 7782$ , or Rs  $(77\ 82 \times 125\ 5)$  cash
- 21 Rs 100 stock costs Rs  $109\frac{1}{2}$  cash,  
Rs 3572 stock costs Rs  $\frac{109\frac{1}{2}}{100} \times 3572$ , or Rs  $(35\ 72 \times 109\ 5)$  cash
- 22 On Rs 100 stock the loss is Rs 5,  
on Rs 780 stock the loss is Rs  $\frac{5}{100} \times 780$
- 23 On £100 stock the profit is £3½,  
on £650 stock the profit is £  $\frac{3\frac{1}{2}}{100} \times 650$
- 24 Rs 131½ cash buys stock afterwards sold for Rs 133 cash,  
hence req<sup>d</sup> proceeds = Rs  $\frac{133}{131\frac{1}{2}} \times 750$
- 25 £105 cash buys stock afterwards sold for £102 cash,  
hence req<sup>d</sup> cash = £  $10\frac{5}{2} \times 155$
- 26 On £100 stock the loss is £5½,  
on £2500 stock the loss is £  $\frac{5\frac{1}{2}}{100} \times 2500$
- 27 On an outlay of Rs 112½ the gain is Rs  $(117 - 112\frac{1}{2})$ , or Rs 4½.  
req<sup>d</sup> gain = Rs  $\frac{4\frac{1}{2}}{112\frac{1}{2}} \times 1187\frac{1}{2}$  = Rs  $\frac{9}{22} \times 1187\frac{1}{2}$  = Rs  $\frac{54}{11}$   
= Rs 37 8½
- 28 On Rs 100 stock the profit is Rs  $(107\frac{1}{2} - 101\frac{1}{2})$ , or Rs 6½,  
on Rs 3720 stock the gain is Rs  $\frac{6\frac{1}{2}}{100} \times 3720$  = Rs  $\frac{1}{6} \times 3720$   
= Rs 213 14 6 p
- 29 As in No 28 we get req<sup>d</sup> loss = £  $\frac{4}{100} \times 1642$  = £50 18 1d

### EXAMPLES XIX b Page 386

1-3 See Art 325, Ex 1

4-6 See Art 325, Ex 2

7 1<sup>st</sup> stock income = Rs  $3 \times 11\frac{3}{4}\%$  = Rs 335 653,

2<sup>nd</sup> stock income = Rs  $11 \times 11\frac{3}{4}\%$  = Rs 346 153,

2<sup>nd</sup> stock gives the greater income by Rs 10 8a

8 Req<sup>d</sup> inc = £22  $\times \frac{100}{100} \times \frac{10}{100}$     9 Req<sup>d</sup> inc =  $\frac{1}{2}$  of £34  $\times \frac{100}{100} \times \frac{20}{100}$

$$10 \text{ Req}^d \text{ inc} = \text{Rs } 2\frac{3}{4} \times \frac{7\frac{3}{8}}{1\frac{1}{8}} = \text{Rs } 2\frac{7}{8} \times 7\frac{3}{8} = \text{Rs } 21\frac{6}{15} = \text{Rs } 21\frac{9}{15} \text{ a } 9 \text{ p}$$

$$11 \text{ Req}^d \text{ inc} = \text{Rs } 4\frac{1}{4} \times \frac{1\frac{7}{8}}{1\frac{1}{8}} = \text{Rs } \frac{9\frac{3}{8}}{2\frac{1}{8}} = \text{Rs } 11\frac{3}{8} = \text{Rs } 11\frac{3}{8} \text{ a } 6 \text{ p}$$

$$12 \text{ Req}^d \text{ inc} = \text{£}2\frac{1}{2} \times \frac{500}{82\frac{1}{4}} = \text{£} \frac{5000}{329} = \text{£}15\frac{19}{75} = \text{£}15\frac{3}{8} \text{ s } 11 \text{ d}$$

$$13 \text{ Req}^d \text{ amt of stock} = \text{Rs } 100 \times \frac{8\frac{1}{2}}{3}$$

$$14 \text{ Req}^d \text{ amt of stock} = \text{Rs } 100 \times \frac{297}{4\frac{1}{2}} \quad 15 \text{ Req}^d \text{ sum} = \text{Rs } 110 \times \frac{60}{5\frac{1}{2}}$$

$$16 \text{ Req}^d \text{ sum} = \text{£}100 \times \frac{24\frac{3}{4}}{5\frac{3}{4}} \quad 17 \text{ Req}^d \text{ sum} = \text{£}115\frac{1}{2} \times \frac{80\frac{3}{4}}{5\frac{3}{4}}$$

$$18 \text{ Req}^d \text{ sum} = \text{£}141\frac{1}{4} \times \frac{17\frac{3}{8}}{5\frac{1}{2} \times \frac{22}{16}} = \text{£} \frac{565 \times 88 \times 2 \times 240}{4 \times 5 \times 11 \times 226} = \text{£}480$$

$$19 \text{ Req}^d \text{ amt} = \text{£}100 \times \frac{35\frac{3}{8}}{2\frac{1}{2}} \quad 20 \text{ Req}^d \text{ sum} = \text{Rs } 104 \times \frac{2\frac{1}{2}}{3}$$

$$21 \text{ As in No 18, req}^d \text{ amt} = \text{£}132\frac{1}{2} \times \frac{77\frac{3}{8}}{4\frac{1}{2} \times \frac{1}{8}} = \text{£} \frac{530 \times 41}{9} = \text{£}2414\frac{4}{9}$$

$$22 \text{ £90 cash buys an income of £}4\frac{1}{2},$$

$$\text{£100 cash buys an income of £}4\frac{1}{2} \times \frac{1\frac{1}{2}}{3}, \text{ giving } 5\%$$

$$23 \text{ As in No 22, £100 cash buys an income of £}3\frac{1}{4} \times \frac{1\frac{1}{2}}{2}, \text{ giving } 3\%$$

$$24 \text{ As in No 22, £100 cash buys an income of £}5\frac{1}{2} \times \frac{1\frac{1}{2}}{4}, \text{ or £}1\frac{11}{8}, \text{ giving } 3\frac{7}{8}\%$$

$$25 \text{ As in No 22, £100 cash buys an income of £}3\frac{1}{2} \times \frac{100}{78\frac{1}{4}}, \text{ or £}4\frac{1}{9}, \text{ giving } 4\frac{4}{9}\%$$

$$26 \text{ As in No 22, £100 cash buys an income of £}4 \times \frac{100}{119\frac{3}{8}}, \text{ or £}3\frac{4}{11}, \text{ giving } 3\frac{3}{8}\%$$

$$27 \text{ 1}^{\text{st}} \text{ stock income from £100} = \text{£}3 \times \frac{1\frac{1}{2}}{3} = \text{£}3\frac{5}{8},$$

$$\text{2}^{\text{nd}} \text{ stock income from £100} = \text{£}4 \times \frac{1\frac{1}{2}}{3} = \text{£}4\frac{1}{6}$$

$$\text{2}^{\text{nd}} \text{ stock is better by £0 } 64, \text{ or } 0\frac{6}{8}\%$$

$$28 \text{ 1}^{\text{st}} \text{ stock income from £100} = \text{£}4 \times \frac{1\frac{1}{2}}{2} = \text{£}3\frac{1}{4},$$

$$\text{2}^{\text{nd}} \text{ stock income from £100} = \text{£}3\frac{1}{2} \times \frac{1\frac{1}{2}}{2} = \text{£}3\frac{3}{4}$$

$$\text{2}^{\text{nd}} \text{ stock is better by £0 } 07, \text{ or } 0\frac{7}{8}\%$$

$$29 \text{ 1}^{\text{st}} \text{ stock income from £100} = \text{£}5 \times \frac{1\frac{1}{2}}{4} = \text{£}3\frac{5}{8},$$

$$\text{2}^{\text{nd}} \text{ stock income from £100} = \text{£}6 \times \frac{1\frac{1}{2}}{4} = \text{£}4\frac{3}{4}$$

$$\text{1}^{\text{st}} \text{ stock is better by £0 } 12, \text{ or by } 0\frac{1}{2}\%$$

30 1<sup>st</sup> stock income from Rs 100 = Rs  $4 \times \frac{100}{100} = \text{Rs } \frac{100}{1}$ ,

2<sup>nd</sup> stock income from Rs 100 = Rs  $4\frac{1}{2} \times \frac{100}{17} = \text{Rs } \frac{100}{2}$

2<sup>nd</sup> stock is the better

For second part of question,

diff of incomes on Rs 100 invested = Rs  $(\frac{100}{1} - \frac{100}{2}) = \text{Rs } \frac{100}{2}$ ,

diff of incomes on Rs 7020 invested = Rs  $\frac{100}{2} \times \frac{7020}{100} = \text{Rs } 10$

31 Let Rs  $x$  be sum invested by each, the income of the first  
= Rs  $3 \times \frac{x}{81}$ , the income of the second = Rs  $4 \times \frac{x}{112}$ ,

$$\frac{3x}{81} - \frac{4x}{112} = 2, \text{ and } x = 1512$$

32 Let Rs  $x$  be the req<sup>d</sup> amt of stock A's income = Rs  $3\frac{1}{2} \times \frac{x}{100}$ ,

B's income = Rs  $2\frac{1}{2} \times \frac{x}{100}$ ,  $\frac{3\frac{1}{2}x}{100} - \frac{2\frac{1}{2}x}{100} = 100$ , and  $x = \text{Rs } 10000$

33 If Rs  $x$  be the req<sup>d</sup> price, then Rs  $x$  cash produces Rs 4 income,

Rs 100 cash produces Rs  $\frac{4}{x} \times 100$  income,

$$\frac{4}{x} \times 100 = 5, \text{ and } x = 80$$

34 As in No 33,  $\frac{2\frac{1}{2}}{x} \times 100 = 2\frac{1}{2}$ , and  $x = 110$

35 As in No 33,  $\frac{4\frac{1}{2}}{x} \times 100 = 6$ , and  $x = 75$

36 Let Rs  $x$  be req<sup>d</sup> price, then income from Rs 1600 invested is

$$3 \times \frac{1600}{x}, \quad 3 \times \frac{1600}{x} = 50, \text{ and } x = 96$$

37 As in No 36,  $5\frac{1}{2} \times \frac{24000}{x} = 1200$ , and  $x = 110$

38 As in No 36,  $5\frac{1}{2} \times \frac{2353}{x} = 95\frac{1}{2}$ , and  $x = 135\frac{2}{3}$

39 If £ $x$  be req<sup>d</sup> price, £ $x$  cash produces £6 income,

£100 cash produces £ $\frac{6}{x} \times 100$  income Thus, after income tax

has been paid, becomes  $\frac{230}{240} \times \frac{6}{x} \times 100$  But this is £5,

$$\frac{230}{240} \times \frac{6}{x} \times 100 = 5, \text{ and } x = 115$$

- 40 4 45 is percentage on £100 cash ,  
 . £21 7s 6d, or £21 375 is percentage on  $£100 \times \frac{21 \frac{3}{4} \frac{7}{8}}{4 \frac{4}{5}}$ ,  
 or £480 (to nearest £)
- 41 If £ $x$  be req<sup>d</sup> amount, income from it is  $£2\frac{3}{4} \times \frac{x}{92}$ ,  
 $2\frac{3}{4} \times \frac{x}{92} = 60$ , and  $x = £2007$  (to nearest £)

### EXAMPLES XIX c Page 392

- 1 Rs 100 stock sells for Rs  $(105\frac{3}{4} - \frac{1}{4})$ , or Rs  $105\frac{1}{2}$  cash ,  
 hence req<sup>d</sup> cash = Rs  $6785 \times \frac{105\frac{1}{2}}{100}$
- 2 Rs 100 stock costs me Rs  $(90\frac{7}{8} + \frac{1}{8})$ , or Rs 91 cash ,  
 hence req<sup>d</sup> price = Rs  $2750 \times \frac{91}{100}$
- 3 Rs  $(101\frac{1}{4} + \frac{1}{4})$ , or Rs  $101\frac{1}{2}$  cash buys Rs 100 stock ,  
 Rs  $3552\frac{1}{2}$  buys Rs  $3552\frac{1}{2} \times \frac{100}{101\frac{1}{2}}$  stock
- 4 Rs  $(93\frac{1}{4} - \frac{1}{8})$ , or Rs  $93\frac{1}{8}$  cash is realised on Rs 100 stock ,  
 req<sup>d</sup> amount of stock = Rs  $100 \times \frac{6844\frac{1}{8}}{93\frac{1}{8}}$
- 5 Rs 100 stock costs Rs  $(125\frac{1}{4} + \frac{1}{4})$ , or Rs  $125\frac{1}{2}$  cash ,  
 req<sup>d</sup> cost = Rs  $5782 \times \frac{125\frac{1}{2}}{100} = Rs 7256 \frac{410}{100}$
- 6 Buying price =  $91\frac{3}{8} + \frac{1}{8}$ , or  $91\frac{1}{2}$ , selling price =  $90\frac{1}{8} - \frac{1}{8}$ , or 90  
 loss on Rs 100 stock = Rs  $1\frac{1}{2}$ , hence req<sup>d</sup> loss = Rs  $1\frac{1}{2} \times \frac{2500}{100}$
- 7 Gain on buying and selling Rs 100 stock = Rs  $(108 - 105)$ , or Rs 3  
 Half year's dividend on Rs 100 stock = Rs  $1\frac{3}{4}$ ,  
 total gain on Rs 100 stock = Rs  $4\frac{3}{4}$   
 Hence on outlay of Rs 105, gain is Rs  $4\frac{3}{4}$   
 " " Rs 3500, gain is Rs  $4\frac{3}{4} \times \frac{3500}{105}$ , or Rs 158 5s 3p
- 8 Gain on buying and selling Rs 100 stock = Rs  $(93\frac{1}{4} - 91\frac{1}{4}) = Rs 2$   
 1 quarter's dividend on Rs 100 stock = Rs  $\frac{5}{8}$ ,  
 total gain on Rs 100 stock = Rs  $2\frac{5}{8}$ ,  
 hence on outlay of Rs  $91\frac{1}{4}$ , gain is Rs  $2\frac{5}{8}$ ,  
 " Rs 4380, " Rs  $2\frac{5}{8} \times \frac{4380}{91\frac{1}{4}}$ , or Rs 126

- 9 Rs 100 stock costs Rs  $96\frac{1}{4}$  cash ,  
 $\text{req}^d \text{ income} = \text{Rs } 2\frac{3}{4} \times \frac{4200}{96\frac{1}{4}}$ , or Rs 120
- 10 Rs 100 stock costs Rs 104 cash , Rs  $3\frac{1}{4}$  income costs Rs 104 cash ,  
 hence  $\text{req}^d \text{ sum} = \text{Rs } 400 \times \frac{104}{3\frac{1}{4}}$ , or Rs 12800
- 11 £100 stock costs £103 $\frac{1}{4}$  cash , £4 income costs £103 $\frac{1}{4}$  cash ,  
 hence  $\text{req}^d \text{ sum} = \text{£}275 \times \frac{103\frac{1}{4}}{\frac{1}{4}} = \text{£}\frac{226875}{32}$ , or £7090
- 12 £100 stock costs £103 $\frac{1}{2}$  cash ,  
 $\text{£}3 \times \frac{2\frac{2}{4} \frac{0}{0}}$  is net income from £103 $\frac{1}{2}$  cash ,  
 hence  $\text{req}^d \text{ sum} = \text{£}500 \times \frac{103\frac{1}{2}}{3 \times \frac{2\frac{2}{4} \frac{0}{0}}} = \text{£}\frac{2070000}{113}$ , or £18319
- 13 Let £ $x$  be  $\text{req}^d$  price, then  $\text{£}(x + \frac{1}{4})$  cash produces £4 $\frac{3}{4}$  income ,  
 hence £100 cash produces  $\text{£}100 \times \frac{4\frac{3}{4}}{x + \frac{1}{4}}$  income ,  $\text{£}100 \times \frac{4\frac{3}{4}}{x + \frac{1}{4}} = 5$ ,  
 or  $100 \times 4\frac{3}{4} = 5(x + \frac{1}{4})$ , hence  $x + \frac{1}{4} = 95$ , and  $x = 94\frac{3}{4}$
- 14 Rs 100 stock can be bought for Rs  $(100 \times \frac{8.01 \frac{0}{0} \frac{7}{10}}{100}) + \text{brokerage}$ ,  
 i.e. Rs  $92\frac{1}{2} + \text{brokerage}$ , or Rs  $92\frac{5}{8}$
- 15 Income from first stock = Rs  $3 \times 60 = \text{Rs } 180$   
 Sale of first stock realises Rs  $60 \times 91$ , or Rs 5460 cash  
 Hence income from second stock = Rs  $4 \times \frac{5.1 \frac{0}{0} \frac{0}{0}}{100} = \text{Rs } 168$ ,  
 $\text{req}^d \text{ loss} = \text{Rs } (180 - 168)$ , or Rs 12
- 16 Income from first stock = Rs  $3\frac{1}{2} \times 55 = \text{Rs } 192\frac{1}{2}$   
 Sale of first stock realises Rs  $91\frac{1}{2} \times 55$  cash  
 Hence income from second stock = Rs  $4 \times \frac{91\frac{1}{2} \times 55}{101\frac{1}{4}} = \text{Rs } 198$ ,  
 $\text{req}^d \text{ change} = \text{Rs } (198 - 192\frac{1}{2})$ , i.e. Rs  $5\frac{1}{2}$  gain
- 17 Income from first stock = Rs  $5 \times \frac{1.4 \frac{0}{0} \frac{0}{0}}{100} = \text{Rs } 620$   
 Sale of first stock realises Rs  $125 \times \frac{1.4 \frac{0}{0} \frac{0}{0}}{100}$ , or Rs 15500  
 Hence income from second stock = Rs  $3 \times \frac{1.5 \frac{0}{0} \frac{0}{0}}{100} = \text{Rs } 500$ ,  
 $\text{req}^d \text{ change is Rs } (620 - 500)$ , i.e. Rs 120 loss
- 18 £1365 cash buys  $\text{£}100 \times \frac{1.7 \frac{0}{0} \frac{5}{100}}{100}$  stock, or £1500 stock  
 £1000 of the stock realises  $\text{£}93\frac{1}{2} \times \frac{1.0 \frac{0}{0} \frac{0}{0}}{100}$  cash, or £935 cash  
 The remaining £500 stock realises  $\text{£}85 \times \frac{1.0 \frac{0}{0} \frac{0}{0}}{100}$  cash, or £425 cash ,  
 $\text{req}^d \text{ loss} = \text{£}(1365 - 935 - 425)$ , or £5



- 19 £4340 cash buys  $£100 \times \frac{4340}{108\frac{1}{2}}$  stock, or £4000 stock ,  
     my first income is  $£3\frac{1}{2} \times 40$ , or £140  
 The £4000 stock realises  $£110 \times 40$ , or £4400 cash  
 £4400 cash then buys an income of  $£5\frac{1}{2} \times \frac{4400}{110}$ , or £220 ,  
     req<sup>d</sup> increase =  $£(220 - 140) = £80$
- 20 £26180 cash buys  $£100 \times \frac{26180}{93\frac{1}{2}}$ , or £28000 stock.  
     his first income is  $£3 \times 280$ , or £840  
 £14000 of this stock realises  $£92\frac{1}{4} \times 140$ , or £12915 cash, which  
     purchases in the 4 p c's an income of  $£4 \times \frac{12915}{100}$ , or £516 6s  
 The remaining £14000 stock brings in  $£\frac{1}{2}$  of 840, or £420 income ,  
     req<sup>d</sup> difference =  $£(516 6s + 420 - 840)$ , or £112 6s increase
- 21 £500 R<sup>r</sup> Stock at 95 costs £475 ,  
     brokerage at  $\frac{1}{2} \%$  on £475 = £2 7s 6d
- 22 £500 R<sup>r</sup> Stock at 128 $\frac{1}{2}$  costs £642 10s ,  
     brokerage at  $\frac{1}{2} \%$  on £642 10s = £3 4s 3d  
 £500 N T D Stock at 111 $\frac{3}{4}$  costs £558 15s ,  
     brokerage at  $\frac{1}{2} \%$  on £558 15s = £2 15s 11d
- 23 £800 Consols at 91 costs £728 , brokerage at  $\frac{1}{8} \%$  on £800 = £1  
 £650 R<sup>r</sup> Stock at 110 $\frac{1}{4}$  realises £719 17s 6d ,  
     brokerage at  $\frac{1}{2} \%$  on £719 17s 6d = £3 12s
- 24 £2375 R<sup>r</sup> Stock at 125 realises £2968 15s ,  
     brokerage at  $\frac{1}{2} \%$  on £2968 15s = £14 16s 11d  
 £4000 India Stock at 97 $\frac{1}{2}$  costs £3900 ,  
     brokerage at  $\frac{1}{2} \%$  on £4000 = £5
- 25 £3600 Consols at 89 $\frac{3}{8}$  realises £3217 10s ,  
     brokerage at  $\frac{1}{8} \%$  on £3600 = £4 10s  
 £2700 R<sup>r</sup> Stock at 119 $\frac{1}{2}$  costs £3226 10s ,  
     brokerage at  $\frac{1}{2} \%$  on £3226 10s = £16 2s 8d
- 26 £400 N T Stock at 103 $\frac{3}{4}$  costs £415 ,  
     brokerage at  $\frac{1}{2} \%$  on £415 = £2 1s 6d

- 27 (i) £300 India Stock at 93 costs £279,  
brokerage at  $\frac{1}{8}\%$  on £300 = 7s 6d  
(ii) £400 Transvaal Stock at 95 $\frac{1}{2}$  costs £382,  
brokerage at  $\frac{1}{4}\%$  on £400 = £1  
(iii) £500 Mexican R<sup>r</sup> Stock at 95 costs £475,  
brokerage at  $\frac{1}{2}\%$  on £475 = £2 7s 6d

EXAMPLES XIX d Page 396

- 1 Rs  $38\frac{3}{4} \times 500$
- 2 Rs  $(200 - 1\frac{1}{16}) \times 60$
- 3 Cost of £1 shares =  $(£2 \text{ } 17s \text{ } 6d + 4\frac{1}{2}d) \times 44 = £127 \text{ } 6s \text{ } 6d$   
Cost of £5 shares =  $(£6 \text{ } 6s \text{ } 3d + 9d) \times 80 = £508$
- 4 Rs  $(15 + 2\frac{1}{2}) \times 50$
- 5  $(£2 + £\frac{3}{4} - 9d) \times 250$
- 6 Rs  $355 - (\text{Rs } 15 + \text{Rs } 2\frac{1}{2} + 4a)$
- 7 (i) Rs  $4\frac{1}{2} \times 35$                   (ii) Rs  $6\frac{1}{2} \times \frac{600}{100} \times 47$   
(iii)  $(\text{Rs } 7\frac{1}{2} \times \frac{150}{100} + 8a) \times 75$
- 8 Req<sup>d</sup> dividend =  $\text{Rs } 100 \times 20 \times \frac{3}{100} \times \frac{1}{13} = \text{Rs } 58 \text{ } 7a$
- 9 (i) On £5 the dividend is  $\frac{10}{100}$  of £3, or  $£\frac{3}{10}$ ,  
req<sup>d</sup> percentage =  $\frac{100}{5} \times \frac{3}{10}$ , or £6  
(ii) On £13½ the dividend is  $\frac{8}{100}$  of £9,  
req<sup>d</sup> percentage =  $\frac{100}{13\frac{1}{2}} \times \frac{8}{100}$  of £9, or £5 6s 8d  
(iii) On £36 the dividend is  $17s \times 2$ ,  
req<sup>d</sup> percentage =  $17s \times 2 \times \frac{100}{360}$ , or £4 14s 5d
- 10 (i) Req<sup>d</sup> no of shares =  $\frac{£375}{£7\frac{1}{2}}$                   (ii) Req<sup>d</sup> cost price =  $£7\frac{1}{2} \times 20$   
(iii) Req<sup>d</sup> dividend =  $£5\frac{1}{2} \times \frac{5 \times 15}{100}$
- 11 (i) Price (with brokerage) per share =  $£10\frac{3}{4} + 1s \text{ } 3d = £10\frac{1}{8}$ ,  
req<sup>d</sup> no of shares =  $£605\frac{1}{2} - 10\frac{1}{8}$   
(ii) Req<sup>d</sup> cost =  $£10\frac{1}{8} \times 36$                   (iii) Req<sup>d</sup> dividend =  $\frac{8\frac{3}{4}}{100}$  of £6 × 50

- 12 On £100 of first stock he realises £96½. If £ $r$  be the price of each share, with £96½ cash, he buys  $\frac{96\frac{1}{2}}{r}$  shares. His income is now £4 ×  $\frac{96\frac{1}{2}}{r}$ , and this by the question is £ $\frac{105}{100} \times 2\frac{3}{4}$

$$\frac{4 \times 96\frac{1}{2}}{r} = \frac{105}{100} \times 2\frac{3}{4}, \text{ and } r = £133\frac{1}{4}$$

### EXAMPLES XIX e Page 397

- I buy £5520 ×  $\frac{100}{90}$ , or £6000 Consols,  
£6000 Consols must sell for £5730 cash,  
or £100       "       "       £95½ cash
- Req<sup>d</sup> income = £2½ ×  $\frac{1200}{95\frac{1}{2}} = £\frac{13200}{383} = £34.465$
- Income on investing Rs 100 in 1<sup>st</sup> stock = Rs  $4\frac{1}{2} \times \frac{100}{100} = \text{Rs } 2.96053$ ,  
"       "       Rs 100   "   2<sup>nd</sup>   "   = Rs  $2\frac{1}{2} \times \frac{100}{98} = \text{Rs } 2.94118$ ,  
the first is more profitable  
For second part of question, we have  
diff for Rs 100 invested = Re 0.01935,  
req<sup>d</sup> diff = Re 0.01935 × 32.30 = Re 0.625
- Let Rs  $x$  be req<sup>d</sup> price, then income = Rs  $2\frac{3}{4} \times \frac{6200}{x + \frac{1}{8}}$  (allowing for brokerage),  
 $2\frac{3}{4} \times \frac{6200}{x + \frac{1}{8}} = 176$ , or  $x + \frac{1}{8} = \frac{2\frac{3}{4} \times 6200}{176}$ , whence  $x = 96\frac{1}{4}$
- Let £ $x$  be the req<sup>d</sup> price, then income from £100 cash  
is £7 ×  $\frac{100}{x + \frac{1}{4}}$  (allowing for brokerage),  
 $7 \times \frac{100}{x + \frac{1}{4}} = 4$ , whence  $x + \frac{1}{4} = \frac{700}{4}$ , and  $x = 174\frac{3}{4}$
- Income = Rs  $3\frac{1}{2} \times \frac{100}{90}$   
For second part of question  
on Rs 90 outlay, the loss is Rs (90 - 87), or Rs 3,  
on Rs 7500       "       "       Rs  $3 \times \frac{100}{90}$
- Let Rs  $x$  be req<sup>d</sup> price. Then Rs 115 cash buys Rs  $4\frac{1}{2} \times \frac{115}{x}$  income  
 $\frac{4\frac{1}{2} \times 115}{x} = 5$ , and  $x = 103\frac{1}{2}$

The half-yearly dividend =  $\frac{1}{2}$  of Rs  $5 \times \frac{340}{100} = \text{Rs } 51$

- 8 Let £42 be his capital    Income from 1<sup>st</sup> stock =  $£2\frac{1}{2} \times \frac{r}{90}$ ,  
     income from 2<sup>nd</sup> stock =  $£3\frac{1}{2} \times \frac{32}{105}$   
     total income =  $£\left(\frac{2\frac{1}{2}}{90} + \frac{3\frac{1}{2} \times 32}{105}\right) = £\frac{r}{8}$   
     Now  $\frac{r}{8} = \frac{1}{32}$  of 42, i.e. his income =  $\frac{1}{32}$  of his capital,  
     or  $3\frac{1}{8}\%$  of his capital
- 9 Req<sup>d</sup> cost =  $£59\frac{1}{2} \times 225 = £(60 - \frac{1}{2}) \times 225 = £(135 - 0.281)$   
     = £134.719  
     Brokerage =  $\frac{1}{100}$  of £134.719 = £0.674      total cost = £135.393
- 10 Total sum due to broker =  $£99\frac{1}{2} 19s + £2 11s = £997 10s$   
     req<sup>d</sup> amt of stock =  $£100 \times \frac{997\frac{1}{2}}{99\frac{1}{2}} = £1000$
- 11 Let Rs  $x$  be the req<sup>d</sup> sum    Income is Rs  $6 \times \frac{2}{141} \times \frac{188}{192}$ ,  
     and this is Rs 1200  
 $6 \times \frac{2}{141} \times \frac{188}{192} = 1200$ , and  $x = \frac{1200 \times 141 \times 192}{6 \times 188} = \text{Rs } 28800$
- 12 If £ $x$  be the amt of 3 p.c. stock, income is  $£3 \times \frac{2}{100} \times \frac{230}{240}$ ,  
     and this = £142 $\frac{1}{2}$   
     Hence  $x = £15400$  This realises  $£78\frac{1}{2} \times 154$ , which is invested in  
     4 p.c. stock    Allowing for brokerage and income tax, it  
     yields an income of  $£4 \times \frac{78\frac{1}{2} \times 154}{102\frac{1}{2}} \times \frac{230}{240}$ , or £453.549, or  
     £453 11s      the req<sup>d</sup> change is £10 16s gain
- 13 Income from the 3 p.c's =  $\text{Rs } 3 \times \frac{5.0.0}{100} \times \frac{1.8.8}{100} = \text{Rs } 146 14a$   
     "      "       $3\frac{1}{2}\%$  " =  $\text{Rs } 3\frac{1}{2} \times \frac{1.8.8}{100} = \text{Rs } 166 4a$   
     the second was the more advantageous by Rs 9 6a
- 14 £4376 G W R stock at 124 costs £5426 4s 10d  
     Brokerage at  $\frac{1}{2}\%$  on £5426 4s 10d = £27 2s 8d
- 15 Suppose he invests Rs  $x$   
     Income from 3 p.c's =  $3 \times \frac{1}{2} \times \frac{r}{97\frac{1}{2}}$ , income from 4 p.c's =  $4 \times \frac{1}{2} \times \frac{r}{125}$ ,  
 $\frac{3}{2 \times 97\frac{1}{2}} + \frac{4}{2 \times 125} = 510$ , and  $x = 16250$

- 16 Let Rs  $x$  be invested in each stock

Income from 6 p c's =  $6 \times \frac{x}{126}$ , income from 9 p c's =  $9 \times \frac{x}{210}$ ,

$$\frac{6x}{126} - \frac{9x}{210} = 225, \text{ and } x = 47250$$

- 17 Let £ $x$  be amt necessary, £ $\frac{91}{10}$  remain after paying legacy duty  
income from 4 p c's (allowing for income tax)

$$= 4\frac{1}{2} \times \frac{91}{10 \times 103\frac{1}{2}} \times \frac{230}{210},$$

$$4\frac{1}{2} \times \frac{9x}{10 \times 103\frac{1}{2}} \times \frac{230}{210} = 60, \text{ and } x = 1600$$

- 18 Amt of 3 p c. stock =  $100 \times \frac{100}{10} = \text{£}10000$

£5000 stock realises £92  $\times$  50, or £1600 If £ $x$  be req<sup>d</sup> percentage,  
then £4000 brings in an income of £ $x \times \frac{4000}{100}$ , or £ $40x$ ,

Since the remaining £5000 stock brings in an income of £150,  
we have  $150 + 40x = 300 + 50$ , and  $x = 2\%$

- 19 Let  $x$  shillings be the rate per £

From first stock income =  $\text{£}2\frac{1}{2} \times \frac{4560}{100} \times \frac{20-x}{20}$

£4560 stock realises £85  $\times$  45 6,

$$\text{income from second stock} = 1 \times \frac{85 \times 156}{114} \times \frac{20-x}{20}$$

$$\frac{4 \times 85 \times 156}{114} \times \frac{20-x}{20} - \frac{2\frac{1}{2} \times 4560}{100} \times \frac{20-x}{20} = 209,$$

$$\text{or } \frac{20-x}{20} \left( \frac{4 \times 85 \times 156}{114} - \frac{2\frac{1}{2} \times 4560}{100} \right) = 209,$$

$$\text{or } \frac{20-x}{20} (136 - 114) = 209,$$

$$(20-x) \times 22 = 118 \text{ and } x = 1$$

*Alter* Income from 2½ p c's =  $\text{£}2\frac{1}{2} \times 4560 = \text{£}114$  (without tax)

£4560 stock realises £85  $\times$  15 6, or £1376

This brings in an income of £4  $\times \frac{1376}{100}$ , or £136 (without tax),

difference of incomes (without tax) = £22,

hence the tax on £22 is (£22 - £20 18s), or 22s,

and tax is 1s in the £

- 20 First income = Rs  $8 \times 50$  = Rs 400

The 50 shares realise Rs 9000, and this buys  $\frac{9000}{50}$  shares at Rs 50  
 These  $\frac{9000}{50}$  shares produce an income of Rs  $3\frac{1}{2} \times \frac{50}{100} \times \frac{1800}{7}$ ,  
 or Rs 450 req<sup>d</sup> difference = Rs 50 gain

- 21 In 1<sup>st</sup> case, Rs  $115\frac{1}{2}$  cash produces Rs  $5\frac{1}{2}$  income,

$$\text{i.e. Rs } 100 \quad \text{,,} \quad \text{,,} \quad \text{Rs } 5\frac{1}{2} \times \frac{100}{115\frac{1}{2}}, \text{ or Rs } \frac{1100}{231} \text{ income}$$

In 2<sup>nd</sup> case, Rs  $7\frac{3}{4}$  cash produces Rs  $3\frac{1}{2} \times \frac{100}{100}$  income,

$$\text{i.e. Rs } 100 \quad \text{,,} \quad \text{,,} \quad \text{Rs } 3\frac{1}{2} \times \frac{100}{100} \times \frac{100}{7\frac{3}{4}}, \text{ or Rs } \frac{700}{155} \text{ income}$$

rates of interest are as  $\frac{1100}{231}$   $\frac{700}{155}$ , or as 155 : 147

*Aliter* The incomes from Re 1 invested in each company are proportional to the respective rates of interest

Re 1 invested in first company produces Re  $\frac{5\frac{1}{2}}{115\frac{1}{2}}$ , or Re  $\frac{11}{231}$  income

Re 1 ,, ,, second ,, ,, Re  $\frac{3\frac{1}{2}}{7\frac{3}{4}}$ , or Re  $\frac{7}{115}$  ,,

rates of interest are as  $\frac{11}{231}$   $\frac{7}{115}$ , or as 145 : 147

- 22 The first income (free of tax) = £8 × 38 4 = £307 4s

On £3840 stock he realises £187½ × 38 4

His second income (free of tax) = £4½ ×  $\frac{187\frac{1}{2} \times 38 4}{108}$  = £300

Hence difference (free of tax) = £7 4s

tax on £7 4s = £7 4s - £6 18s = 6s,

whence tax = 10d in the £

- 23 His first income = Rs  $2\frac{3}{4} \times \frac{80000}{100} \times \frac{188}{100}$  = Rs 2291 666

On selling out he realises Rs 80000 ×  $\frac{90}{100}$

His second income = Rs  $2\frac{1}{4} \times \frac{80000}{100} \times \frac{100}{94}$  = Rs 2632 978

difference = Rs 341 312 = Rs 341 5a

- 24 Let £x be spent on the Stock

$$\text{Net income from this} = £3 \times \frac{2}{97\frac{1}{8}} \times \frac{19}{20}$$

$$\frac{3 \times 2 \times 19}{97\frac{1}{8} \times 20} = 52, \text{ and } r = \frac{52 \times 783 \times 20}{8 \times 19 \times 3} = \frac{2610 \times 13}{19} = £1785 7894$$

Adding 1s or £0 05 for Contract Note Stamp,

req<sup>d</sup> answer = £1785 8394 = £1785 16s 9d

- 25 He realised Rs  $(100+150) \times 100$ , or Rs 25000 Each Rs 10 share  
cost Rs  $\frac{83\frac{1}{10}}{100} \times 10$ , he bought  $(25000 - \frac{83\frac{1}{10}}{10})$ , or 3000 shares
- 26 £500 Canada Stock costs £104 $\frac{1}{4}$   $\times 5$ , or £521 $\frac{1}{4}$   
Thus £478 $\frac{3}{4}$  is left to be invested in N Z Stock  
This buys £100  $\times \frac{478\frac{3}{4}}{97\frac{3}{4}}$ , or £489 770, or £489 15s 5d N Z Stock
- 27 £20000 Jamaica Stock costs £101 $\frac{1}{8}$   $\times 200$ , or £20225  
£10000 Cardiff Stock costs £97  $\times 100$ , or £9700  
£20075 remains This buys £100  $\times \frac{200075}{97\frac{1}{4}}$ ,  
or £20642 Bristol Stock  
Net income on £50000 invested  
= £ $(3\frac{1}{2} \times 200 + 3 \times 100 + 3 \times 20642) \times \frac{1}{20}$   
= £ $(700 + 300 + 61926) \times \frac{1}{20}$  = £1619 26  $\times \frac{1}{20}$  = £1538 297  
average income per cent = £ $\frac{1538297}{10000}$  = £3 08
- 28 Rs 40000 stock costs him Rs 65  $\times 400$ , or Rs 26000  
He owes the bank 4 % of  $\frac{400}{100}$  of Rs 26000, or Rs 416  
He receives Rs  $2\frac{1}{4} \times 400$ , or Rs 900 dividend  
By sale of Rs 40000 stock, he realises Rs 70  $\times 400$ , or Rs 28000  
his profit is Rs  $(28000 + 900 - 26000 - 416)$ , or Rs 2484  
Now Rs 2484 is profit on an outlay of Rs 26000,  
giving Rs  $\frac{2484}{2600}$  profit on Rs 100 outlay, or 9 6 %

## MISCELLANEOUS EXAMPLES V Page 401

## A

- 1 22046 lbs = 1 Kg ,      1 lb =  $\frac{1}{22046}$  Kg ,  
1 ton or 2240 lbs =  $\frac{2240}{22046}$  Kg = 1016 Kg
- 2 10 tons 13 cwt 74 lbs =  $(10 + \frac{13}{20} + \frac{74}{160})$  tons =  $10\frac{153}{224}$  tons  
req<sup>d</sup> cost = £ $1\frac{1}{6} \times 10\frac{153}{224}$  = £ $\frac{7}{6} \times (10 + \frac{153}{224})$  = £ $\frac{70}{6} + \frac{7 \times 153}{6 \times 224}$   
= £ $11\frac{2}{3} + £\frac{51}{64}$  = £ $11\frac{2}{3} + 3\frac{3}{4}d \times 51$  = £11 13s 4d + 15s 11 $\frac{3}{4}d$   
= £12 9s 3d to nearest penny

- 3 Let Rs  $x$  be the partner's share of the profits  
Then  $0.24 \times x = 7800$ , and  $x = 32500$ , req<sup>d</sup> fraction  $= \frac{32500}{148750} = \frac{2}{3}$
- 4 £5 17s 6d  $= 117\frac{1}{2}s$ ,  $2s\ 3d = 2\frac{1}{4}s$ ,  
area of room  $= \frac{117\frac{1}{2}}{2\frac{1}{4}} \text{ sq yds} = \frac{470}{9} \text{ sq yds} = 470 \text{ sq ft}$ ,  
and req<sup>d</sup> breadth  $= \frac{470}{23\frac{1}{2}} \text{ ft} = 20 \text{ ft}$
- 5 As in Art 292, Ex 1, req<sup>d</sup> S P  $= \frac{114}{5}$  of Rs 80 = Rs 96
- 6 Let Rs  $x$  be the req<sup>d</sup> amount of stock His income is Rs  $3\frac{3}{4} \times \frac{x}{100}$   
 $\frac{3\frac{3}{4} \times x}{100} = \text{Rs } 84\frac{5}{8}$ , and  $x = \text{Rs } 2250$

## B

- 7 Cost for 20 yds  $= £2\ 11s\ 5d = £2\frac{117}{40}$   
cost for 1 yd  $= \frac{1}{20}$  of  $£2\frac{117}{40}$ ,  
no of yds in perimeter  $= \frac{£339\frac{7}{10}}{\frac{1}{20} \text{ of } £2\frac{117}{40}} = 2640$ ,  
and no of yds in side  $= 660$ , req<sup>d</sup> acreage  $= \frac{660 \times 660}{4840} = 90$
- 8 Total C P  $= 240a$ , total S P  $= 250a$   
(i) actual gain  $= 10a$ , (ii) percentage gain  $= \frac{10}{240} \times 100 = 4\frac{1}{6}$
- 9 112 lbs of coffee cost 168s, 52 lbs of coffee, or 12 lbs of tea  
cost  $168s \times \frac{52}{112}$ ,  
22 lbs of tea cost, or 572 lbs of sugar cost  $\frac{168s \times 52 \times 22}{112 \times 12}$ , or 143s  
req<sup>d</sup> weight of a cask of sugar costing 42s  $= 572 \text{ lbs} \times \frac{42}{143} = 168 \text{ lbs}$
- 10 Let the req<sup>d</sup> area  $= x \text{ sq cm}$ , then  $1800 \times x = 350 \times 0.02$ ,  
whence  $x = \frac{7}{18000}$  Now  $\frac{7}{18000} \text{ sq cm} = \frac{7}{18} \text{ sq mm} = 0.39 \text{ sq mm}$
- 11 Convenient scales are 0 1" to 1d horizontally, and 0 1" to 1 lb vertically Since 1 cwt  $= 112 \text{ lbs}$ , and £1 6s 8d  $= 320d$ , as in Art 277, Example,  $y = \frac{112}{320}$ , or  $y = \frac{7}{20}$  Join the origin to the point (20, 7), and the graph is obtained The required values may then be read off



12 From Art 313,

$$\text{the amount in 3 yrs} = \text{Rs } 1756 \left(1 + \frac{2\frac{1}{4}}{100}\right)^3 = \text{Rs } 1756 \times 1.0225^3$$

Rs	
1756	
1 022	5
<hr/>	
1756	
35 12	
3 512	
878	
<hr/>	
1795 510	

Rs	
1795 510	
1 0225	
<hr/>	
1795 51	
35 910	2
3 591	02
897	76
<hr/>	
1835 908	98

Rs	
1835 908	98
1 0225	
<hr/>	
1835 908	98
36 718	18
3 671	82
917	95
<hr/>	
1877 216	93

$$\text{req'd CI} = \text{Rs } 1877 \text{ 3 a 6 p} - \text{Rs } 1756 = \text{Rs } 121 \text{ 3 a 6 p}$$

C

$$13 \text{ Req'd fraction} = \frac{\text{Rs } 20 \text{ 11 a 6 p}}{\text{Rs } 121 \text{ 14 a}} = \frac{\text{Rs } 20\frac{3}{4}}{\text{Rs } 121\frac{7}{8}} = \frac{17}{100} = 0.17$$

$$14 \quad \begin{array}{r} 9 \text{ 07.07 (3 0117596} \\ 9 \quad \quad \quad = 3 \text{ 011760} \end{array}$$

601	707
6021	10600
60227	457900
60234	35911
	5749
	373

$$15 \text{ Interest} = 1\frac{1}{2} \text{ a} \times 52 \text{ on } 240 \text{ a} \\ = 78 \text{ on } 240 \\ = \frac{78}{240} \times 100 \text{ on } 100 \\ = 32\frac{1}{2} \%$$

$$16 \text{ Req'd fraction} = \frac{16\frac{1}{2} \times 12\frac{1}{2} \text{ sq ft}}{21\frac{2}{3} \times 18\frac{1}{4} \text{ sq ft}} = \frac{1}{2}$$

17 (8+10+12), or 30 shovelfuls are thrown in per 1 min ,  
 the first man would take 90 min  $\times \frac{3}{9}$ , the second, 90 min  $\times \frac{3}{10}$ ,  
 the third, 90 min  $\times \frac{3}{12}$ , or 337 $\frac{1}{2}$  min , 270 min , and 225 min  
 respectively , i.e. 5 $\frac{5}{8}$  hours, 4 $\frac{1}{2}$  hours, and 3 $\frac{1}{4}$  hours respectively

$$18 \text{ 2nd SP} = \text{1st SP} \times \frac{\text{Rs } 6 \text{ 3 a}}{\text{Rs } 5 \text{ 1 a}} = \text{1st SP} \times \frac{99}{81} = \frac{11}{9} \text{ of SP}$$

Also 1st SP = 135 % of CP , 2nd SP = 1 $\frac{1}{9}$  of 135 % of CP ,  
 or 2nd SP = 165 % of CP , hence req'd gain = 65 %

(See Art 292)

## D

19 Volume of each brick =  $\frac{9 \times 4\frac{1}{2} \times 3}{1728}$  cu ft ,

weight of each brick =  $\frac{9 \times 4\frac{1}{2} \times 3}{1728} \times 145$  lbs

Hence, if  $x$  be req<sup>d</sup> no of bricks,  $\frac{9 \times 4\frac{1}{2} \times 3}{1728} \times 145 \times x = 5 \times 2240$  ,

whence  $x = \frac{5 \times 2240 \times 1728}{9 \times 4\frac{1}{2} \times 3 \times 145} = \frac{2240 \times 128}{9 \times 29} = 1100$  (nearly)

20 In walking  $4\frac{1}{2}$  miles,  $B$  can give  $A$   $\frac{1}{2}$  mile start

„ 1 mile,  $B$  „  $A$   $\frac{1}{3}$  „ „

or „ 1 yd,  $B$  „  $A$   $\frac{1}{3}$  yd „

„ 1350 yds,  $B$  „  $A$   $\frac{1}{3}$  of 1350 yds, or 450 yds start

21 Let  $x$  shillings be req<sup>d</sup> C P , the marked price =  $\frac{130}{100}$  of  $x$  shillings,  
and S P =  $\frac{95}{100}$  of  $\frac{130}{100}$  of  $x$  shillings Also £8 4s 8d = 164 $\frac{2}{3}$ s

$\frac{95 \times 130 \times x}{100 \times 100} = 164\frac{2}{3}$ , and  $x = \frac{494 \times 100^2}{3 \times 95 \times 130} = 133\frac{1}{3}$ , i.e. £6 13s 4d

22 2 cwt 26 lbs = 250 lbs , req<sup>d</sup> cost = 250  $\times$  0 4536  $\times$  4 75 fr  
= 1000  $\times$  0 1134  $\times$  4 75 fr = 113 4  $\times$  4 75 fr = 539 fr (nearly)

## E

23 1<sup>st</sup> dividend = Rs 4  $\times$   $\frac{9000}{1000}$  = Rs 360

Rs 9000 stock realises Rs 112 $\frac{1}{2}$   $\times$   $\frac{9000}{1000}$ , or Rs 10125 ,

2<sup>nd</sup> dividend = Rs 5 $\frac{1}{2}$   $\times$   $\frac{10125}{168\frac{3}{4}}$  = Rs 330, giving Rs 30 loss

24 Let Rs  $x$  be one part, then Rs (1130 -  $x$ ) is the other part

$\frac{7x}{100} + \frac{2(1130 - x)}{100} = \frac{5}{100}$  of 1130, and  $x = 678$  ,

hence we obtain Rs 678 at 7 % , and Rs 452 at 2 %

26 Req<sup>d</sup> ratio =  $\frac{\text{Rs } 269 \frac{8}{9}}{\text{Rs } 500 \frac{8}{9}} = \frac{\text{Rs } 269\frac{1}{2}}{\text{Rs } 500\frac{1}{2}} = \frac{7}{13}$

27

	£38 6
	47
	1544
	270 2
	1814 2
440 yds = $\frac{1}{2}$ mi	9 65
220 yds = $\frac{1}{4}$ mi	4 825
55 yds = $\frac{1}{8}$ of 220 yds	1 206   25
55 yds = $\frac{1}{2}$ of 220 yds	1 206   25
5 yds = $\frac{1}{11}$ of 55 yds	109   66
	1831 197   16

giving £1831 4s

28 Let Rs  $x$  be req<sup>d</sup> price

Then

$$30x + \frac{3x}{4} \times 20 = \frac{135}{100} \text{ of } 5000,$$

$$\text{and } x = 150$$

29.

{ 2 men with 1 boy do  $\frac{1}{6}$  of the work in 1 day,

{ 1 man „ 2 boys „  $\frac{1}{8}$  „ „ 1 „

{ 4 men with 2 boys do  $\frac{2}{6}$  of the work in 1 day,

{ 1 man „ 2 „ „  $\frac{1}{8}$  „ „ 1 „

by subtraction, 3 men do  $\frac{2}{6} - \frac{1}{8}$ , or  $\frac{7}{24}$  of the work in 1 day,

that is, 1 man does  $\frac{7}{24}$  of the work in 1 day

1 boy does  $\frac{1}{8} - \frac{1}{24}$ , or  $\frac{1}{24}$  of the work in 1 day

Since their daily rates of work are proportional to their weekly

rates, the wages of a boy must be Rs  $7 \times \frac{\frac{1}{24}}{\frac{7}{24}} = \text{Rs } 4$

30 Let  $x$  be the original population of each part,

$$\text{then } \frac{80x}{100} + \frac{115x}{100} = 39390,$$

$$195x = 39390 \times 100, \text{ and } x = 20200$$

F

31 30 eggs are bought for 25 a and sold for 36 a,

11 a is gained on an outlay of 25 a,

i.e. 44 a „ „ 100 a, or 44 %

32 His estimate of 30 in is really 32 in, of 1 in is really  $\frac{32}{30}$  in,  
of 1 sq in is really  $\frac{32}{30} \times \frac{32}{30}$  sq in Now 2 ac 3 r = 11 r,

his estimate of 11 r is really  $\frac{32}{30} \times \frac{32}{30} \times 11 \text{ r} = 12516 \text{ r} = 313 \text{ ac}$

33 His first income = Rs  $2\frac{3}{4} \times \frac{10000}{100} = \text{Rs } 275$

The stock realises Rs 10000

If he invests Rs  $x$  in the 4 per cent stock, his income is Rs  $4 \times \frac{x}{140}$

$$4 \times \frac{x}{140} = 275, \text{ and } x = 9625,$$

he retains Rs (10000 - 9625) or Rs 375

34. Time = 73 days Interest =  $\text{£}3\frac{1}{2} \times \frac{389\frac{1}{2}}{100} \times \frac{73}{365}$

$$= \text{£}\frac{7}{2} \times \frac{1168}{3000} \times \frac{1}{8} = \text{£}\frac{917}{3000} = \text{£}2\ 725 = \text{£}2\ 14s\ 6d$$

$$\text{Amount} = \text{£}389\ 6s\ 8d + \text{£}2\ 14s\ 6d = \text{£}392\ 1s\ 2d$$

35 Cutting the 17 in edge from the 18 in edge, the 9 in edge from the 30 in edge, he gets 3 pieces out of each sheet, and requires 12 sheets. Each sheet has an area of  $(18 \times 30)$  sq in, or  $3\frac{3}{4}$  sq ft

Hence 12 sheets cost  $2a \times 3\frac{3}{4} \times 12$ , or 90a. The area of the 36 pieces is  $\frac{17 \times 9}{144}$  sq ft  $\times 36$ , and then value is  $3a \times \frac{17 \times 9}{144} \times 36$ , or  $114\frac{3}{4}a$ . His profit is therefore  $23\frac{3}{4}a$  on 90a outlay, or  $27\frac{1}{2}\%$

36 Let the man's daily wage be  $7x$  annas, the wife's  $4x$  annas, the son's  $3x$  annas, then  $7x \times 2 + 4x \times 3 + 3x \times 4 = 80$ , and  $x = 2a$ , the man's wage is  $14a$ , the woman's  $8a$ , the boy's  $6a$ .

### G

37 103 tons 5 cwt 3 qrs

$$= (103 + \frac{5}{20} + \frac{3}{40 \times 4}) \text{ tons}$$

$$= 103\ 2875 \text{ tons}$$

req<sup>d</sup> price

$$= \text{£}5\ 425 \times 103\ 2875$$

$$= \text{£}560\ 335 = \text{£}560\ 6s\ 8d$$

103 287	5
516 437	5 425
41 315	0
2 065	7
516	4
560 334	6

$$\text{Volume of casting} = \frac{103\ 287 \times 2240}{488} = \frac{103\ 287 \times 2240}{61}$$

$$= \frac{2320000}{61} = 474\ 1 \text{ cu ft}$$

38 Volume of wood =  $(26 \times 19 \times 18 - 25 \times 18 \times 17)$  cu in

$$= 18(26 \times 19 - 25 \times 17) \text{ cu in} = 18 \times 69 \text{ cu in} = \frac{18 \times 69}{12} \text{ cu ft},$$

$$\text{weight of box} = \frac{18 \times 69}{12} \times 40 \text{ lbs} = \frac{117}{4} \text{ lbs} = 28\frac{3}{4} \text{ lbs}$$

- 39 Let each woman do  $2x$  units of work per day,  
 then each man does  $3x$  " "  
 50 men working for 80 days do  $3x \times 50 \times 80$ , or  $12000x$  units,  
 and the cost is  $5\frac{1}{2}s \times 50 \times 80$ , or £1100  
 Again, 20 men and 30 women do  $(3x \times 20 + 2x \times 30)$ ,  
 or  $120x$  units per day,  
 they would do  $12000x$  units in 100 days,  
 and the cost would be  $(5\frac{1}{2}s \times 20 + 2\frac{3}{4}s \times 30) \times 100$ , or £962 10s,  
 the req<sup>d</sup> difference is £1100 - £962 10s, or £137 10s less
- 40 Let £P be req<sup>d</sup> sum, then from Art 313,  $P \times (\frac{105}{100})^3 = 308\frac{1}{2}$ ,  
 $P = 308\frac{7}{10} \times (\frac{100}{105})^3 = \frac{3087}{10} \times (\frac{20}{21})^3 = \frac{890}{3} = £296\frac{2}{3}$
- 41 Cost Price = Rs 38, Selling Price = Rs  $38 \times \frac{105}{100}$   
 If Rs  $x$  be the "marked" price, then Selling Price = Rs  $\frac{95x}{100}$ ,  
 $\frac{95x}{100} = 38 \times \frac{105}{100}$ , and  $x = \text{Rs } 42$
- 42 Suppose he invests Rs  $v$  in the first stock, and Rs  $23100 - v$  in  
 the second His income from the first stock = Rs  $3 \times \frac{x}{81}$ ,  
 from the second = Rs  $4\frac{1}{2} \times \frac{23100 - x}{135}$ ,  
 $\frac{3v}{81} + \frac{4\frac{1}{2}}{135}(23100 - v) = 800$ , or  $\frac{v}{27} + \frac{23100 - x}{30} = 800$ ,  
 $x = 8100$ , giving Rs 8100 at 81, and Rs 15000 at 135

H

43	1 2345	
	1 2345	
	1 2345	
	2469	0
	370	35
	49	38
	6	17
	1 5239	90
	1 2345	
	1 5239	90
	3047	98
	457	20
	60	96
	7	62
	1 8813	66

giving 1 8814

$$\begin{aligned}
 44 \quad \text{Exp}^n &= \frac{(4\frac{1}{3} - 1\frac{1}{3}) \times 12}{(6\frac{2}{3} + 4\frac{2}{3}) \times 12} - \frac{21 - 194}{93 + 87} \\
 &= \frac{52 - 13}{80 + 55} - \frac{016}{080} \\
 &= \frac{39}{135} - \frac{16}{80} \\
 &= \frac{13}{45} - \frac{8}{40} \\
 &= \frac{1}{5}
 \end{aligned}$$

45 100 yds weigh 14 08 lbs,

$$\text{or } \frac{100}{100} \text{ m } \quad \text{,,} \quad \frac{1408}{00022} \text{ gm } ,$$

$$1 \text{ m } \quad \text{,,} \quad \frac{1408 \times 100}{00022 \times 100}, \text{ or } \frac{128 \times 100}{200}, \text{ or } 6976 \text{ gm}$$

46 Let  $x$  gallons of the first, and  $(84-x)$  gallons of the second be taken

Water in first + water in second = water in mixture,

$$\frac{10x}{100} + \frac{3}{100}(84-x) = \frac{5}{100} \text{ of } 84, \text{ and } x=24,$$

req<sup>d</sup> answer is 24 gallons of first, with 60 gallons of second

47 Since 24 hours = 86400 secs, and  $2\frac{1}{4}$  min = 135 secs, the min hand moves over  $(86400 + 135)$ , or 86535 sec spaces in 86400 actual secs,

it indicates 1 sec of time in  $\frac{86400}{86535}$  actual secs,

,, 1 hr ,,  $\frac{86400}{86535}$  ,, hrs

When it shews 5 p m on Wednesday, it has moved over

53 hour-spaces since noon on Monday,

$$\text{correct time} = 53 \times \frac{86400}{86535} = 53 \times \frac{64}{61} \text{ hrs}$$

This reduces to 52 hrs 55 min 2 secs after 12 noon on Monday, giving 4 hrs 55 min 2 secs after 12 noon on Wednesday

48 If profits are Rs  $x$ , since 40 % of Rs  $x$  is shared equally by them, we need only consider the remaining 60 % of Rs  $x$ , or Rs  $\frac{3x}{5}$

This is divided in the ratio of 13500 7500, or 9 5,

their shares of this are  $\frac{9}{14}$  of Rs  $\frac{3x}{5}$ , and  $\frac{5}{14}$  of Rs  $\frac{3x}{5}$  resp<sup>ly</sup>

$$\frac{9}{14} \text{ of } \frac{3x}{5} = \frac{5}{14} \text{ of } \frac{3x}{5} + 450, \text{ and } x = \text{Rs } 2625$$

# I

49 Discount = Rs 9 8a on Rs 190, i e Rs  $9\frac{1}{2}$  on Rs 190, or 5 %

50 Expenses =  $\frac{75}{100}$  of £41647310 =  $\frac{7}{8}$  of £41647310

$$= £6073566 = £6074000 \text{ (to nearest thousand)}$$

For second part of question,

the old rateable value of £1 is now £1  $\times \frac{105}{100}$

The cost to collect £1  $\times \frac{105}{100}$  is now £  $\frac{75}{100} \times \frac{110}{100}$ ,

,, £1 ,, £  $\frac{75}{100} \times \frac{110}{100} \times \frac{100}{100}$ , or £  $1\frac{1}{2}$

This = 36 $\frac{3}{4}$ d, giving 3s 0 $\frac{1}{2}$ d

- 51 216 eggs are bought for  $\frac{1}{12} \times 216$  annas, or 180 a

If  $x$  eggs are broken,

then  $216 - x$  are sold for  $\frac{1}{8} \times (216 - x)$  annas

Since a profit of 20% is made,  $\frac{1}{8} \times (216 - x) = 180 \times \frac{120}{100}$ ,

$$216 - x = \frac{180 \times 120}{100 \times 8}, \text{ and } x = 36$$

- 52 Let £P be the req<sup>d</sup> sum, then from Art 313,

$$P = \frac{£800}{(1.04)^4} = \frac{£800}{1.16986}$$

$$= £683.8 = £684 \text{ to nearest } £$$

$$\begin{array}{r} 683.8 \\ 1.16986 \overline{) 800.000} \\ \underline{9808} \\ 450 \\ 99 \end{array}$$

- 53 Space left uncovered is  $(32 \times 25 - 29\frac{1}{2} \times 22\frac{1}{2})$ , or  $136\frac{1}{2}$  sq ft  
Since  $(32 \times 25)$ , or 800 sq ft cost £28 to cover,

$$\text{Amount saved} = £28 \times \frac{136\frac{1}{2}}{800} = £4\frac{13}{20} = £4 \text{ } 15\text{ } 4\frac{1}{2}d$$

- 54 A, B and C together fill  $\frac{1}{8}$  of a cist in 1 min

A and C "  $\frac{1}{12}$  " 1 "

A and B "  $\frac{1}{13\frac{1}{2}}$  " 1 "

B fills  $(\frac{1}{8} - \frac{1}{12})$ , or  $\frac{1}{24}$  of a cist in 1 min

C "  $(\frac{1}{8} - \frac{1}{13\frac{1}{2}})$ , or  $\frac{1}{20}$  " 1 "

A "  $(\frac{1}{12} - \frac{1}{20})$ , or  $\frac{1}{30}$  " 1 "

Rs 7 8a should be divided in the proportion of  $\frac{1}{30}$   $\frac{1}{24}$   $\frac{1}{20}$ ,  
or as 4 5 6, A getting Rs 2, B Rs 2 8a, C Rs 3

J

- 55 (i)  $13\text{ } 3\frac{1}{2}d \times 480 = (13\text{ } 4d - \frac{1}{2}d) \times 480 = (£\frac{2}{3} - \frac{1}{4 \times 20}) \times 480$   
 $= £320 - £1 = £319$

$$(ii) 24 \times 25 \times 26 = 6 \times 4 \times 25 \times 26 = 6 \times 100 \times 26 = 15600$$

$$(iii) \text{Income} = \frac{2\frac{3}{10}}{110} \text{ of Rs } 12000 = \frac{1}{40} \text{ of Rs } 12000 = \text{Rs } 300$$

$$(iv) \text{The incomes are as } \frac{7\frac{1}{2}}{250} \quad \frac{11\frac{1}{4}}{400} \quad \text{Now } \frac{7\frac{1}{2}}{250} = \frac{15}{500} = \frac{3}{100} = \frac{12}{400}$$

showing the first investment gives the larger income

- 56 7 men and 4 boys do  $\frac{1}{2}$  the work of (14 men and 8 boys),  
or (14 men and 3 men), or 17 men in the same time,

7 men and 4 boys will take 4 times as long  
to do a piece of work twice as great,

$$\text{req'd time} = 17 \text{ days} \times \frac{3 \text{ men}}{17 \text{ men}} \times 4 = 12 \text{ days}$$

- 57 If £P be the Principal, the Interest = £  $\frac{4P}{100}$ ,

$$\text{req'd time} = \text{£P} - \text{£} \frac{4P}{100} \text{ yrs} = 25 \text{ years}$$

- 58 Let  $d$  feet be the req'd depth, 100 gals =  $\frac{100}{6 \cdot 25}$  cu ft,

$$9 \times d = \frac{100}{6 \cdot 25}, \text{ or } d = \frac{100}{60 \cdot 07} \text{ ft} = \frac{100}{60 \cdot 07} \text{ in} = 1 \text{ ft } 9 \text{ in (nearly)}$$

- 59 Let £ $x$  be price of 3%, and let £2A be invested

The income from 1<sup>st</sup> stock is £4  $\times \frac{A}{90}$ , from 2<sup>nd</sup> stock is £3  $\times \frac{A}{x}$ ,

$$\text{and the total income is } \text{£}5 \times \frac{2A}{100}, \quad \frac{4A}{90} + \frac{3A}{x} = \frac{10A}{100}$$

Divide by A, and

$$\frac{4}{90} + \frac{3}{x} = \frac{10}{100}, \text{ or } \frac{3}{x} = \frac{10}{100} - \frac{4}{90}, \text{ or } \frac{3}{x} = \frac{5}{90}, \text{ and } x = 54$$

- 60 B runs 75 mi per hr, or 1 mi in  $\frac{60}{75}$ , or 8 min,

in 6 min B runs  $\frac{3}{4}$  mi, or 1320 yds

In 6 min A runs 1 mi  $\times \frac{6}{72}$ , or 1408 yds,

$$\text{req'd distance} = (1408 - 1320), \text{ or } 88 \text{ yds}$$

### K.

- 61 Let  $x$  be req'd number, in a year he sends 12 $x$  rupees, or  
£12 $x \times \frac{1}{240}$ ,

$$12x \times \frac{1}{240} = 100, \text{ whence } x = 125$$

- 62 Req'd weight = 7 207 gm  $\times 23 \times 23 \times 1000$   
= 5 29  $\times 7$  207 Kg  
= 38 Kg

$$\begin{array}{r|l} 7 & 207 \\ \hline & 529 \\ 36 & 0 \\ 1 & 4 \\ & 6 \\ \hline 38 & 0 \end{array}$$



63 The two kinds of eggs are worth 135 at  $(2s\ 3d + 1s\ 9d)$ ,  
or 135 at  $4s$  a score  $= 4s \times 6\frac{3}{4} = \pounds 1\ 7s$

220 lbs of lard at  $8\frac{1}{2}d = 110$  lbs of lard at  $1s\ 4\frac{1}{2}d$  Combining  
this with the butter, we get 110 lbs at  $(1s\ 4\frac{1}{2}d + 1s\ 7\frac{1}{2}d)$ ,  
or 110 lbs at  $3s$  a lb  $= \pounds 16\ 10s$

106 cakes at  $11s\ 3d$  a doz  $= 318$  cakes at  $3s\ 9d$  a doz

Combining these with the biscuits,

we get 318 articles at  $(3s\ 9d + 4s\ 3d)$  a doz,

or 318 at  $8s$  a doz  $= 318 \times \frac{8}{12}s = \frac{2}{3}\ 318s = \pounds 10\ 12s$

Total  $= \pounds 1\ 7s + \pounds 16\ 10s + \pounds 10\ 12s = \pounds 28\ 9s$

Discount  $= 10d \times 28\frac{9}{10} = (280 + 4\frac{1}{2})d = \pounds 1\ 3s\ 4\frac{1}{2}d$ ,

giving  $\pounds 28\ 9s - \pounds 1\ 3s\ 4\frac{1}{2}d$ , or  $\pounds 27\ 5s\ 3\frac{1}{2}d$

for the discounted bill

64 Suppose I hold Rs  $x$  in the 3 per cents and Rs  $(7200 - x)$  in the  
5 per cents, the incomes from these are Rs  $3 \times \frac{x}{100}$ ,

and Rs  $5 \times \frac{7200 - x}{100}$  Since the total income  $=$  Rs  $3\frac{1}{2} \times \frac{7200}{100}$ ,

$\frac{3x}{100} + \frac{5(7200 - x)}{100} = 3\frac{1}{2} \times \frac{7200}{100}$ , or  $3x + 5(7200 - x) = 3\frac{1}{2} \times 7200$ ,

and  $x = 6000$

I hold Rs 6000 at 3%, and Rs 1200 at 5%

65 In the mixture of 9 gallons,

alcohol  $= (\frac{2}{100}$  of  $3 + \frac{5}{100}$  of  $5)$  gals  $= 4\frac{1}{2}$  gals, giving 50%

66 Let his wages be  $x$  rupees per week His weekly savings  $= x \times \frac{12\frac{1}{2}}{100}$

rupees, and his weekly expenses  $= x \times \frac{87\frac{1}{2}}{100}$  rupees,

his increased weekly expenses  $= x \times \frac{87\frac{1}{2}}{100} \times \frac{110}{100}$  rupees

Since his increased weekly wage is  $(x + 1\frac{1}{2})$  rupees,

he saves  $(x + 1\frac{1}{2} - x \times \frac{87\frac{1}{2}}{100} \times \frac{110}{100})$  rupees per week

$52(x + 1\frac{1}{2} - \frac{87\frac{1}{2}x}{100} \times \frac{110}{100}) = 52x \times \frac{12\frac{1}{2}}{100} - 13$

Divide by 52, and  $x + 1\frac{1}{2} - \frac{87\frac{1}{2}x}{100} \times \frac{110}{100} = \frac{12\frac{1}{2}x}{100} - \frac{1}{4}$ ,

giving  $x + \frac{3}{2} - \frac{77x}{80} = \frac{x}{8} - \frac{1}{4}$  Multiply by 80, and we get

$80x + 120 - 77x = 10x - 20$ , whence  $x = 20$

## L

$$67 \quad (i) \text{Exp}^n = \frac{15\frac{3}{4} - 8\frac{5}{8}}{4\frac{3}{4} + 2\frac{1}{8}} = 1 \quad (ii) \text{Exp}^n = \frac{\frac{3}{70} + \frac{1}{33}}{9 - \frac{3}{70} \times \frac{1}{33}} = \frac{\frac{169}{70 \times 33}}{\frac{6929}{770}} = \frac{1}{123}$$

$$68 \quad (i) \text{Req}^d \text{ speed} = 2 \text{ mi} \times \frac{60 \text{ min}}{40 \text{ min}} = 3 \text{ mi per hour}$$

$$(ii) \text{Req}^d \text{ speed} = 3 \text{ mi in 1 hr} = (3 \times 1760 \times 3) \text{ ft in } (60 \times 60) \text{ secs} \\ = \frac{3 \times 1760 \times 3}{60 \times 60} \text{ ft, or } 44 \text{ in 1 sec}$$

After going half a mile, 30 min remain in which he must walk  $2\frac{1}{2}$  mi he must walk 5 mi in 60 min, or 1 hour

$$69 \quad \text{Let the watch cost Rs } x, \text{ he sells it for Rs } x \times \frac{109\frac{1}{2}}{100}, \text{ but this is the} \\ \text{same as Rs } 115 \times \frac{95}{100} \quad x \times \frac{109\frac{1}{2}}{100} = 115 \times \frac{95}{100}, \text{ and } x = 100$$

$$70 \quad \text{Let } P_{10} \text{ be the population 10 years ago, } P \text{ the population to day,} \\ \text{then } P = P_{10} \times \frac{110}{100} \quad \text{Also } P = 22000000 \times \frac{105}{100} = 23100000, \\ P_{10} \times \frac{110}{100} = 23100000 \text{ and } P_{10} = 23100000 \times \frac{100}{110} = 21000000 \\ \text{Req}^d \text{ increase} = P - P_{10} = 23100000 - 21000000 = 2100000$$

$$71 \quad 4 \text{ qts weigh 10 lbs, } 1 \text{ qt weighs } 2\frac{1}{2} \text{ lbs, and 1 ht weighs 1 Kg,} \\ \text{or } 2\frac{205}{10} \text{ lbs}$$

$$\frac{1 \text{ qt}}{1 \text{ ht}} = \frac{2\frac{1}{2} \text{ lbs}}{2\frac{205}{10} \text{ lbs}} = \frac{2500}{2205} = \frac{500}{441} = \frac{10}{88} \text{ (nearly),}$$

$$\text{and } 88 \text{ qts} = 10 \text{ ht}$$

$$\text{If } y \text{ quarts are equivalent to } x \text{ litres, then } \frac{y}{x} = \frac{88}{10} \text{ or } y = \frac{88}{10} x$$

Take 0.5" to 1 litre horizontally, and 1" to 1 quart vertically  
As in Art 277, Example, if we join the origin to the point (10, 88), the graph is obtained and the required values may be read off

$$72 \quad \text{Total cost} = 6441000 \text{ fl} = \pounds \frac{6441}{1 - \frac{1}{11}} \times 1000 \\ = \pounds 5318 \times 1000 = \pounds 532000 \text{ (nearly)}$$

$$\text{Since } 110 \text{ Km} = 110 \times \frac{5}{8} \text{ Km,}$$

$$\text{cost per mile} = \pounds 532000 \times \frac{8}{5 \times 110} = \pounds \frac{5320 \times 8}{5 \times 110} \times 1000 \\ = \pounds \frac{4256}{55} \times 1000 = \pounds 77 \times 1000 \\ = \pounds 8000 \text{ (nearly)}$$

$$\begin{array}{r} 5318 \\ 1211 \overline{) 644100} \\ \underline{3860} \\ 227 \\ \underline{106} \end{array}$$

M

$$\begin{aligned}
 73 \quad \text{Reqd int} &= \text{Rs } 8740 \times \frac{220}{365} \times \frac{45}{100} \\
 &= \text{Rs } 8740 \times \frac{440}{730} \times \frac{45}{100} \\
 &= \text{Rs } \frac{173052}{73} \\
 &= \text{Rs } 237057 = \text{Rs } 237 \text{ 1a}
 \end{aligned}$$

$$\begin{array}{r}
 237 \text{ 057} \\
 73 \overline{) 173052} \\
 \underline{270} \\
 515 \\
 \underline{420} \\
 550 \\
 \underline{39}
 \end{array}$$

74 Let him hold £x stock, his income (after deduction of income tax)

$$\text{is } \frac{x \times 2\frac{1}{2}}{100} \times \frac{19}{20}, \quad \frac{x \times 2\frac{1}{2} \times 19}{100 \times 20} = 539\frac{1}{20},$$

$$\text{or } \frac{19 \times x}{20 \times 40} = \frac{10792}{20}, \text{ and } x = \frac{10792 \times 40}{19} = \text{£}22720$$

75 Area of larger =  $10 \times 101 \times 101$  ac = 10201 ac.  
diff of area = 0201 ac = 972 sq yds 756 sq ft

$$\begin{array}{r}
 484 \\
 \underline{201} \\
 968 \\
 \underline{484} \\
 97284 \text{ sq yds} \\
 \underline{9} \\
 756 \text{ sq ft}
 \end{array}$$

76 1 sq mi, or 640 ac = 625 sq in, or 30 ac =  $\frac{625 \times 3}{64}$  sq in,  
reqd length of side =  $\sqrt{\frac{625 \times 3}{64}}$  in =  $\frac{25}{8} \times \sqrt{3}$  in =  $\frac{25}{8} \times 1.73205$  in  
=  $\frac{43.301}{8}$  in = 5.41 in (nearly)

77 Let x rupees be prime cost, then, as in Art 293,

$$x \times \frac{120}{100} \times \frac{125}{100} \times \frac{140}{100} = 13\frac{1}{2}, \text{ and } x = \frac{105 \times 100 \times 100 \times 100}{8 \times 120 \times 125 \times 140} = 6\frac{1}{4}$$

78 Net profits + balance = £5000 = £26613 8s 6d

Thus £26613 8s 6d is paid on a capital of £500000,

$$\text{or } \frac{26613 \text{ 8s 6d}}{5000} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{£100}$$

This gives a dividend of 5%, and the remainder, £1613 8s 6d, is carried forward

N

79 1 cm per 1 sec = 19.8 Km per 1980000 secs

$$\begin{aligned}
 \text{Now } 1980000 \text{ secs} &= \frac{1980000}{60 \times 24 \times 365} \text{ yrs} = \frac{19800}{5256} \text{ yrs} \\
 &= 3.8 \text{ yrs}
 \end{aligned}$$

$$\begin{array}{r}
 376 \\
 5256 \overline{) 19800} \\
 \underline{4032} \\
 353
 \end{array}$$

80 Let req<sup>d</sup> price be  $x$  francs

$$\begin{aligned}\text{Then } x \times 100 &= (275 \times 80 + 95 \times 20) \times \frac{110}{100} \\ &= (220 + 190) \times \frac{110}{100}, \\ x &= \frac{410 \times 110}{100} = 451 \text{ fr}\end{aligned}$$

81 The internal volume of the box is  $(17-2)^3$ , or  $15^3$  cu in ,

$$15^3 \text{ cu in would contain ore worth } £421\frac{7}{8}, \text{ or } £\frac{15^3}{2^3},$$

hence  $2^3$  cu in " " " £1

But the ore in the box is actually worth  $£(421\frac{7}{8} - 78\frac{7}{8})$ , or  $£7^3$ ,  
the internal volume of the box must be  $2^3 \times 7^3$  cu in ,  
or  $14^3$  cu in ,

and the thickness of the material  $= \frac{1}{2}(17-14)$  in , or  $1\frac{1}{2}$  in

82 Re 1 per day = Rs 365 per annum ,

$$\text{req<sup>d</sup> percentage} = 100 \times \frac{16\frac{2}{3}}{120} = 13\frac{1}{3} = 302$$

83 The second cyclist rides 1760 yds in 200 secs ,

$$\text{" " } 1760 \times \frac{100}{200}, \text{ or } 1716 \text{ yds in 195 secs}$$

But the first rides 1760 yds in 195 secs ,

$$\text{req<sup>d</sup> start} = (1760 - 1716) \text{ yds} = 44 \text{ yds}$$

84 Diff of the two interests = interest on first year's interest ,

$$\text{Rs } 115 \text{ } 5\text{a} - \text{Rs } 112 \text{ } 8\text{a}, \text{ or Rs } 2\frac{1}{2} \text{ is interest on Rs } 112 \text{ } 8\text{a} ,$$

$$\text{Rs } 2\frac{1}{2} \times \frac{100}{112\frac{1}{2}}, \text{ or Rs } 2\frac{1}{2} \text{ " " Rs } 100$$

$$\text{Thus req<sup>d</sup> rate} = 2\frac{1}{2} \%, \text{ and req<sup>d</sup> Principal} = \text{Rs } 112\frac{1}{2} \times \frac{100}{2\frac{1}{2}} = \text{Rs } 4500$$

O

85 £11 18s 1d = £12 - 1s 8d = £12 -  $\frac{1}{2}$  of £1

$$£2 \text{ } 12\text{s } 6\text{d} = £3 - 7\text{s } 6\text{d} = £3 - \frac{1}{4} \text{ of } £3$$

86 As in Art 302, Ex 2, we have

$$P + P \times \frac{2\frac{1}{2}}{100} \times \frac{63}{365} = 4518 \text{ } 5625 ,$$

$$P \left( 1 + \frac{2\frac{1}{2} \times 63}{100 \times 365} \right) = 4518 \text{ } 5625 ,$$

$$\text{or } P \times \frac{100 \times 365}{100 \times 365} = 4518 \text{ } 5625 ,$$

$$P = 4518 \text{ } 5625 \times \frac{100 \times 365}{100 \times 365}$$

$$\begin{array}{r} 45485625 \\ \underline{146} \\ 45485625 \\ 181942500 \\ \underline{272913750} \\ 146567)6640901250(4530 \text{ } 9 \\ \underline{77822} \\ 1538 \\ \underline{141} \\ 10 \end{array}$$

87  $(\frac{5}{8})^3$  cu in weigh 0.625 lb or  $\frac{5}{8}$  lb

$(\frac{5}{8})^2$  weighs 1 lb

$(\frac{5}{8})^2 \times 40$ , or  $\frac{5^2}{2^3}$  cu in weigh 40 lbs

length of edge =  $\frac{5}{2}$  in =  $2\frac{1}{2}$  in

88 The engine must run a number of feet equal to the LCM of 135 ft, 275 ft, and 1320 ft ( $\frac{1}{4}$  mile)

$$\text{This LCM} = 2^3 \cdot 3^3 \cdot 5^2 \cdot 11 \text{ ft} = \frac{2^3 \cdot 3^3 \cdot 5^2 \cdot 11}{1760 \cdot 3} \text{ mi}$$

$$= \frac{2^3 \cdot 3^3 \cdot 5^2 \cdot 11}{2^5 \cdot 5 \cdot 11 \cdot 3} \text{ mi} = 11\frac{1}{4} \text{ mi}$$

89 The first kind costs him  $\frac{100}{125}$  of 20a, or 16a per lb,

the second „ „  $\frac{100}{112\frac{1}{2}}$  of 27a, or 24a per lb

Selling at 23a per lb, his gain on a mixture of 2 lbs is

$$46a - (16 + 24)a, \text{ or } 6a$$

on an outlay of 40a he will gain 6a, giving 15%

90 His income =  $\frac{3}{100}$  of £19250 = £577 $\frac{1}{2}$

An increase of 10% on this = £57 $\frac{1}{2}$

Each £100 stock sold realises £99 cash

This buys stock at 101 and brings in

an income of £4  $\times$   $\frac{99}{101}$ , or £3 $\frac{9}{10}$

increase of income per £100 stock sold = £3 $\frac{9}{10}$  - £3 = £ $\frac{9}{10}$

Hence an increase of £57 $\frac{1}{2}$  in the income requires the sale of

$$£100 \times \frac{\frac{9}{10}}{\frac{9}{100}} \times 57\frac{1}{2}, \text{ or } £7150 \text{ stock}$$

91 Suppose  $y$  ac. are reaped by  $x$  men. Then, as in Art 277, Ex.,

$y = \frac{2}{4}x$  Taking scales of 0.1" to 1 ac. horizontally, and 0.1" to 1 man vertically, if we join the origin to the point (24, 29), we obtain a graph from which we can read off the required values

92 Let req<sup>d</sup> cost be  $x$  shillings. Then, from Art 293, we have

$$x \times \frac{1 \cdot 20}{100} \times \frac{105}{100} = 70, \quad x = \frac{70 \times 100 \times 100}{120 \times 105} = \frac{500}{9} = 55\frac{5}{9}$$

req<sup>d</sup> cost = Rs 56

## P

$$\begin{aligned}
 93 \quad \text{Req}^1 \text{ interest} &= \text{Rs } \frac{27}{100} \times \frac{68}{365} \times 2378 \\
 &= \text{Rs } \frac{14 \times 2 \times 2 \times 2}{100 \times 365} \\
 &= \text{Rs } 12 \text{ } 183 \\
 &= \text{Rs } 12 \text{ } 3 \text{ a}
 \end{aligned}$$

$$\begin{array}{r}
 2378 \\
 \underline{187} \\
 2378 \\
 19024 \\
 \underline{16646} \\
 365 \overline{) 114686} \quad (12 \text{ } 183 \\
 \underline{796} \\
 668 \\
 \underline{3036} \\
 116 \\
 \underline{6}
 \end{array}$$

$$\begin{aligned}
 94 \quad 36 \text{ in cost } 59d, \text{ or } \underline{\pounds} \frac{59}{240} \\
 \frac{16}{59 \text{ } 17} \text{ m cost } \frac{59}{240} \times 252 \text{ fr,} \\
 \text{and 1 m cost } \frac{59 \times 252}{240 \times 17} \text{ fr,} \\
 \text{or 1 m cost } \frac{59 \times 252 \times 12}{24} \text{ fr,} \\
 \text{or } 6 \text{ } 77 \text{ fr}
 \end{aligned}$$

$$\begin{array}{r}
 59 \times 07 \times 39 \text{ } 37 \\
 \underline{21} \\
 = 4 \text{ } 13 \times 39 \text{ } 37 \\
 \underline{24} \\
 = 162 \text{ } 5981 \\
 \underline{21} \\
 = 20 \text{ } 3217 = 6 \text{ } 774
 \end{array}$$

$$\begin{aligned}
 95 \quad 1000 \text{ gals} &= 1000 \times 0 \text{ } 16 \text{ cu ft} = 160 \text{ cu ft,} \\
 160 \times 60 \times 24 \text{ cu ft} &\text{ are discharged in a day} \\
 \text{If } x \text{ is req}^1 \text{ no of days, then } 160 \times 60 \times 24 \times x &= 300 \times 300 \times 20, \\
 \text{and } x &= 7 \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 96 \quad \text{The outlay on 120 eggs} &= \text{Rs } 3 \text{ } 12 \text{ a} + \text{Rs } 1 \text{ } 4 \text{ a} = \text{Rs } 5 \\
 \text{are broken, the selling price of 100 must be Rs } 5 &+ \text{Rs } 1 \text{ } 14 \text{ a,} \\
 \text{i.e. Rs } 6 \text{ } 14 \text{ a, or } 110 \text{ a. Measure 10 inches to the inch hori-} \\
 \text{zontally, and 10 eggs to the inch vertically. The equation of} \\
 \text{the graph will be } \frac{y}{100} = \frac{x}{110}, \text{ or } y = \frac{10}{11}x. \text{ Join the point (11, 10)} \\
 \text{to the origin, and from this graph read off the required values}
 \end{aligned}$$

$$\begin{aligned}
 97 \quad 32 \text{ sq yds } 1 \text{ sq ft} &= 289 \text{ sq ft} = 17^2 \text{ sq ft,} \quad \text{length} = 17 \text{ ft} \\
 \text{area to be painted} \\
 &= (17 \times 11 \frac{1}{2} \times 4 + 17 \times 17 - 7 \frac{1}{2} \times 3 \frac{1}{2} - 5 \frac{1}{2} \times 3 \frac{1}{2}) \text{ sq ft} \\
 &= (17 \times 46 + 17 \times 17 - 25 - 20) \text{ sq ft} = 1026 \text{ sq ft} \\
 \text{Hence cost} &= \frac{13}{9} \text{ a } \times 1026 = \frac{17}{2} \times 57 \text{ a} = \text{Rs } 30 \text{ } 4 \text{ a } 6 \text{ p}
 \end{aligned}$$

$$\begin{aligned}
 98 \quad \text{Let } \pounds x \text{ be req}^1 \text{ gross income. Then net income} &= x \times \frac{87}{100} \times \frac{234}{100} \\
 \text{Now } \pounds 513 \text{ } 16 \text{ s } 6 \text{ d} &= \pounds 513 \frac{1}{4} = \pounds \frac{2053}{4} \\
 x \times \frac{87}{100} \times \frac{234}{100} &= \frac{2053}{4}, \quad x = \frac{2053 \times 100 \times 100}{4 \times 87 \times 234} = \pounds 620
 \end{aligned}$$

99 Let  $V$  cu cm be the volume of each pellet,

then  $58 V = 2^2 \times 1 \times 0.7854$ ,

$$V = \frac{4 \times 0.7854}{58} = \frac{1.5708}{29} = 5.416 = 54.2 \text{ cu mm}$$

100 Let  $L$  cwt be the load when  $AP = x$  ft ( $BP = (10 - x)$  ft)

Then, from Art 283,  $L = \frac{l}{x(10-x)}$  When  $x = 5$ ,  $L = 12$ ,

and we find  $l = 300$ ,  $L = \frac{300}{x(10-x)}$

To find the req<sup>d</sup> loads, we put  $x = 1$  ft, 2 ft, 3 ft, 4 ft successively, and find  $L = 33\frac{1}{3}$  cwt,  $18\frac{3}{4}$  cwt,  $14\frac{2}{7}$  cwt,  $12\frac{1}{2}$  cwt respectively

### EXAMPLES XX a Page 411

- 1  $\text{Fract}^n = \frac{1}{2 + \frac{1}{3}}$
- 2  $\text{Fract}^n = \frac{3}{1 + \frac{1}{3\frac{1}{2}}}$
- 3  $\text{Exp}^n = 2 + \frac{4}{1 + \frac{7}{9}}$
- 4  $\text{Exp}^n = 3 - \frac{1}{2 + \frac{1}{4}}$
- 5  $\text{Fract}^n = \frac{6}{2 + \frac{1}{3\frac{1}{2}}}$
- 6  $\text{Fract}^n = \frac{10}{1 + \frac{3}{2}}$
- 7  $\text{Exp}^n = 7 - \frac{2}{8 - \frac{7}{4}}$
- 8  $\text{Fract}^n = \frac{11}{3 + \frac{1}{5\frac{1}{2}}}$
- 9  $\text{Exp}^n = 5 - \frac{2}{4 - \frac{5}{4}}$
- 10  $\text{Fract}^n = \frac{2 + \frac{5}{7}}{6 + \frac{7}{9\frac{1}{2}}}$
- 11  $\text{Fract}^n = \frac{6 + \frac{6}{3\frac{1}{2}}}{4 - \frac{4}{1\frac{1}{5}}} \times 10\frac{8}{9}$
- 12  $\text{Fract}^n = \frac{2 + \frac{5}{1\frac{1}{6}}}{2 + \frac{1}{3\frac{1}{6}}}$
- 13  $\text{Exp}^n = 2 - \frac{3}{9 - \frac{1}{1\frac{1}{3}}} = 2 - \frac{3}{4\frac{2}{3}}$
- 14  $\text{Exp}^n = 2 - \frac{2}{8 - \frac{7}{1\frac{1}{6}}} = 2 - \frac{2}{3\frac{1}{4}}$

### EXAMPLES XX b Page 415

- 1 The first two convergents are  $\frac{2}{1}$  and  $\frac{7}{3}$  3<sup>rd</sup> convergent =  $\frac{7 \times 1 + 2}{3 \times 1 + 1} = \frac{9}{4}$ ,  
4<sup>th</sup> convergent =  $\frac{9 \times 2 + 7}{4 \times 2 + 3} = \frac{25}{11}$ , 5<sup>th</sup> convergent =  $\frac{25 \times 3 + 9}{11 \times 3 + 4} = \frac{174}{38}$
- 2 The first two convergents are  $\frac{3}{1}$  and  $\frac{13}{4}$  The others are  
 $\frac{13 \times 2 + 3}{4 \times 2 + 1} = \frac{29}{9}$ ,  $\frac{29 \times 1 + 13}{9 \times 1 + 4} = \frac{42}{13}$ ,  $\frac{42 \times 3 + 29}{13 \times 3 + 9} = \frac{155}{58}$ ,  $\frac{155 \times 5 + 42}{48 \times 5 + 13} = \frac{777}{243}$
- 3 The first two convergents are  $\frac{1}{2}$  and  $\frac{2}{5}$  The others are  
 $\frac{2 \times 3 + 1}{5 \times 3 + 2} = \frac{7}{17}$ ,  $\frac{7 \times 1 + 2}{17 \times 1 + 5} = \frac{9}{22}$ ,  $\frac{9 \times 4 + 7}{22 \times 4 + 17} = \frac{43}{105}$ ,  $\frac{43 \times 2 + 9}{105 \times 2 + 22} = \frac{95}{232}$ ,  $\frac{95 \times 6 + 43}{232 \times 6 + 105} = \frac{593}{1407}$

4 As in Art 336, we have

$$\begin{array}{r|rr} 1 & 79 & 291 \\ 6 & 25 & 54 \\ & 1 & 4 \end{array} \begin{array}{l} 3 \\ 2 \\ 4 \end{array}$$

This gives  $\frac{1}{3+}$   $\frac{1}{1-}$   $\frac{1}{2+}$   $\frac{1}{6+}$   $\frac{1}{4}$

The first two convergents are  $\frac{1}{3}$  and  $\frac{1}{4}$ ,

the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{1 \times 2 - 1}{4 \times 2 + 3}$ ,  $\frac{3 \times 6 + 1}{11 \times 6 + 4}$

5 As in Art 336, we have

$$\begin{array}{r|rr} 2 & 97 & 625 \\ 1 & 11 & 43 \\ & 1 & 10 \end{array} \begin{array}{l} 6 \\ 3 \\ 10 \end{array}$$

This gives  $\frac{1}{6+}$   $\frac{1}{2+}$   $\frac{1}{3+}$   $\frac{1}{1+}$   $\frac{1}{10}$

The first two convergents are  $\frac{1}{6}$  and  $\frac{2}{13}$

the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{2 \times 7 + 1}{13 \times 5 + 6}$ ,  $\frac{- \times 1 + 2}{45 \times 1 + 13}$

6 As in Art 336, we have

$$\begin{array}{r|rr} 2 & 233 & 313 \\ 10 & 73 & 80 \\ 3 & 3 & 7 \\ & & 1 \end{array} \begin{array}{l} 1 \\ 1 \\ 2 \\ 1 \end{array}$$

This gives  $1 + \frac{1}{2+}$   $\frac{1}{1-}$   $\frac{1}{10+}$   $\frac{1}{2+}$   $\frac{1}{3}$

The first two convergents are  $\frac{1}{1}$  and  $\frac{3}{4}$ ,

the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{1 \times 1 + 1}{2 \times 1 - 1}$ ,  $\frac{4 \times 10 + 3}{3 \times 10 + 2}$

7 As in Art 336, we have

$$\begin{array}{r|rr} 4 & 349 & 1128 \\ 4 & 25 & 81 \\ & 1 & 6 \end{array} \begin{array}{l} 3 \\ 3 \\ 6 \end{array}$$

This gives  $3 + \frac{1}{4+}$   $\frac{1}{3+}$   $\frac{1}{4+}$   $\frac{1}{6}$

The first two convergents are  $\frac{3}{1}$  and  $\frac{17}{4}$ ,

the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{13 \times 7 + 1}{4 \times 5 + 1}$ ,  $\frac{42 \times 4 + 13}{13 \times 4 + 4}$

8 As in Art 336, we have

$$\begin{array}{r|rr} 7 & 1000 & 1139 \\ 6 & 27 & 139 \\ 3 & 3 & 4 \\ & & 1 \end{array} \begin{array}{l} 1 \\ 5 \\ 1 \\ 1 \end{array}$$

This gives  $1 + \frac{1}{7+}$   $\frac{1}{5+}$   $\frac{1}{6+}$   $\frac{1}{1+}$   $\frac{1}{3}$

The first two convergents are  $\frac{1}{1}$  and  $\frac{8}{5}$ ,

the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{8 \times 5 + 1}{7 \times 5 + 1}$ ,  $\frac{41 \times 6 + 8}{36 \times 6 + 8}$

9 As in Art 336, we have

$$\begin{array}{r|rr} 2 & 1000 & 1414 \\ 2 & 172 & 414 \\ 5 & 32 & 70 \\ & 2 & 6 \end{array} \begin{array}{l} 1 \\ 2 \\ 2 \\ 3 \end{array}$$

This gives  $1 + \frac{1}{2+}$   $\frac{1}{2+}$   $\frac{1}{2+}$   $\frac{1}{2+}$   $\frac{1}{5+}$   $\frac{1}{3}$

The first two convergents are  $\frac{1}{1}$  and  $\frac{3}{2}$ ,

the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{3 \times 2 + 1}{2 \times 2 + 1}$ ,  $\frac{- \times 2 + 3}{5 \times 2 + 2}$



- 10 As in Art 336, we have

$$\begin{array}{c|c|c} 1 & 393 & 1000 \\ 5 & 179 & 214 \\ 1 & 4 & 35 \\ & 1 & 3 \end{array}$$

This gives  $\frac{1}{2+} \frac{1}{1+} \frac{1}{1+} \frac{1}{5+} \frac{1}{8+} \frac{1}{1+} \frac{1}{3}$ The first two convergents are  $\frac{1}{2}$  and  $\frac{1}{3}$ ,the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{1 \times 1 + 1}{3 \times 1 + 2}$ ,  $\frac{2 \times 5 + 1}{5 \times 5 + 3}$ 

- 11 As in Art 336, we have

$$\begin{array}{c|c|c} 3 & 3029 & 10000 \\ 6 & 290 & 913 \\ 2 & 32 & 43 \\ 10 & 10 & 11 \\ & & 1 \end{array}$$

This gives  $\frac{1}{3+} \frac{1}{3+} \frac{1}{3+} \frac{1}{6+} \frac{1}{1+} \frac{1}{2+} \frac{1}{1+} \frac{1}{10}$ The first two convergents are  $\frac{1}{3}$  and  $\frac{2}{10}$ ,the 3<sup>rd</sup> and 4<sup>th</sup> are  $\frac{3 \times 3 + 1}{10 \times 3 + 3}$ ,  $\frac{10 \times 6 + 1}{33 \times 6 + 10}$ 

- 12 See Art 339, Ex 2, and Art 340

The 4<sup>th</sup> and 5<sup>th</sup> Convergent are  $\frac{9}{22}$  and  $\frac{43}{105}$ the error in taking  $\frac{43}{105}$  for the C F is less than  $\frac{1}{2 \times 105 \times 22}$ ,  
or  $\frac{1}{1620}$ 

- 13 1 metre = 39 37 in = 1 0936 yds Express this as a C F

We obtain  $\frac{35}{2}$  for the 4<sup>th</sup> Convergent1 metre =  $\frac{35}{2}$  yds (nearly), or 32 metres = 35 yds (nearly)

- 14 One div
- <sup>n</sup>
- of the first =
- $\frac{200}{102}$
- div
- <sup>n</sup>
- of the second

Expressing this as a C F, we obtain  $\frac{40}{31}$  as the 4<sup>th</sup> Convergentone div<sup>n</sup> of the first =  $\frac{40}{31}$  div<sup>n</sup> of the second (nearly),or 31 div<sup>n</sup> of the first = 40 div<sup>n</sup> of the second (nearly)

- 15 See Art 339, Ex 2, and Art 340 The 6
- <sup>th</sup>
- and 7
- <sup>th</sup>
- Convergent
- <sup>s</sup>
- of 1 41421 are
- $\frac{20}{70}$
- and
- $\frac{230}{160}$
- Of these,
- $\frac{20}{70}$
- is greater, and
- $\frac{230}{160}$
- is less, than the C F Since, then, the C F lies between them in value, it follows that the difference of
- $\frac{20}{70}$
- and the C F is less than
- $\frac{20}{70} - \frac{230}{160}$
- , or
- $\frac{1}{70 \times 160}$

- 16 See Art 339, Ex 2, and Art 340

The 4<sup>th</sup> and 5<sup>th</sup> Convergent<sup>s</sup> are  $\frac{21}{10}$  and  $\frac{151}{115}$ the C F differs from  $\frac{151}{115}$  by less than  $\frac{1}{2 \times 115 \times 115}$ , or  $\frac{1}{3680}$ Thus  $\frac{151}{115}$  is the req<sup>d</sup> approximation

$$17 \quad \text{The CF} = 3 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{9 + \frac{1}{4}}}}} = \frac{1784}{457}$$

The 4<sup>th</sup> and 5<sup>th</sup> Converge<sup>ts</sup> are  $\frac{76}{11}$  and  $\frac{77}{10}$ . Since the CF lies between these in value, it follows that the CF differs from  $\frac{76}{11}$  by less than  $\frac{76}{11} - \frac{77}{10}$ , or  $\frac{1}{11 \times 103}$ , which is less than 0.001

Now  $\frac{77}{10}$  is nearer the CF in value than  $\frac{76}{11}$   
the error in this case is also less than 0.001

$$19 \quad 1 \text{ fr} = \frac{50}{27} - \frac{1}{6} = \frac{50}{27}$$

The 3<sup>rd</sup> and 4<sup>th</sup> converge<sup>ts</sup> of  $\frac{50}{27}$  are  $\frac{4}{3}$  and  $\frac{23}{13}$

As in No 17 it may be shewn that 1 fr differs from  $\frac{4}{3}$ ,  
or from  $\frac{23}{13}$  by a quantity less than  $\frac{1}{13 \times 2}$ , or  $\frac{1}{146}$ ,  
for 1 fr the diff is less than 1 fr

$$20 \quad \sqrt{26} \text{ lies between 5 and 6} \quad \text{Put } \sqrt{26} = 5 + r \\ (5+r)^2 = 26, \text{ or } 10r + r^2 = 1 \\ r = \frac{1}{10+r} = \frac{1}{10 + \frac{1}{10 + \frac{1}{10 + \frac{1}{10 + \dots}}}}$$

The 3<sup>rd</sup> and 4<sup>th</sup> converge<sup>ts</sup> are  $\frac{501}{100}$  and  $\frac{506}{100}$

From Art 310, it follows that  $\frac{501}{100}$  differs from  $\sqrt{26}$

by a fraction less than  $\frac{1}{2 \times 100 \times 10301}$  or  $\frac{1}{2060 \times 10301}$

21 Consider the decimal portion 0.242218 It reduces to a CF whose first and third convergents are  $\frac{1}{4}$  and  $\frac{8}{33}$ , thus shewing 8 days in 33 yrs to be more correct than 1 day in 4 yrs

Now 8 days in 33 yrs = 0.242424 day in 1 yr

error for 1 yr = (0.242424 - 0.242218) day = 0.000206 day

1 day would be the error for  $\frac{1}{0.000206}$  yrs, or 4854 yrs

### EXAMPLES XX c Page 418

1 (i)	2773 31	(ii)	327 168
	6259		54936
	$\frac{1}{2}$ 39329		31
	865229		164808
Subt $\frac{1}{10000}$	346		$\frac{1}{2}$ 7848
	864883		172656
re 8649		Subt $\frac{1}{10000}$	69
			172587
			re 1726

$$\begin{aligned}
 2 \text{ Reqd area} &= (62.3)^2 \times \pi \\
 &= 3881.29 \times \pi \\
 &= 12193.46 \text{ sq m}
 \end{aligned}$$

$$\begin{array}{r}
 3881.29 \\
 \times 3.14 \\
 \hline
 11613.87 \\
 7762.58 \\
 \hline
 12196.45 \\
 \text{Subt } 12193.46 \\
 \hline
 3.00
 \end{array}$$

$$\begin{array}{r}
 0.31830989 \\
 3 \text{ (i) } 3.14159265 \overline{) 1.00000000} \\
 \underline{57522205} \\
 26106278 \\
 \underline{973537} \\
 31079 \\
 \underline{2785} \\
 272
 \end{array}$$

$$\begin{aligned}
 NB \text{ From Art 313,} \\
 \approx 3.14159265
 \end{aligned}$$

$$\begin{aligned}
 (ii) \frac{1}{1000} - \left( \frac{1}{1000} + \frac{1}{1000} + \frac{1}{1000000} \right) \\
 = 0.3333 - (0.01 + 0.005 + 0.000002) \\
 = 0.31831
 \end{aligned}$$

$$\begin{aligned}
 (iii) \text{ Reqd diameter} \\
 = 291.85 \text{ m} \times \frac{1}{\pi} \\
 = 291.85 \times 0.3183 \text{ m} \\
 = 93 \text{ m (nearly)}
 \end{aligned}$$

$$\begin{array}{r}
 4 \text{ (i)} \quad \begin{array}{r}
 \text{m} \\
 3724 \\
 \times 3.14159 \\
 \hline
 11693.68 \\
 74481.8 \\
 \hline
 40731
 \end{array} \\
 \text{giving } 1073 \text{ yds}
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad \begin{array}{r}
 \text{m} \\
 561.41 \\
 \times 3.14159 \\
 \hline
 1763.76 \\
 1763.76 \\
 \hline
 614.01
 \end{array} \\
 \text{giving } 614 \text{ yds}
 \end{array}$$

5 It may be easily shown that  $1 - \frac{1}{10} + \frac{1}{10} \approx \frac{1}{2}$

$$\begin{array}{r}
 (i) \quad \begin{array}{r}
 \text{yds} \\
 3650 \\
 \text{Subtract } \frac{1}{10} \quad 365 \\
 \hline
 3285 \\
 \text{Add } \frac{1}{10} \quad 52.14 \\
 \hline
 3337.1
 \end{array} \\
 \text{giving } 3337 \text{ m}
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad \begin{array}{r}
 \text{yds} \\
 8400 \text{ (=5 miles)} \\
 \text{Subtract } \frac{1}{10} \quad 840 \\
 \hline
 7560 \\
 \text{Add } \frac{1}{10} \quad 125.71 \\
 \hline
 8046.71
 \end{array} \\
 \text{giving } 8046 \text{ m}
 \end{array}$$

$$\begin{aligned}
 6 \quad 22 \text{ yds} &= 20 \text{ m}, \\
 1760 \text{ yds} &= 1600 \text{ m} \\
 \text{But } 1760 \text{ yds} &= 1760 \times 0.91439 \text{ m} \\
 &= 1609.3 \text{ m} \\
 \text{Error} &= (1609.3 - 1600) \text{ m} = 9.3 \text{ m}
 \end{aligned}$$

$$\begin{array}{r}
 911.39 \\
 \times 1.76 \\
 \hline
 1604.24 \\
 1604.24 \\
 \hline
 1609.32
 \end{array}$$

For second part of question, we have

$$1 \text{ mile} = 80 \text{ chains} = 80(20 + \frac{1}{10} + \frac{1}{100}) \text{ m} = (1600 + 8 + 1) \text{ m} = 1609 \text{ m}$$

$$7 \quad (i) \quad 5 \text{ ft } 3 \text{ in} = 63 \text{ in} \quad \text{reqd equivalent} = 63(\frac{1}{4} + \frac{4}{100}) \text{ cm} \\ = (157.5 + 2.52) \text{ cm} = 160 \text{ m},$$

$$(ii) \quad \text{reqd equivalent} = 39.375(\frac{1}{4} + \frac{4}{100}) \text{ cm} = (98.44 + 1.57) \text{ cm} = 100 \text{ m}$$

$$8 \quad \begin{array}{r} 14 \\ \text{Add } \frac{1}{100} \quad 014 \\ \text{Add } \frac{1}{10000} \quad 00014 \\ \text{Add } \frac{1}{20000} \quad 00007 \\ \hline 1.41421 \\ = \sqrt{2} \end{array} \quad \begin{array}{r} 353.55 \\ \hline 14 \\ 353.55 \\ 141.42 \\ \hline 494.97 \\ \text{Add } \frac{1}{100} \quad 4.95 \\ \text{Add } \frac{1}{10000} \quad .05 \\ \text{Add } \frac{1}{20000} \quad .00 \\ \hline 499.97 \\ = 500 \text{ m} \end{array}$$

$$9 \quad (i) \quad \begin{array}{l|l} \frac{1}{5} = 0.20000 & 00 \\ \frac{1}{5^2} = 0.04000 & 00 \\ \frac{1}{5^3} = 0.00800 & 00 \\ \frac{1}{5^4} = 0.00160 & 00 \\ \frac{1}{5^5} = 0.00032 & 00 \\ \frac{1}{5^6} = 0.00006 & 40 \\ \frac{1}{5^7} = 0.00001 & 28 \\ \frac{1}{5^8} = 0.00000 & 26 \\ \frac{1}{5^9} = 0.00000 & 05 \\ \frac{1}{5^{10}} = 0.00000 & 00 \\ \hline & 0.25000 \end{array} \quad (ii) \quad \begin{array}{l|l} \frac{1}{7} = 0.14285 & 71 \\ \frac{1}{7^2} = 0.02040 & 82 \\ \frac{1}{7^3} = 0.00291 & 55 \\ \frac{1}{7^4} = 0.00041 & 65 \\ \frac{1}{7^5} = 0.00005 & 95 \\ \frac{1}{7^6} = 0.00000 & 85 \\ \frac{1}{7^7} = 0.00000 & 12 \\ \frac{1}{7^8} = 0.00000 & 02 \\ \frac{1}{7^9} = 0.00000 & 00 \\ \hline & 0.16666 \end{array} \quad \begin{array}{l} 67 \\ 67 \\ 67 \\ 67 \\ 67 \\ 67 \\ 67 \\ 67 \\ 67 \\ 67 \end{array}$$

giving 0.166667

10.

$$\begin{array}{r|l} \frac{1}{2} = 0.500000 & 00 \\ \frac{1}{2 \cdot 4} = 0.125000 & 00 \\ \frac{1}{2 \cdot 16} = 0.020833 & 33 \\ \frac{1}{2 \cdot 4 \cdot 8} = 0.002604 & 17 \\ \frac{1}{2 \cdot 4 \cdot 8 \cdot 10} = 0.000260 & 42 \\ \frac{1}{2 \cdot 4 \cdot 8 \cdot 10 \cdot 12} = 0.000021 & 70 \\ \frac{1}{2 \cdot 4 \cdot 8 \cdot 10 \cdot 12 \cdot 14} = 0.000001 & 55 \\ \frac{1}{2 \cdot 4 \cdot 8 \cdot 10 \cdot 12 \cdot 14 \cdot 16} = 0.000000 & 10 \\ \frac{1}{2 \cdot 4 \cdot 8 \cdot 10 \cdot 12 \cdot 14 \cdot 16 \cdot 18} = 0.000000 & 00 \\ \hline & 0.618721 \\ & 27 \end{array}$$

giving 0.618721

11

$$\begin{array}{r|l} \frac{1}{1 \cdot 3} & = 0.333333 \quad 33 \\ \frac{1}{3 \cdot 3^2} = 0.33333333 - (3 \times 9) = 0.11111111 - 9 & = 0.012345 \quad 67 \\ \frac{1}{5 \cdot 3^4} = 0.01234567 - (3 \times 5) = 0.00111522 - 5 & = 0.000823 \quad 01 \\ \frac{1}{7 \cdot 3^6} = 0.00111522 - (9 \times 7) = 0.00015725 - 7 & = 0.000065 \quad 32 \\ \frac{1}{9 \cdot 3^8} = 0.00015725 - (9 \times 9) = 0.00005081 - 9 & = 0.000005 \quad 65 \\ \frac{1}{11 \cdot 3^{10}} = 0.00000565 - 11 & = 0.000000 \quad 51 \\ \frac{1}{13 \cdot 3^{12}} = 0.00000565 - (9 \times 13) = 0.00000063 - 13 & = 0.000000 \quad 05 \\ \hline & 0.316573 \quad 57 \\ & 57 \end{array}$$

giving 0.316574

## EXAMPLES XXI b Page 425

- 9  $\log \frac{a}{bc} = \log a - \log (bc) = \log a - \log b - \log c$
- 10  $\log \frac{a^2}{bc^3} = \log a^2 - \log (bc^3) = \log a^2 - \log b - \log c^3$   
 $= 2 \log a - \log b - 3 \log c$
- 11  $\log \frac{a^4 b^3}{c^5} = \log (a^4 b^3) - \log c^5 = \log a^4 + \log b^3 - \log c^5$   
 $= 4 \log a + 3 \log b - 5 \log c$
- 12  $\log \frac{\sqrt[4]{b} \sqrt[3]{c}}{\sqrt{a^3}} = \log \frac{b^{\frac{1}{4}} c^{\frac{1}{3}}}{a^{\frac{3}{2}}} = \log (b^{\frac{1}{4}} c^{\frac{1}{3}}) - \log a^{\frac{3}{2}} = \log b^{\frac{1}{4}} + \log c^{\frac{1}{3}} - \log a^{\frac{3}{2}}$   
 $= \frac{1}{4} \log b + \frac{1}{3} \log c - \frac{3}{2} \log a$
- 13  $\log 36 = \log (2^2 \cdot 3^2) = \log 2^2 + \log 3^2 = 2 \log 2 + 2 \log 3$
- 14  $\log \frac{1}{108} = \log \frac{1}{2^3 \cdot 3^3} = \log 1 - \log (2^3 \cdot 3^3) = -2 \log 2 - 3 \log 3$
- 15  $\log \sqrt{648} = \log \sqrt{2^3 \cdot 3^5} = \log (2^{\frac{3}{2}} \cdot 3^{\frac{5}{2}}) = \frac{3}{2} \log 2 + 2 \log 3$
- 16  $\log \sqrt{54} \times \sqrt[3]{243} = \log (\sqrt{2 \cdot 3^3} \sqrt[3]{3^5}) = \log (2^{\frac{1}{2}} \cdot 3^{\frac{3}{2}} \cdot 3^{\frac{5}{3}}) = \log (2^{\frac{1}{2}} \cdot 3^{\frac{11}{2}})$   
 $= \log (2^1 \cdot 3^{5.5}) = \frac{1}{2} \log 2 + \frac{11}{2} \log 3$
- 17  $\log (\frac{2^2 \cdot 1^7}{3^2 \cdot 1^6} - \frac{1^1 \cdot 1^1}{2^1 \cdot 1^1}) = \log 49 = \log 7^2 = 2 \log 7$
- 18  $\log \frac{2^6}{3^1} + \log \frac{1^1 \cdot 1^0}{2^1 \cdot 1^0} = \log (\frac{2^6}{3^1} \times \frac{1^1 \cdot 1^0}{2^1 \cdot 1^0}) = \log \frac{2^5}{3} = \log 2 - \log 3$
- 19 We have  $\log PR^n = \log A$ , or  $\log P + n \log R = \log A$ ,  
 $n \log R = \log A - \log P$ , and  $n = \frac{\log A - \log P}{\log R}$
- 20 We have  $\log \frac{\pi d^3}{6} = \log V$ , or  $\log \pi + 3 \log d - \log 6 = \log V$ ,  
whence  $\log d = \frac{1}{3} (\log V + \log 6 - \log \pi)$

## EXAMPLES XXI c Page 429

4-11 See Art 368

$$12 \quad \bar{4} 5703 - 5 = \frac{(\bar{5} + 1 \frac{5}{6} - 0.3)}{6} = \bar{1} 3141$$

$$13 \quad \frac{1}{8}(\bar{3} 8123) = \frac{1}{8}(\bar{6} + 3 8123) = \bar{1} 6354$$

$$14 \quad \frac{2}{5}(\bar{1} 5632) = \frac{1}{5}(\bar{1} 1264) = \frac{1}{5}(\bar{5} + 4 1264) = \bar{1} 8253$$

$$15 \quad \frac{3}{4}(\bar{2} 1305) = \frac{1}{4}(\bar{6} 3915) = \frac{1}{4}(\bar{8} + 2 3915) = \bar{2} 5979$$

## EXAMPLES XXI d Page 432

$$1 \quad \begin{array}{l} \log 283 = \bar{1} 4518 \\ \text{diff for } 4 = 6 \\ \log 176 = 1 2455 \\ \text{diff for } 2 = 5 \\ \log \text{ product} = 6984 \end{array}$$

$$\begin{array}{l} \text{antilog } 698 = 4 989 \\ \text{diff for } 4 = 5 \\ \text{antilog } 6984 = 4 994 \end{array}$$

$$2 \quad \begin{array}{l} \log 803 = 9047 \\ \text{diff for } 4 = 2 \\ \log 189 = \bar{1} 2765 \\ \text{diff for } 3 = 7 \\ \log \text{ product} = 1821 \end{array}$$

$$\begin{array}{l} \text{antilog } 182 = 1 521 \\ \text{diff for } 1 = 0 \\ \text{antilog } 1821 = 1 521 \end{array}$$

$$3 \quad \begin{array}{l} \log 470 = 2 6721 \\ \text{diff for } 8 = 7 \\ \log 639 = 8055 \\ \log \text{ product} = 3 4783 \end{array}$$

$$\begin{array}{l} \text{antilog } 3 478 = 3006 \\ \text{diff for } 3 = 2 \\ \text{antilog } 3 4783 = 3008 \end{array}$$

$$4 \quad \begin{array}{l} \log 37 = 5682 \\ \log 89 = 9494 \\ \log 023 = \bar{2} 3617 \\ \log \text{ product} = \bar{1} 8793 \end{array}$$

$$\begin{array}{l} \text{antilog } \bar{1} 879 = 7568 \\ \text{diff for } 3 = 5 \\ \text{antilog } \bar{1} 8793 = 7573 \end{array}$$

$$5 \quad \begin{array}{l} \log 319 = 1 5038 \\ \log 151 = 1790 \\ \log 97 = 9868 \\ \log \text{ product} = 2 6696 \end{array}$$

$$\begin{array}{l} \text{antilog } 2 669 = 466 7 \\ \text{diff for } 6 = 6 \\ \text{antilog } 2 6696 = 467 3 \end{array}$$

$$6 \quad \begin{array}{l} \log 43 = 1 6335 \\ \log 807 = 9069 \\ \log 0392 = \bar{2} 5933 \\ \log \text{ product} = 1337 \end{array}$$

$$\begin{array}{l} \text{antilog } 133 = 1 358 \\ \text{diff for } 7 = 2 \\ \text{antilog } 1337 = 1 360 \end{array}$$

	Numerator	Denominator
7	log 17 3 = 1 2380	log 294 0 = 2 4683
		diff for 8 = 12
	log numerator = 1 2380	log denominator = 2 4695
	1 2380	antilog 2 768 = 05861
	subtract 2 4695	diff for 5 = 7
	log fraction = 2 7685	antilog 2 7685 = 05868
8	Numerator	Denominator
	log 487 = 2 6875	log 6390 = 3 8055
		diff for 8 = 5
	log numerator = 2 6875	log denominator = 3 8060
	2 6875	antilog 2 881 = 07603
	subtract 3 8060	diff for 5 = 9
	log fraction = 2 8815	antilog 2 8815 = 07612
9	Numerator	Denominator
	log 2 17 = 3365	log 897 = 1 9528
	diff for 9 = 18	diff for 3 = 1
	log numerator = 3383	log denominator = 1 9529
	3383	antilog 385 = 2 427
	subtract 1 9529	diff for 4 = 2
	log fraction = 0 3854	antilog 3854 = 2 429
10	Numerator	Denominator
	log 0125 = 2 0969	log 410 = 0 6128
	diff for 4 = 14	diff for 5 = 5
	log numerator = 2 0983	log denominator = 0 6133
	2 0983	
	subtract 0 6133	antilog 3 4850 = 0 003055
	log fraction = 3 4850	
11	Numerator	Denominator
	log 2 38 = 0 3766	log 4 83 = 0 6839
	log 3 90 = 0 5911	
	diff for 1 = 1	
	log numerator = 0 9678	log denominator = 0 6839
	0 9678	antilog 0 283 = 1 919
	subtract 0 6839	diff for 9 = 4
	log fraction = 0 2839	antilog 0 2839 = 1 923



	Numerator	Denominator
12	$\log 147 = 1.1673$ $\text{diff for } 2 = 4$ $\log 380 = 1.5798$ $\text{diff for } 5 = 6$ $\log \text{ numerator} = 2.7483$ $\quad 2.7483$ $\text{subtract } 2.5887$ $\quad \underline{\quad} 0.1596$	$\log 387 = 2.5877$ $\text{diff for } 9 = 10$ $\log \text{ denominator} = 2.5887$ $\text{antilog } 0.159 = 1.442$ $\text{diff for } 6 = 2$ $\text{antilog } 0.1596 = 1.444$

	Numerator	Denominator
13	$\log 925 = 2.9661$ $\text{diff for } 9 = 4$ $\log 159 = 0.2014$ $\text{diff for } 7 = 20$ $\log \text{ numerator} = 3.1699$ $\quad 3.1699$ $\text{subtract } 1.8694$ $\log \text{ fraction} = 1.3005$	$\log 740 = 1.8692$ $\text{diff for } 3 = 2$ $\log \text{ denominator} = 1.8694$ $\text{antilog } 1.300 = 19.95$ $\text{diff for } 5 = 2$ $\text{antilog } 1.3005 = 19.97$

14 Let  $x = \sqrt{51}$ , or  $(51)^{\frac{1}{2}}$  Then  $\log x = \frac{1}{2} \log 51 = \frac{1}{2}(0.7076)$   
 $= 0.3538 = \log(2.258)$

15 Let  $x = \sqrt[3]{11}$ , or  $(11)^{\frac{1}{3}}$  Then  $\log x = \frac{1}{3} \log 11 = \frac{1}{3}(1.0414)$   
 $= 0.3471 = \log(2.224)$

16 Let  $x = (0.97)^4$  Then  $\log x = 4 \log(0.97) = \bar{2}.9868 \times 4$   
 $= \bar{5}.9472 = \log(0.0008855)$

17 Let  $x = \sqrt[4]{1015}$ , or  $(1015)^{\frac{1}{4}}$  Then  $\log x = \frac{1}{4} \log(1015)$   
 $= \frac{1}{4}(1.0064) = 0.2516 = \log 1.784$

18 [153.76 = 153.8 correct to four significant figures]

Numerator	Denominator
$\log 153.8 = 2.1869$	$\log 276 = 2.4409$
$\log 0.137 = \bar{2}.1367$	$\log 0.038 = \bar{3}.5798$
$\log \text{ numerator} = 0.3236$	$\log \text{ denominator} = 0.0207$
$\quad 0.3236$	
$\text{subtract } 0.0207$	$\text{antilog } 0.3029 = 2.008$
$\log \text{ fraction} = 0.3029$	

19. [3303 7=3303 correct to four significant figures]

Numerator	Denominator.
$\log 3303 = 3\ 5189$	$\log 0561 = \bar{2}\ 7490$
$\log 143 = 1\ 1553$	$\log 387 = \bar{1}\ 5877$
$\log \text{numerator} = 4\ 6742$	$\log 0091 = \bar{3}\ 9590$
$4\ 6742$	$\log \text{denominator} = 4\ 2957$
subtract $\bar{4}\ 2957$	$\text{antilog } 8\ 3785 = 2\ 391 \times 10^3$
$\log \text{fraction} = 8\ 3785$	

20 Let  $x = 5^8$

$$\text{Then } \log x = 8 \log 5 = 0\ 6990 \times 8 = 5\ 5920 = \log (3\ 908 \times 10^5)$$

21 Let  $x = 11^6$

$$\text{Then } \log x = 6 \log 11 = 1\ 0414 \times 6 = 6\ 2484 = \log (1\ 772 \times 10^6)$$

22 Let  $x = 7^7$

$$\text{Then } \log x = 7 \log 7 = 0\ 8451 \times 7 = 5\ 9157 = \log (8\ 235 \times 10^5)$$

23 Let  $x = 13^5$

$$\text{Then } \log x = 5 \log 13 = 1\ 1139 \times 5 = 5\ 5695 = \log (3\ 711 \times 10^5)$$

24 Let  $x = \sqrt[3]{82\ 558}$ , or  $(82\ 558)^{\frac{1}{3}}$  Then  $\log x = \frac{1}{3} \log (82\ 558)$   
 $= \frac{1}{3} \log (82\ 56) = \frac{1}{3} (1\ 9168) = 0\ 6389 = \log (4\ 354)$

25 Let  $x = 17^3 \times 29^2$

$$\text{Then } \log x = 3 \log 17 + 2 \log 29 = 1\ 2304 \times 3 + 1\ 4624 \times 2 \\ = 6\ 6160 = \log (4\ 130 \times 10^6)$$

26 Let  $x = (2\ 301)^5$

$$\text{Then } \log x = 5 \log (2\ 301) = 0\ 3619 \times 5 = 1\ 8095 = \log (6\ 449 \times 10)$$

27 Let  $x = (089)^{\frac{4}{7}}$  Then  $\log x = \frac{4}{7} \log (089) = \frac{4}{7} (\bar{2}\ 9494)$

$$= \frac{4}{7} (\bar{2}\ 9796) = \frac{\bar{11} + 2}{7} = \bar{1}\ 3997 = \log (2\ 510 \times 10^{-1})$$

28 Let  $x = \frac{5^3 \times 19^3}{6^5}$  Then  $\log x = 3 \log 5 + 3 \log 19 - 5 \log 6$

$$= 0\ 6990 \times 3 + 1\ 2788 \times 3 - 0\ 7782 \times 5 = 0\ 7636 = \log 5\ 802$$

29 Let  $x = \frac{1}{(3\ 47)^3}$ , or  $(3\ 47)^{-3}$

$$\text{Then } \log x = -3 \log 3\ 47 = 0\ 5403 \times (-3) \\ = -(2\ 1612) = -3 + (3 - 2\ 1612) \quad [\text{See Art } 367] \\ = \bar{3}\ 8388 = \log (6\ 900 \times 10^{-3})$$

- 30 Let  $v$  = fraction Then, correcting denom<sup>r</sup> to four significant figures,

$$\begin{aligned}\log v &= \frac{1}{2}(\log 01367 + \log 0296 - \log 8735) \\ &= \frac{1}{2}(\bar{2}1358 + \bar{2}4713 - 29412) \\ &= \frac{1}{2}(\bar{7}6659) = \frac{1}{2}(\bar{8} + 16659) = \bar{4}8329 = \log(6.806 \times 10^{-4})\end{aligned}$$

- 31 Let  $v$  = fraction Then  $\log v = \frac{1}{2}(\log 678 + \log 901 - \log 0234)$   
 $= \frac{1}{2}(\bar{1}8312 + 9547 - \bar{2}3692) = \frac{1}{2}(24167) = 12084$   
 $= \log(1585)$ , giving 16 for answer

### EXAMPLES XXI e Page 435

- 1 From Art 376,  $\log A = \log P + n \log R$   
 $\log A = \log 370 + 25 \log 104 = 25682 + 25 \times 00170$   
 $= 29932 = \log 9845$ , giving Rs 985
- 2  $A = \text{Rs } 250 \times (1.05)^5 \times (1.025)^4$   
 $\log A = \log 250 + 5 \log 1.05 + 4 \log 1.025$   
 $= 23979 + 5 \times 00212 + 4 \times 00107$   
 $= 25467 = \log 3522$ , giving Rs. 352
- 3 From Art 376, it follows that  
 $\log P = \log A - n \log R = \log 3000 - 15 \log 1035 = 34771 - 15(00149)$   
 $= 32536 = \log 1793$ , giving £1793
- 4 From Art 376,  $n = \frac{\log A - \log P}{\log R} = \frac{\log 3000 - \log 1130}{\log 105}$   
 $= \frac{34771 - 30531}{0012} = \frac{04240}{0012} = 20 \text{ years}$
- 5 If  $n$  be the req<sup>d</sup> no of years, then, as in 4,  
 $n = \frac{\log 450 - \log 270}{\log 104} = \frac{26532 - 24314}{00170} = \frac{02218}{0017} = 13$ ,  
 son will be (8 + 13), or 21 years old
- 6 Let  $N$  be the req<sup>d</sup> population Then  $N = 4459000 \times (0.9477)^3$ ,  
 $\log N = \log 4459000 + 3 \log (0.9477) = 66493 + 3 \times \bar{1}9766$   
 $= 65791 = \log(3794000)$ , giving 3794000
- 7 If  $v$  be the mean proportional, then  $v^2 = 287 \times 3008$   
 $2 \log v = \log 287 + \log 3008 = 04579 + 14782 = 19361$ ,  
 $\log v = 09680$ , and  $v = 9290$
- If  $y$  be the third proportional, then  $y = \frac{(7805)^2}{00238}$ ,  
 and  $\log y = 2 \log 7805 - \log (00238)$   
 $= 17848 - \bar{2}3766 = 34082$ ,  
 $y = 2560$

$$8 \quad f = \frac{2s}{t^2} = \frac{2 \times 289 \cdot 3}{(31)^2} = \frac{578 \cdot 6 \times 8^2}{31^2},$$

$$\log f = \log 578 \cdot 6 + 2 \log 8 - 2 \log 31 = 2 \cdot 7624 + 1 \cdot 8062 - 2 \cdot 9828 \\ = 1 \cdot 5858, \text{ and } f = 38 \cdot 53$$

$$9 \quad \text{If } w \text{ kilograms be req'd weight, } w = 0 \cdot 00776 \times 540 \times 36 \times 22$$

$$\log w = \log(0 \cdot 00776) + \log 540 + \log 36 + \log 22 \\ = \bar{3} \cdot 8899 + 2 \cdot 7324 + 1 \cdot 5563 + 1 \cdot 3424 = 3 \cdot 5210 \\ w = 3319 \text{ Kg}$$

$$10 \quad \text{From formula we get}$$

$$\log t = \log \tau + \frac{1}{2}(\log l - \log g) = \log 3 \cdot 142 + \frac{1}{2}(\log 150 - \log 981) \\ = 0 \cdot 4972 + \frac{1}{2}(2 \cdot 1761 - 2 \cdot 9917) = 0 \cdot 4972 + \frac{1}{2}(\bar{1} \cdot 1844) \\ = 0 \cdot 4972 + \frac{1}{2}(\bar{2} + 1 \cdot 1844) = 0 \cdot 4972 + \bar{1} \cdot 5922 = 0 \cdot 0894, \\ t = 1 \cdot 228 = 1 \cdot 23 \text{ secs (nearly)}$$

$$11 \quad \text{We have } r^3 = \frac{3V}{4\pi}, \quad \log r = \frac{1}{3}(\log 3 + \log V - \log 4 - \log \pi)$$

$$\log r = \frac{1}{3}(\log 3 + \log 248 \cdot 6 - \log 4 - \log 3 \cdot 142) \\ = \frac{1}{3}(0 \cdot 4771 + 2 \cdot 3956 - 0 \cdot 6021 - 0 \cdot 4972) \\ = \frac{1}{3}(1 \cdot 7734) = 0 \cdot 5911, \\ r = 3 \cdot 9 \text{ cm}$$

$$12 \quad \text{As in No 11, } \log r = \frac{1}{3}(\log 3 + \log V - \log 4 - \log \pi)$$

$$\log r = \frac{1}{3}(\log 3 + \log(18 \cdot 2)^3 - \log 4 - \log \pi) \\ = \frac{1}{3}(0 \cdot 4771 + 3 \times 1 \cdot 2601 - 0 \cdot 6021 - 0 \cdot 4972) \\ = \frac{1}{3}(3 \cdot 1581) = 1 \cdot 0527, \\ r = 11 \cdot 29 \text{ cm and diam}^r = 22 \cdot 58 \text{ cm}$$

$$13 \quad \text{If } \tau \text{ be req'd value, then } \log x = \log m + 2 \log v - \log 2$$

$$\log x = \log 9 \cdot 17 + 2 \log 17 \cdot 64 - \log 2 \\ = 0 \cdot 9624 + 2 \times 1 \cdot 2465 - 0 \cdot 3010 \\ = 3 \cdot 1544, \text{ and } x = 1427$$

$$14 \quad \text{We have } \log F = \log m + 2 \log v - \log g - \log r$$

$$= \log 24 \cdot 7 + 2 \log 60 - \log 32 \cdot 19 - \log 8 \cdot 4 \\ = 1 \cdot 3927 + 2 \times 1 \cdot 7782 - 1 \cdot 5077 - 0 \cdot 9243 \\ = 2 \cdot 5171 \quad F = 329$$

$$15 \quad \text{We have } 2 \log v = \log g + \log t - \log 289$$

$$= \log \frac{32 \cdot 2}{5 \cdot 2880} + \log 4000 - \log 289 \\ = \log 32 \cdot 2 - \log 5280 + \log 4000 - \log 289 \\ = 1 \cdot 5079 - 3 \cdot 7226 + 3 \cdot 6021 - 2 \cdot 4609 = \bar{2} \cdot 9265, \\ \log v = \frac{1}{2}(\bar{2} \cdot 9265) = \bar{1} \cdot 4632 \text{ and } v = 0 \cdot 2905$$

If  $v = \frac{2\pi r}{V \times 60 \times 60}$ ,  $\log v = \log 2 + \log \pi + \log r - \log v - 2 \log 60$ ,  
 or  $\log v = \log 2 + \log 3.142 + \log 4000 - \log(0.2905) - 2 \log 60$   
 $= 0.3010 + 0.4972 + 3.6021 - \bar{1}.4632 - 3.5564$   
 $= 1.3807$  Hence  $x = 24$  (nearly)

- 16 Let  $W$  lbs be the req<sup>d</sup> weight of water

Then  $W = 62.3 \text{ lbs} \times (\frac{1}{5} \text{ of vol of roller in cu ft})$

$$= 62.3 \times \frac{4}{5} \times \frac{\pi r^2 l}{1728}$$

$$\begin{aligned} \log W &= \log 62.3 + \log 4 + \log \pi + 2 \log r + \log l - \log 1728 - \log 5 \\ &= \log 62.3 + \log 4 + \log 3.142 + 2 \log 13 + \log 36 \\ &\quad - \log 1728 - \log 5 \\ &= 1.7945 + 0.6021 + 0.4972 + 2.2278 + 1.5563 \\ &\quad - 3.2375 - 0.6990 \end{aligned}$$

$$= \left\{ \begin{array}{r} 1.7945 \\ 0.6021 \\ 0.4972 \\ 2.2278 \\ 1.5563 \\ \hline 6.6779 \end{array} \right\} - \left\{ \begin{array}{r} 3.2375 \\ 0.6990 \\ \hline 3.9365 \end{array} \right\} = 2.7414$$

$$W = 551 \text{ lbs}$$

- 17 Let  $h$  cm be the req<sup>d</sup> height Then  $\frac{h^3}{(14.2)^3} = \frac{1}{2}$  (See Ait 284, III)

$$\begin{aligned} \log h &= \frac{1}{3}(3 \log 14.2 - \log 2) = \frac{1}{3}(3.4569 - 0.3010) \\ &= \frac{1}{3}(3.1559) = 1.0520, \quad h = 11.3 \text{ cm} \end{aligned}$$

- 18 Let  $h$  cm be the req<sup>d</sup> height

For the 3 dl pot we have  $\frac{h^3}{(12.6)^3} = \frac{(1.76 \times 0.3) \text{ pints}}{1 \text{ pint}}$ , and

$$\begin{aligned} 3 \log h &= 3 \log 12.6 + \log 1.76 + \log 0.3 = 3.3012 + 0.2455 + \bar{1}.4771 \\ &= 3.0238, \quad \log h = 1.0079, \text{ and } h = 10.2 \text{ cm} \end{aligned}$$

For the 5 dl pot we shall have

$$\begin{aligned} 3 \log h &= 3 \log 12.6 + \log 1.76 + \log 0.5 = 3.3012 + 0.2455 + \bar{1}.6990 \\ &= 3.2457, \quad \log h = 1.0819, \text{ and } h = 12.1 \text{ cm} \end{aligned}$$

For the 1 lit pot we shall have

$$\begin{aligned} 3 \log h &= 3 \log 12.6 + \log 1.76 + \log 1 = 3.3012 + 0.2455 \\ &= 3.5467, \quad \log h = 1.1822, \text{ and } h = 15.2 \text{ cm} \end{aligned}$$

- 19 We have from Ait 376,  $n = \frac{\log A - \log P}{\log R}$ ,

$$n = \frac{\log 100 - \log 50.38}{\log 1.025} = \frac{2 - 1.7023}{0.0107} = \frac{0.2977}{0.0107} = 28 \text{ yrs.}$$

20 We have  $d^2 = \frac{V}{0.7854h}$ , where  $V$  is volume of cylinder

$$\begin{aligned} 2 \log d &= \log V - \log(0.7854) - \log h \\ &= \log \frac{2 \times 6}{136} - \log(0.7854) - \log 153 \\ &= \log 256 - \log 136 - \log(0.7854) - \log 153 \\ &= 1.4082 - 1.1335 - 1.8951 - 1.1847 \\ &= \bar{1}.1949, \quad \log d = \bar{1}.5974, \text{ and } d = 0.3958 \text{ cm} \end{aligned}$$

21 From formula in No 20,

$$\begin{aligned} \log h &= \log V - \log(0.7854) - 2 \log d \\ &= \log \frac{150 \times 10^3}{888} - \log(0.7854) - 2 \log(0.3) \\ &= \log(450 \times 10^3) - \log 888 - \log(0.7854) - 2 \log(0.3) \\ &= 5.6532 - 0.9484 - 1.8951 - 1.0512 \\ &= 7.8555, \quad h = 71690000 \text{ cm} = 716900 \text{ m} \end{aligned}$$

22 Let  $v = \sqrt{(3d)^2 \times H - L}$        $\log v = \frac{1}{2}(5 \log(3d) + \log H - \log L)$ ,

$$\begin{aligned} \text{or } \log v &= \frac{1}{2}\{5(\log 3 + \log d) + \log H - \log L\} \\ &= \frac{1}{2}\{5(\log 3 + 1.25) + \log 38 - \log 1760\} \\ &= \frac{1}{2}\{5 \times (0.4771 + 0.6284) + 1.5798 - 3.2455\} \\ &= \frac{1}{2}\{5.5275 + 1.5798 - 3.2455\} \\ &= \frac{1}{2}(3.8618) = 1.9309, \\ v &= 85.29 \text{ gallons} \end{aligned}$$

23 Let  $r$  in be req<sup>d</sup> pressure. Then  $r = \left(\frac{1}{147}\right)^{16} \times 30$  in first case

$$\begin{aligned} \log r &= 15(\log 130 - \log 147) + \log 30 \\ &= 15(2.1139 - 2.1673) + 1.4771 \\ &= 15 \times (\bar{1}.9466) + 1.4771 = \bar{1}.1990 + 1.4771 = 0.6761 \\ r &= 4.743 \text{ in} \end{aligned}$$

In second case we get

$$\begin{aligned} \log r &= 50 \times \bar{1}.9466 + 1.4771 = \bar{3}.3300 + 1.4771 = \bar{2}.8071, \\ r &= 0.06113 \end{aligned}$$

24 If  $x$  sq ft be req<sup>d</sup> area, then  $x = \frac{4 \times 1.2}{3} \sqrt{(0.626 \times 12)^2 + 16^2}$

$$\begin{aligned} \text{i.e. } x &= 16 \sqrt{(0.626)^2 \times 3^2 \times 4^2 + 16^2} = 16 \times 4 \sqrt{(0.626)^2 \times 3^2 + 4^2} \\ &= 16 \times 4 \times 2 \sqrt{(0.313)^2 \times 3^2 + 2^2} = 128 \sqrt{4.882} \\ \log x &= \log 128 + \frac{1}{2} \log(4.882) = 2.1072 + \frac{1}{2}(0.6886) \\ &= 2.1072 + 0.3443 = 2.4515, \\ x &= 280 \text{ sq ft} \end{aligned}$$

Most of the graphical examples in this set depend on the methods of Arts 377 and 378. In many cases a reference to a diagram will be all the solution necessary.

- 1 Mark off  $OY = 60'$  to represent 60 miles. Through  $Y$  draw  $YZ$  parallel to  $OX$ .  
Join  $O$  to the point whose abscissa along  $OX$  represents 1 hour, and whose ordinate measured up from  $OX$  is 12 miles. The ordinate of any point on this line will measure the distance travelled towards  $Y$  by the first rider.  
Join  $Y$  to the point whose abscissa along  $YZ$  represents 1 hour, and whose ordinate measured downwards from  $YZ$  represents 9 miles. The resulting line will show the distance travelled towards  $O$  at the end of any time. (For this line all ordinates must be drawn to  $YZ$ .)  
The two lines cut at the point whose abscissa is  $2.86''$ , which on the given scale represents 2 hrs 52 min.  
The vertical distance between the lines is  $1.8'$  for the abscissa  $2.0'$ , i.e. the riders are 18 miles apart after 2 hrs.
- 2 Take the scale as in Ex 1. The graph representing  $A$ 's motion is the line joining the origin to the point  $(1.0', 0.6')$ .  $B$ 's graph is found by joining  $(1.5', 0)$  to  $(2.5', 0.8')$ . These two lines meet at the point  $(6.0'', 3.6')$ . Hence the time of meeting is 6.0 p.m.  
Measuring the difference between the ordinates of corresponding points on the two lines, we find that  $A$ 's graph is  $0.5'$  above  $B$ 's for the abscissa marking 3.30 p.m., and that therefore  $A$  is 5 miles ahead of  $B$  at that time. Similarly  $B$  is 3 miles ahead at 7.30 p.m.
- 3 A convenient scale is 25 miles to the inch vertically, and 1 hour to the inch horizontally. The graphs are shown on half this scale in Fig 17. The answers are given by the coordinates of  $P$  and the abscissa of  $LM$ ,  $HK$ .

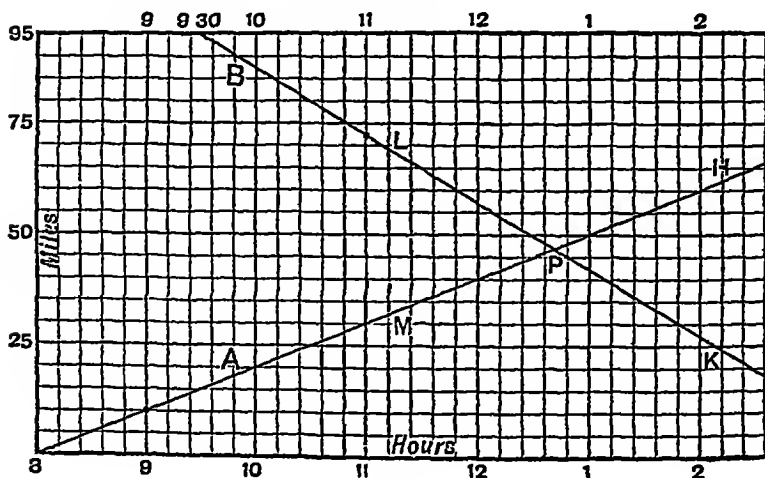


FIG 17

4. Let Rs  $y$  represent the salary after  $x$  years, Rs  $a$ , Rs  $b$  the annual increase and the initial salary respectively

Then we have the relation  $y = ax + b$ , which is the equation of a straight line. A scale of Rs 800 to the inch vertically, and 10 years to the inch horizontally will be found convenient. The line can then be drawn by joining the points (6, 1280) and (15, 2000). The initial salary is given by the intercept on OY, and the salary for the 21st year is given by the ordinate corresponding to an abscissa 20.

$$5 \quad CP + \frac{1}{100} \text{ of } CP = 16s \ 4d, \quad CP = \frac{100}{101} \text{ of } 16s \ 4d = 14s \ 7d$$

$$\text{Profit on each article} = £10 \ 18s \ 9d - 100 = 2s \ 2\frac{1}{2}d$$

$$\text{Selling Price} = 14s \ 7d + 2s \ 2\frac{1}{2}d = 16s \ 9\frac{1}{2}d$$

6. Reckoning in thousands, the total no. of gallons supplied  
 $= 165 \times 610 + 179 \times 730 + 153 \times 520 = 10065 + 13067 + 7956 = 31088$

$$\text{The total population} = 610 + 730 + 520 = 1860,$$

$$\text{average supply per head} = \frac{31088}{1860} = 16 \frac{7}{8}$$

7. Suppose he holds  $£x$  stock, his income from this is  $£\frac{21}{100}$  of  $x$ ,

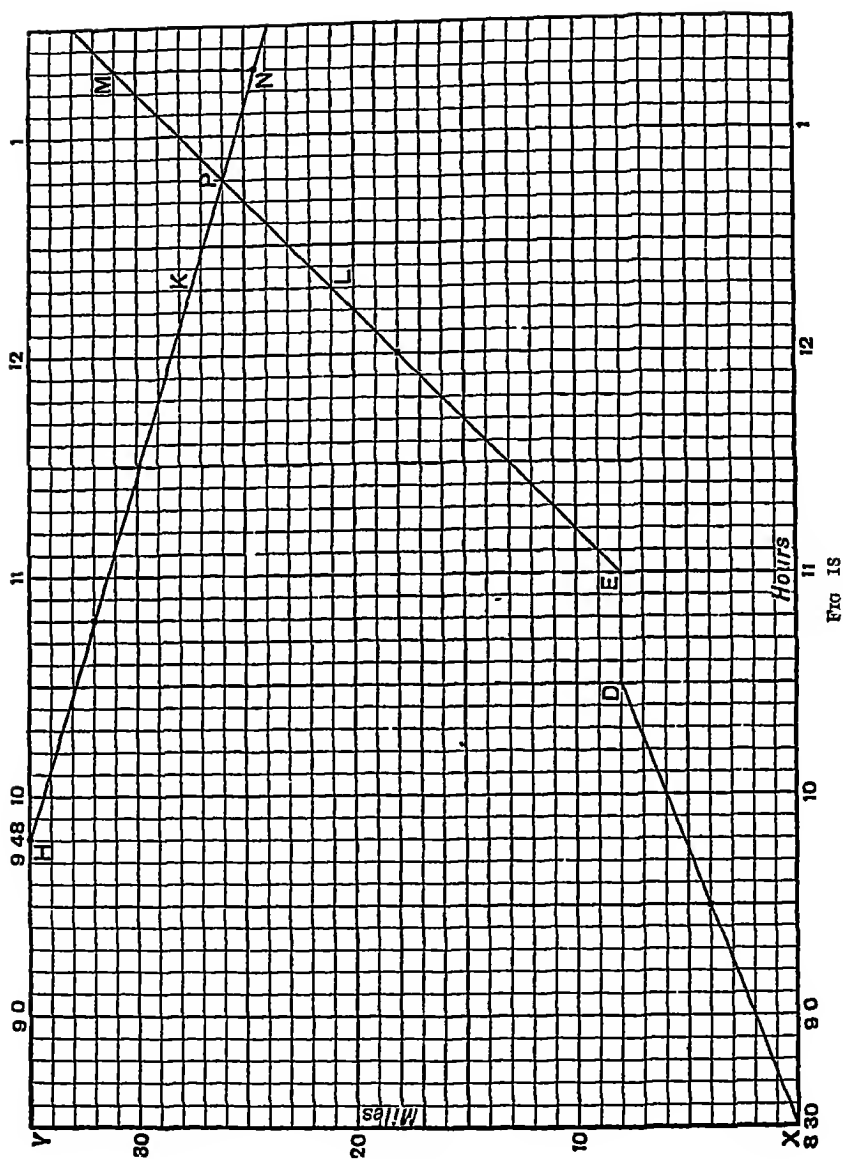
$$\text{and after income-tax has been deducted, it} = £\frac{19}{20} \text{ of } \frac{21}{100} \text{ of } x,$$

$$\frac{19}{20} \text{ of } \frac{21}{100} \text{ of } x = 448\frac{8}{10}, \text{ and } x = \frac{8908 \times 200 \times 20}{20 \times 5 \times 19} = £18880$$

8.  $A$  runs  $3\frac{1}{2} \times 9$  ft while  $B$  runs  $4\frac{1}{2} \times 7$  ft,  
 or  $A$  „  $14 \times 9$  ft „  $B$  „  $17 \times 7$  ft,  
 or  $A$  „ 126 ft „  $B$  „ 119 ft,  
 $A$  catches  $B$  in 126 ft when  $B$  has a start of 7 ft,  
 or  $A$  „  $B$  „ 18 ft „  $B$  „ „ 1 ft,  
 or  $A$  „  $B$  „  $18 \times 25$  yds „  $B$  „ „ 25 yds  
 $A$  must run  $18 \times 25$  yds, or 450 yds

9. The graphs are shewn in Fig 18.  $A$ 's graph is the broken line XDEM, his rest of half an-hour being shewn by the horizontal line DE.  $B$ 's graph is the line HKN. The solutions are given by the abscissæ of P and of the points where KL, MN meet the hour-axis.





- 10 See Fig 19.  $OY$  is of any convenient length and represents the time taken by  $A$  to run 120 yards. In the same time  $B$  and  $C$  would run 100 yards and 80 yards respectively. Hence the graphs of the three runners are found by joining  $O$  to the points marked  $A, B, C$  in the figure. Find the point  $Q$  on  $OB$  which corresponds to an abscissa 80 yards, and draw a horizontal cutting  $OC, OA$  in  $R$  and  $P$ . Then the distances run by  $C$  and  $A$  when  $B$  has gone 80 yards are given by the abscissae of  $R$  and  $P$ .

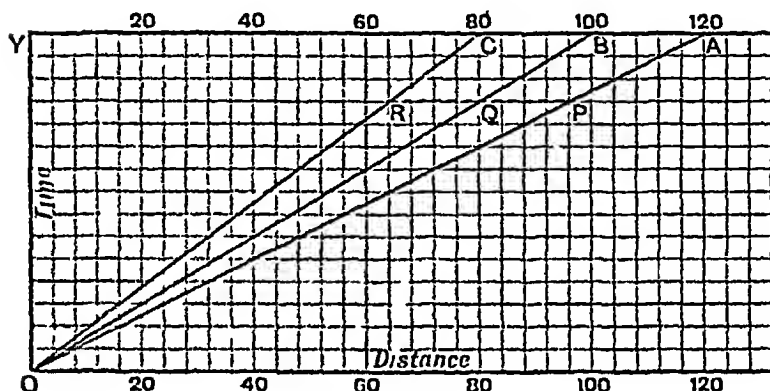


FIG 19

11. Fig 20 is drawn on a reduced scale

$A$ 's graph is the line  $OP$

Since  $B$  has a start of 8 yds.  $R$  is a point on his graph

Also  $B$  beats  $A$  by 1 second,  $S$  is a point on his graph

Thus  $B$ 's graph is the line  $RS$

$A$  beats  $C$  by 40 yds.,  $Q$  is a point on  $C$ 's graph

Find  $T$  the point on  $B$ 's graph corresponding to 15 seconds, and measure  $TK$  downwards to represent 18 yds. Then  $K$  is also a point on  $C$ 's graph

Thus  $C$ 's graph is the line  $QK$ . When produced this meets the time axis at  $L$ . Then, since  $OL$  represents 5 seconds,  $C$  must have started 5 seconds after  $A$ .

As the graphs are three *parallel* lines, the speeds of the runners, which are measured by the slopes of the lines, are equal and the race would end in a dead heat.

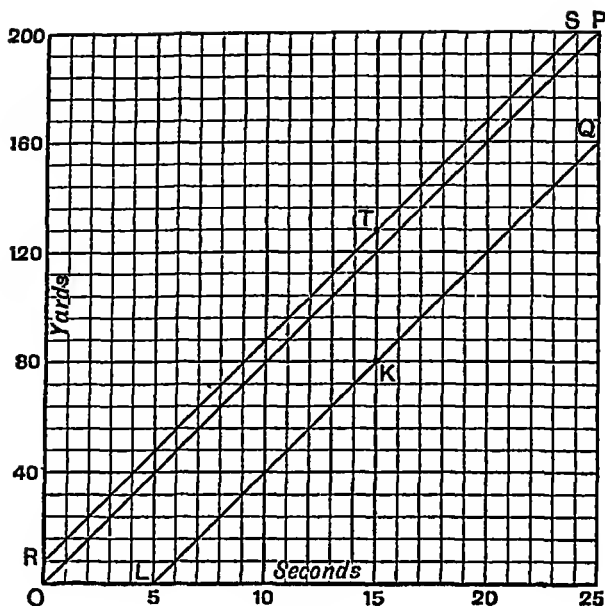


FIG 20

- 12 Let  $d$  ft be the req<sup>d</sup> depth

$$\text{cu ft of water in cistern} = 5 \times 4 \times d,$$

$$\text{cu ft of water on roof} = 20 \times 12 \times \frac{2}{12}$$

$$5 \times 4 \times d = 20 \times 12 \times \frac{2}{12}, \text{ and } d = 2$$

- 13 fr 25 105 = £1,  
fr 430 50 = £ $\frac{43050}{25105}$

$$\begin{array}{r} 17\ 148 \\ 25103 \overline{) 43050} \\ \underline{179450} \\ 3715 \\ \underline{1204} \\ 200 \end{array}$$

- 14 The S I for 3 yrs at 5% =  $\frac{5}{100}$  of Rs  $401\frac{5}{8} \times 3 = \text{Rs } 60\ 4a$

The amount at C I for 3 yrs at 5%

$$= \text{Rs } 410\frac{5}{8} \times (1.05)^3 = \text{Rs } \frac{3213}{8} \times 1.157625$$

$$= \text{Rs } 3213 \times 0.144703125 = \text{Rs } 464\ 931$$

$$= \text{Rs } 464\ 15a$$

$$\begin{array}{r} 0\ 144703125 \\ 3213 \\ \hline 431\ 109\ 4 \\ 28\ 940\ 6 \\ 1\ 447\ 0 \\ 434\ 1 \\ \hline 464\ 931\ 1 \end{array}$$

$$\text{C I} = \text{Rs } 464\ 15a - \text{Rs } 401\ 10a = \text{Rs } 63\ 5a$$

$$\text{req<sup>d</sup> diff} = \text{Rs } 63\ 5a - \text{Rs } 60\ 4a = \text{Rs } 3\ 1a$$

15 Space left uncovered =  $(30 \times 28 - 27\frac{1}{2} \times 25\frac{1}{2})$  sq ft =  $\frac{555}{4}$  sq ft

This would cost Rs  $420 \times \frac{\frac{555}{4}}{30 \times 28}$ , or Rs  $69\frac{3}{8}$ , to carpet

Rs  $69\frac{3}{8}$ , or Rs 69 6a, is saved

16 1 lb av = 7000 gr of pure gold

Each £ contains  $\frac{1}{12}$  of 123 gr of pure gold If  $x$  be req<sup>d</sup> number,

we have  $\frac{1}{12}$  of  $123 \times x = 7000$ , whence  $x = \frac{84000}{123} = 62\frac{114}{123}$

Thus 62 sovereigns can be coined

- 17 (Fig 21)  $A$ 's graph is found by joining  $X$  to  $P$  Find the point  $R$  on this line which corresponds to 4 p.m. Then  $R$  is a point on  $B$ 's graph Thus  $B$ 's graph is the line joining  $Q$  and  $R$  The solutions can now be read off from the figure

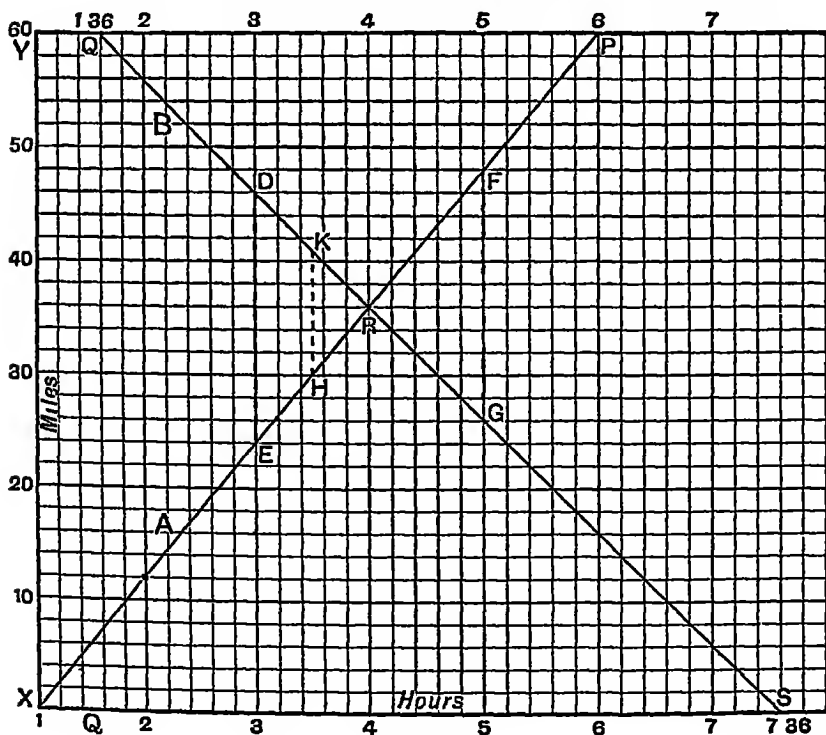


FIG 21

- 18 Since one goes 8 miles while the other goes 4 miles, the speed of the first is twice that of the second. Let their speeds be  $2a$  miles and  $a$  miles per hour respectively, and let them meet  $x$  miles from B.

The first goes  $(12+x)$  miles in  $\frac{12+x}{2a}$  hours, the second goes

$$(12-x) \text{ miles in } \frac{12-x}{a} \text{ hours, } \quad \frac{12+x}{2a} = \frac{12-x}{a}$$

Multiply by  $2a$ , then  $12+x=2(12-x)$ , and  $x=4$

For graphical verification, take a scale of 2 miles to the inch vertically and suppose A at the origin. The rates are as 2:1. Take as the unit of time that interval during which the faster goes 8 miles, the slower 4 miles. Represent this unit by 4 in. on the  $x$  axis, and along it mark 4 in., 8 in., 12 in., from the origin, 1, 2, 3 respectively.

The graph of the faster will then be obtained by joining the point (1, 8) to the origin. It passes through the point (1.5, 12). Now join the last point to the point (3, 0), and we clearly obtain the graph for the return journey of the faster traveller. Similarly the graph of the slower traveller is the line joining the point (1, 4) to the origin. It cuts the 'return' graph of the faster at the point (2, 8), shewing the men meet 8 miles from A i.e. 4 miles from B.

- 19 See Fig. 22. A scale of 30 miles to the inch vertically, and 1 hr. to the inch horizontally will be found more convenient than the one here employed.

The beginning and end of his journey, being at different speeds, will be represented by different graphs. These are obtained by joining A to P and B to Q. The abscissa of the point of intersection of AP and BQ will give the required time.

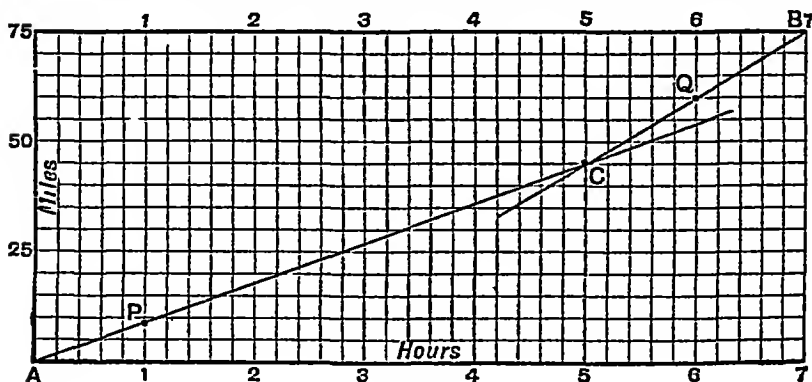


FIG. 22

$$20 \quad \text{Total increase} = 185000 \times \frac{0.482}{100} + 235000 \times \frac{0.724}{100} + 325000 \times \frac{0.516}{100} \\ = 8917 + 7614 + 1677 = 33301$$

If  $x\%$  be the average increase,

$$\text{the total increase} = \frac{x}{100} \text{ of } (185000 + 235000 + 325000)$$

$$= x(1850 + 2350 + 3250) = 7450x,$$

$$7450x = 33301 \text{ and } x = 0.447\%$$

$$21 \quad \text{The total receipts for the first four months} = \text{£}1775 \text{ 1s}$$

The receipts for the remaining eight months must be

$$\text{£}5000 - \text{£}1555 \text{ 1s, or } \text{£}3444 \text{ 19s},$$

$$\text{reqd average takings} = \frac{\text{£}3444 \text{ 19s}}{8} = \text{£}430 \text{ 12s } 4\frac{1}{2}d$$

$$22 \quad \text{Since } 7931520 = 765 \times 2^7 \times 3^4, \text{ it follows that the only pairs are}$$

$$\{765, 765 \cdot 2^7 \cdot 3^4\} \text{ and } \{765 \cdot 2^7, 765 \cdot 3^4\},$$

because, when we divide the numbers forming each pair by 765, the quotients are prime to each other. For if they were not, their H.C.F. would not be 765. This may be seen in the case of such a pair as  $765 \cdot 2^3 \cdot 3^2$  and  $765 \cdot 2^7 \cdot 3^4$ , the H.C.F. is not 765 but  $765 \cdot 2^3 \cdot 3^2$ . Again taking another pair,  $765$  and  $765 \times 2^7$ , their L.C.M. is  $765 \times 2^7$ , or 6120, not 7931520. In the same way any other case may be dealt with.

$$23 \quad \text{Let the sides be } 2x \text{ ft and } 3x \text{ ft} \quad \text{Area} = 6x^2 \text{ sq ft}$$

$$\text{and cost} = \text{Rs } \frac{4}{10 \times 10} \times 6x^2, \quad \frac{4}{10 \times 10} \times 6x^2 = 24,$$

$$\text{and } x^2 = 144, \text{ or } x = 12,$$

hence lengths are 24 ft and 36 ft respectively

$$24. \quad \text{The company pays out } \frac{41}{100} \text{ of Rs } 20000000 \text{ as interest on mortgage.}$$

If Rs  $x$  be the gross annual receipts, then expenses are  $\frac{40}{100}$ ,

$$\text{then net annual receipts are } x - \frac{40}{100}x = \frac{41}{100} \text{ of Rs } 20000000$$

this must be equal to the interest on the shareholders' capital,  
viz 5 % of Rs 40000000

$$x - \frac{40x}{100} - \frac{4x}{100} \text{ of Rs } 20000000 = \frac{5}{100} \text{ of Rs } 40000000,$$

$$\text{or } \frac{60x}{100} - 900000 = 2000000, \text{ and } x = \text{Rs } \frac{29000000}{6}$$

the average weekly receipts

$$= \text{Rs } \frac{20000000}{6 \times 52}$$

$$= \text{Rs } \frac{20000000}{312}$$

$$= \text{Rs } 92948 \text{ } 11 \text{ a}$$

$$\begin{array}{r} 92948 \text{ } 72 \\ 312 \overline{) 29000000} \\ \underline{920} \\ 2960 \\ \underline{1520} \\ 2720 \\ \underline{2240} \\ 560 \end{array}$$

25 18 eggs cost 12a, or 12 eggs cost 8a, but 12 eggs are sold for  $10\frac{1}{2}a$ ,  
gain is  $2\frac{1}{2}a$  on an outlay of 8a, giving  $\frac{2\frac{1}{2}}{8} \times 100$ , or  $31\frac{1}{4} \%$

26 £17 5s 7d = £17 279|1666 = value of 1 ton  
40

	£691 166 6	= value of 40 tons
2 cwt = $\frac{1}{10}$ of 1 ton	1 727 9	= " 2 cwt
8 lbs = $\frac{1}{14}$ of 1 cwt	061 7	= " 8 lbs
4 lbs = $\frac{1}{2}$ of 8 lbs	030 9	= " 4 lbs
	£692 987 1	= value of 40 tons 2 cwt 12 lbs

Giving £692 19s 9d

27 Let the weights of unit volumes be 4s, 5s and 6s ounces respectively, and let the substances contain 3v, 4v and 5v units of volume respectively. Then the respective weights are 4s × 3v, 5s × 4v and 6s × 5v ounces, or 12sv, 20sv and 30sv ounces. Now 5 lbs 13 oz = 93 oz

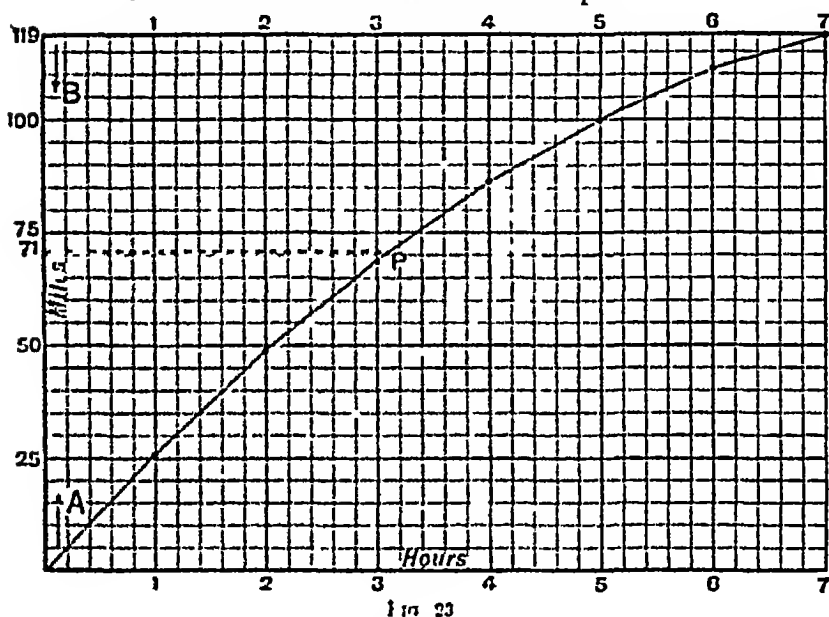
$$12sv + 20sv + 30sv = 93, \text{ and } sv = \frac{3}{2} \text{ oz},$$

$$\text{hence } 12sv = 18 \text{ oz}, 20sv = 30 \text{ oz}, 30sv = 45 \text{ oz}$$

Thus the weights are 1 lb 2 oz, etc

- 28 (Fig 23) Take 25 miles to the inch vertically, and 1 hour to the inch horizontally. The figure shown is drawn on half these scales.

Measure off AB along the vertical axis to represent 119 miles and draw a horizontal through B. Plot the points whose ordinates represent (1 hr, 26 miles), (2 hrs, 49 miles), (3 hrs, 69 miles) etc, and join each successive point as in the diagram. The horizontal through B is seen to be cut at a point on the '7' ordinate. Hence B is reached at 7 p.m.



The horizontal through the point whose ordinate represents 48 miles from B (or 71 miles from A) cuts the graph at P, whose abscissa represent 3.7 p.m. approximately.

- 29 Increase of acreage between 1895 and 1900 = 103770 ac., and this corresponds to 480000000 lbs

1 ac. contains  $\frac{480000000}{103770}$  lbs,  
or 218 lbs

For second part of question,  
decrease of acreage = 1197 ac

decrease of crop =  $\frac{480000000}{103770} \times 1197$  lbs  
= 296500 lbs nearly

$$\begin{array}{r}
 217.7 \\
 19,377,0 \overline{) 48000000} \\
 \underline{9216} \\
 1196 \\
 \underline{110} \\
 1197 \times 18 = 57156 \\
 \underline{296517} \\
 19377 \overline{) 5715600000} \\
 \underline{187020} \\
 126270 \\
 \underline{10008} \\
 208 \\
 \underline{16}
 \end{array}$$



- 30 (Fig 24) The graph for the X to Y train must be obtained by joining the points whose coordinates, measured from X are (0, 2 33) and (35, 3 33) For at 2 33 p m the train is still at X, and at 3 33 p m has travelled 35 miles

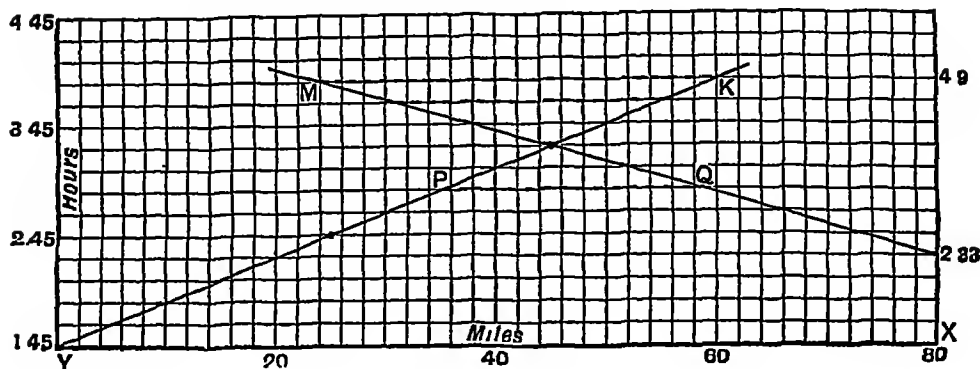


Fig 24

- 31 1 bigha is rented at Rs 600

Now 1 bigha =  $(1600 \times 9)$  sq ft

hence  $(1600 \times 9)$  sq ft are rented at Rs 600

$(34 \times 125)$  sq ft are rented at Rs  $\frac{600 \times 34 \times 125}{1600 \times 9}$ , or Rs  $\frac{2125}{12}$ ,  
or Rs 177 083, giving for answer Rs 177 1a 3p

- 32 Let  $x$  rupees be the marked price He sells at  $\frac{90}{100}x$  rupees  
But he gains 15% on an outlay of Rs 72,

he sells at Rs  $72 \times \frac{115}{100}$ ,

hence  $\frac{90}{100} = 72 \times \frac{115}{100}$ , and  $x = \text{Rs } 92$

- 33 Since A's wages are greater than B's for the same time, we must

multiply £11 10s by  $\frac{\text{£}1 \text{ } 16\text{s } 8\text{d}}{\text{£}1 \text{ } 8\text{s } 9\text{d}}$ , or  $\frac{440\text{d}}{345\text{d}}$

Again, since A takes less time than B for the same piece of work,  
we must multiply £11 10s by  $\frac{2\frac{1}{2}}{3\frac{1}{3}}$  (See Art 279)

$$\text{reqd charge} = \text{£}11\frac{1}{2} \times \frac{440}{345} \times \frac{2\frac{1}{2}}{3\frac{1}{3}} = \text{£}11$$

- 34 For first period total crop  
 $= (13\ 224 \times 15\ 2)$  million bushels  
 $= 201$  million bushels

For second period total crop  
 $= (17\ 198 \times 18)$  million bushels  
 $= 310$  million bushels

13	224
1	5 2
132	2
66	1
2	6
200	9
17	198
1	8
172	0
137	5
309	5

For first period, amount of wheat per head

$$= \frac{201}{114} \text{ bushels} = 6 \text{ bushels}$$

For second period, amount of wheat per head

$$= \frac{310}{38} \text{ bushels} = 8 \text{ bushels}$$

- 35 If the total expenses for  $x$  boys are represented by  $\pounds y$ , the variable part may be denoted by  $\pounds ax$ , and the constant part by  $\pounds b$ . Hence  $x$  and  $y$  satisfy a linear equation  $y = ax + b$ , where  $a$  and  $b$  are constant quantities. Hence the graph is a straight line. (See Art. 378.)

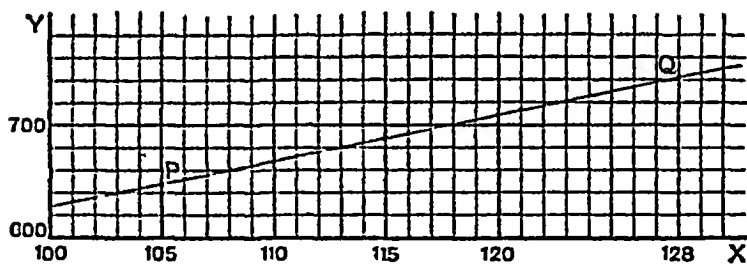


FIG. 25

As the numbers are large, it will be convenient if we begin measuring ordinates at 600, and abscissae at 100. This enables us to bring the requisite portion of the graph into a smaller compass. When  $x = 105$ ,  $y = 650$ , and when  $x = 128$ ,  $y = 742$ . Thus two points P and Q are found, and the line PQ is the required graph.

By measurement we find that when  $x = 115$ ,  $y = 690$ , and that when  $y = 710$ ,  $x = 120$ . Thus the required answers are £690, and 120 boys.

- 36 As in Ex. 35 the graph must be a straight line. Hence we need only join the points (12, 3840) and (16, 4320) and then read off the ordinate corresponding to an abscissa 15

A convenient scale is that of 5 patients to the inch horizontally and Rs 200 to the inch vertically. The vertical graduation may commence at Rs 3500

- 37 Let the respective populations be  $3x$  and  $8x$ , then the total population is  $11x$ ,

the total number of males is  $\frac{42}{100} \times 3x + \frac{46}{100} \times 8x$  in  $11x$ ,

$$\text{or } \frac{494x}{100} \text{ in } 11x, \text{ or } \frac{494x}{100} \times \frac{100}{11x} \text{ in } 100, \text{ or } \frac{494}{11} \text{ in } 100, \text{ giving } 45\%$$

- 38 Internal edge = 1 m = 100 cm, external edge = 102 cm,

$$\text{volume of metal} = (102^3 - 100^3) \text{ cu cm} = 61208 \text{ cu cm}$$

$$\text{weight of metal} = 61208 \text{ gm} \times 7 = 428456 \text{ Kg}$$

- 39 Error =  $\pounds\left(\frac{567}{980} - \frac{567}{1000}\right)$  in  $\pounds\frac{567}{980} = \pounds\left(\frac{1}{980} - \frac{1}{1000}\right)$  in  $\pounds\frac{1}{980}$   
 $= \pounds(100 - 96)$  in  $\pounds 100$ , or 4%

$$\text{Again error} = \frac{4}{100} \text{ of } 567 \text{ f} = 4 \times 5.67 \text{ f} = 22.68 \text{ f, or } 23 \text{ f}$$

- 40 After the first replacement,

the first cask contains $\left\{ \begin{array}{l} 48 \text{ gals of wine} \\ 12 \text{ gals of water} \end{array} \right\}$ , i.e. $\left\{ \begin{array}{l} \frac{4}{5} \text{ of cask is wine} \\ \frac{1}{5} \text{ of cask is water} \end{array} \right\}$	the second cask contains $\left\{ \begin{array}{l} 12 \text{ gals of wine} \\ 18 \text{ gals of water} \end{array} \right\}$ , i.e. $\left\{ \begin{array}{l} \frac{2}{5} \text{ of cask is wine} \\ \frac{3}{5} \text{ of cask is water} \end{array} \right\}$
--	---

The 12 gallons then drawn,

from the first cask contain $\frac{4}{5}$ of 12 gals of wine	from the second cask contain $\frac{2}{5}$ of 12 gals of wine
---	--

The amount left,

in the first cask is 48 gals of wine and water, of which $\frac{4}{5}$ of 48 gals is wine	in the second cask is 18 gals of wine and water, of which $\frac{2}{5}$ of 18 gals is wine
---	--

after the second replacement,

the first cask contains $(\frac{1}{6}$ of $48 + \frac{2}{3}$ of $12$ ) gals of wine, or $\left\{ 43\frac{1}{6}$ gals of wine $16\frac{4}{6}$ gals of water $\right\}$ , $\frac{43\frac{1}{6}}{60}$ or $\frac{1\frac{2}{3}}{5}$ of cask is wine $\frac{16\frac{4}{6}}{60}$ or $\frac{7}{15}$ of cask is water	the second cask contains $(\frac{2}{3}$ of $18 + \frac{1}{6}$ of $12$ ) gals of wine, or $\left\{ 16\frac{4}{6}$ gals of wine $13\frac{1}{6}$ gals of water $\right\}$ , $\frac{16\frac{4}{6}}{30}$ or $\frac{1\frac{4}{6}}{3}$ of cask is wine $\frac{13\frac{1}{6}}{30}$ or $\frac{1\frac{1}{6}}{5}$ of cask is water
---	--

The 12 gallons then drawn,

from the first cask contain $\frac{1\frac{2}{3}}{6}$ of 12 gals of wine	from the second cask contain $\frac{1\frac{4}{6}}{6}$ of 12 gals of wine
--	---

The amount left,

in the first cask is 48 gals of wine and water, of which $\frac{1\frac{2}{3}}{6}$ of 48 gals is wine	in the second cask is 18 gals of wine and water, of which $\frac{1\frac{4}{6}}{6}$ of 18 gals is wine
--	---

after the third replacement,

the first cask contains $(\frac{1\frac{2}{3}}{6}$ of $48 + \frac{1\frac{4}{6}}{6}$ of $12$ ) gals of wine, or 41 28 gals of wine	the second cask contains $(\frac{1\frac{4}{6}}{6}$ of $18 + \frac{1\frac{2}{3}}{6}$ of $12$ ) gals of wine, or 18 72 gals of wine
--	---

41 3 men and 4 boys do  $\frac{1}{8}$  of the work in 1 day,

but 3 men „ 1 boy „  $\frac{1}{8}$  „ „ 1 day,

3 boys do  $(\frac{1}{8} - \frac{1}{8})$ , or  $\frac{1}{16}$  „ „ 1 day,

and 8 boys „  $\frac{1}{16} \times \frac{8}{3}$ , or  $\frac{1}{6}$  „ „ 1 day

But 4 women and 8 boys do  $\frac{1}{6}$  „ „ 1 day,

4 women do  $(\frac{1}{6} - \frac{1}{6})$ , or  $\frac{4}{48}$  „ „ 1 day,

and 1 woman does  $\frac{1}{48}$  „ „ 1 day,

giving 15 days for req<sup>d</sup> time



In the same way the third instalment amounts to Rs 97240 5(1 05)  
at the end of the time

With the last instalment, his payments amount to

Rs 97240 5(1 05<sup>3</sup> + 1 05<sup>2</sup> + 1 05 + 1),  
or Rs 97240 5(4 310125),  
or Rs 419118 7 as above

97240	5
388962	4 310125
29172	2
972	4
9	7
1	9
	5
419118	7

$$\begin{aligned}
 47 \quad 1 \text{ Nap} &= \frac{0.180}{.1} \text{ Kg} = \frac{0.180}{31} \times \frac{r}{.268} \text{ lbs} = \frac{0.180 \times r}{31 \times .268} \times 7000 \text{ grs} \\
 &= \pounds \frac{0.180 \times r \times .000}{31 \times .268} \times \frac{.0211}{.70410}, \\
 &= \pounds \left( \frac{.6-31}{31} \right) \times \left( \frac{0.180}{.268} \right) \times \frac{r \times .000}{.70410}, \\
 &= \pounds 201 \times \frac{1}{1-.6} \times \frac{r \times .000}{.70410} = \pounds \frac{27.000}{.316}, \\
 &\quad \text{reqd number of Napoleons} = 31527
 \end{aligned}$$

48 Suppose he mixes  $x$  lbs of the first kind with  $y$  lbs of the second  
It costs him  $(3\frac{1}{2}x + 5y)$  annas, he sells it at  $4\frac{1}{2}(x + y)$   
 $4\frac{1}{2}(x + y) = \frac{10}{6} \times (3\frac{1}{2}x + 5y)$ , and multiplying by 100,  
 $450(x + y) = 102(3\frac{1}{2}x + 5y)$ , or  $450x + 450y = 357x + 510y$ ,  
 $93x = 60y$ , and  $\frac{x}{y} = \frac{60}{93} = \frac{20}{31}$ ,  
 he mixes 20 of first kind with 31 of second

49 Exp<sup>n</sup> is greater than  $\frac{70 \times 24}{1-500000}$ , or  $\frac{70 \times 8 \times 24}{100000000}$ , or  $\frac{7488}{100000000}$ ,  
 or 0 0000748  
 Exp<sup>n</sup> is less than  $\frac{40 \times 25}{1-400000}$ , or  $\frac{100}{1-400000}$ , or 0 00008

50 Let  $\pounds r$  be the buying price, then  $x \times \frac{105\frac{1}{2}}{100}$  is the selling price

Now a loss of  $5\frac{1}{2}d$  in the  $\pounds$  leaves  $\pounds r \times \frac{234\frac{1}{2}}{240}$  out of  $\pounds r$

$$\begin{aligned}
 \frac{105\frac{1}{2}}{100} - 23375 &= \frac{234\frac{1}{2}}{240}, \text{ or } \frac{2112}{200} - 23375 = \frac{469}{480}x, \\
 \text{or } \left( \frac{211}{200} - \frac{469}{480} \right) x &= 23375, \text{ or } \frac{187}{2400} x = 23375, \\
 \text{or } x &= \frac{23375 \times 2400}{187} = 125 \times 24 = \pounds 30
 \end{aligned}$$

- 51 Let  $e$  shillings be the price of an English piece,  
and  $f$  " " a French piece

$$\text{Then } \frac{e}{f} = \frac{\text{area of English piece}}{\text{area of French piece}} = \frac{22\frac{1}{2} \times 12}{18 \times 9} = \frac{5}{3}$$

$e = \frac{5}{3}$  of  $f$ , and we must multiply the prices of the French pieces by  $\frac{5}{3}$

- On Level Uphill Downhill
- 52 In miles  $\left\{ \begin{array}{l} X's \text{ rate} = 4, \\ Y's \text{ rate} = 3 \end{array} \right\} \left\{ \begin{array}{l} X's \text{ rate} = 3, \\ Y's \text{ rate} = \frac{3}{2} \text{ of } 3 \end{array} \right\} \left\{ \begin{array}{l} X's \text{ rate} = 5, \\ Y's \text{ rate} = \frac{3}{2} \text{ of } 5 \end{array} \right\}$   
 per hr  
 $Y$  reaches  $C$  in  $\frac{6}{\frac{3}{2} \text{ of } 3}$  hrs, or  $2\frac{2}{3}$  hrs,  $X$  reaches  $B$  in  $\frac{4}{3}$  hrs,  
 or 2 hrs, and in  $\frac{2}{3}$  hr more he goes  $(3 \times \frac{2}{3})$  mi, or 2 mi  
 beyond  $B$   
 at the end of  $2\frac{2}{3}$  hrs,  $X$  and  $Y$  are  $2\frac{1}{2}$  mi apart  
 $X$  and  $Y$  then go towards each other at 3 mi an hr, and  $\frac{3}{2}$  of  
 5 mi an hr respectively, their rate of approach (see Arts  
 216, 217) is  $(3 + \frac{3}{2} \text{ of } 5)$ , or  $6\frac{3}{4}$  mi per hr, they meet in  
 $\frac{2\frac{1}{2}}{6\frac{3}{4}}$  hr, or  $\frac{1}{2}$  hr In this time  $Y$  will be  $(\frac{3}{2} \text{ of } 5) \times \frac{1}{2}$ , or  $1\frac{1}{2}$  mi  
 from  $C$ , they meet 11 miles from  $A$

- 53 Take a scale of 1 in to 10 yds along the axis of  $y$   
 Mark along the axis of  $x$  the points 0, 1, 2, 3, 4, etc, an inch  
 apart  
 Let 1 in along the  $x$ -axis represent the time  $B$  takes to walk  
 20 yds Now, while  $B$  walks 6 yds,  $A$  walks  $7\frac{1}{2}$ , hence,  
 while  $B$  walks 20 yds,  $A$  walks  $20 \times \frac{7\frac{1}{2}}{6}$ , or 25 yds  
 Since  $B$  has a start of 20 yds, we shall obtain his graph by  
 joining the points (0, 20) and (1, 40) Similarly  $A$ 's graph  
 is obtained by joining the points (0, 0) and (1, 25)  
 (i) When  $B$  has walked 50 yds, he is  $(20+50)$ , or 70 yds from  
 the start From the graph  $A$  is then  $62\frac{1}{2}$  yds from the start  
 Hence  $A$  is  $(70 - 62\frac{1}{2})$ , or  $7\frac{1}{2}$  yds behind  $B$   
 (ii) When  $A$  has walked 130 yds,  $B$  will be seen from the graph  
 to be 124 yds from the start Hence  $A$  is  $(130 - 124)$ , or  
 6 yds ahead  
 (iii) It will also be seen that when  $B$  is 116 yds from the start he  
 is 4 yds behind  $A$   $B$  must have walked  $(116 - 20)$ , or  
 96 yds

54 Along OX measure off  $OA=26''$  to represent 2 hr 10 min. On the vertical through A mark the point B, such that  $AB=10''$ . Then since the man's speed against the stream is  $(4\frac{1}{2}-1\frac{1}{2})$ , i.e. 3, mls per hr, the first part of his journey is represented by the line joining O to  $(12'', 15'')$ . He returns at the rate of  $(4\frac{1}{2}+1\frac{1}{2})$ , i.e. 6, mls per hr, and at 2.10 is 2 mls from his starting point. Hence the second part of his motion is shown by a line drawn from B to represent 6 mls per hr. This is found by joining B to the point whose abscissa, measured to the left of A, is  $12''$ , and whose ordinate is  $40'$ . The graphs cut at a point whose co-ords mark 5 mls and 1 hr 40 min. Hence the distance rowed up stream was 5 miles.

55 Let req<sup>d</sup> height be  $h$  cm, then  $\frac{h^3}{(12.4)^3} = \frac{1.76}{1}$ ,

$$h^3 = 1.76 \times (12.4)^3, \text{ and } 3 \log h = \log 1.76 + 3 \log 12.4$$

$$\text{or } \log h = \frac{1}{3}(2.455 + 3.2802) = 1.1752, \text{ and } h = 14.97 \text{ cm.}$$

56 Its value in 1 yr = Rs 27400  $\times$  0.95

$$\text{Its value in 6 yrs} = \text{Rs } 27400 \times (0.95)^6$$

$$= \text{Rs } 27400 \times 0.735092$$

$$= \text{Rs } 20141.5$$

57 Suppose a man does  $5a$  units of work per day

„ woman „  $3a$  „ „ „

„ boy „  $2a$  „ „ „

Then 10 men in 25 days do  $5a \times 25 \times 10$ , or  $1250a$  units,

„ 8 women „ 25 days „  $3a \times 25 \times 8$ , or  $600a$  units,

„ 6 boys „ 25 days „  $2a \times 25 \times 6$ , or  $300a$  units

Thus the work consists of  $(1250a + 600a + 300a)$ , or  $2150a$  units,  
and three times the work consists of  $(2150a \times 3)$  units

Again 19 men in 1 day do  $(5a \times 19)$ , or  $95a$  units,

20 women „ 1 day „  $(3a \times 20)$ , or  $60a$  units,

30 boys „ 1 day „  $(2a \times 30)$ , or  $60a$  units,

giving a total of  $(95a + 60a + 60a)$ , or  $215a$  units

they will do a work consisting of  $(2150a \times 3)$  units in  
 $\left(\frac{2150a \times 3}{215a}\right)$  days, or 30 days



58 4 lbs of A should be mixed with 7 lbs of B,

weight of A should be  $\frac{4}{11}$  of weight of mixture

But 4 cu in of A is actually mixed with 7 cu in of B,

and since  $\frac{\text{weight of 1 cu in of A}}{\text{weight of 1 cu in of B}} = \frac{151}{117}$ ,

$$\frac{\text{weight of 4 cu in of A}}{\text{weight of 7 cu in of B}} = \frac{151 \times 4}{117 \times 7} = \frac{604}{819}$$

and the weight of A actually is  $\frac{604}{604+819}$ , or  $\frac{604}{1423}$  of weight of mixture

the percentage of A's weight actually is  $\frac{604}{1423} \times 100$ , or 42.445,

but the percentage of A's weight should be  $\frac{4}{11} \times 100$ , or 36.363,

error =  $(42.445 - 36.363)\%$ , or 6.08%

59  $a=0.8$  in,  $b=0.7$  in,  $c=1$  in

area of section in figure =  $(0.8 \times \frac{1.7}{2})$  sq in = 0.68 sq in

area of section of haystack

$$= (0.68 \times 36) \text{ sq yds} = (0.68 \times 36 \times 9) \text{ sq ft},$$

and its volume =  $(0.68 \times 36 \times 9 \times 30)$  cu ft

Hence its weight =  $\frac{0.68 \times 36 \times 9 \times 30 \times 10}{2240} = \frac{4131}{14} = 295$  tons,  
giving 30 tons

60 External volume =  $(3\frac{3}{4} \times 2\frac{7}{12} \times 1\frac{1}{8})$  cu ft =  $\frac{1081}{8}$  cu ft  
= 11.302 cu ft

Since 1 gal of water occupies  $\frac{10}{62.3}$  cu ft, 38 gals occupy  
 $\frac{10}{62.3} \times 38$  cu ft

interval volume =  $\frac{10 \times 38}{62.3}$  cu ft = 6.099 cu ft,

whence volume of stone =  $(11.302 - 6.099)$  cu ft = 5.203 cu ft

Now 5.203 cu ft of stone weigh 874 lbs,

and 5.203 cu ft of water weigh  $5.203 \times 62.3$  lbs

req<sup>d</sup> number of times =  $\frac{874}{5.203 \times 62.3}$   
=  $\frac{874}{324}$   
= 2.69 = 2.7

$$\begin{array}{r} 5 \overline{) 203} \\ \underline{6 \ 2 \ 3} \\ 312 \ 0 \\ \underline{10 \ 4} \\ 1 \ 6 \\ \underline{324 \ 0} \end{array} \quad \begin{array}{r} 2 \ 69 \\ 32.4 \overline{) 874} \\ \underline{226} \\ 32 \end{array}$$

- 61 Let  $\pounds x$  be the req<sup>d</sup> sum The Amt of  $\pounds x$  for 3 yrs =  $\pounds x \times (1.03)^3$   
(See Art 313)

The respective Amts of the  $\pounds 100$  payments are

$$\pounds 100 \times (1.03)^2, \pounds 100 \times (1.03) \text{ and } \pounds 100$$

Thus the total payment =  $\pounds x \times (1.03)^3 + \pounds 100(1.03^2 + 1.03 + 1)$

Similarly if he paid  $\pounds 150$  a year, the total payment would be  $\pounds 150(1.03^2 + 1.03 + 1)$

$$x \times (1.03)^3 + 100(1.03^2 + 1.03 + 1) = 150(1.03^2 + 1.03 + 1),$$

$$\text{or } (1.03)^3 x = 50(1.03^2 + 1.03 + 1) = 50(1.0609 + 1.03 + 1) = 50 \times 3.0909$$

$$\begin{aligned} x &= \frac{50 \times 3.0909}{1.03^3} \\ &= \frac{154.545}{1.092727} \\ &= 141.4306 \\ &= \pounds 141.9s \end{aligned}$$

$$\begin{array}{r} 141.4306 \\ 1.092727 \overline{) 154.5450} \\ \underline{452723} \\ 15632 \\ \underline{4705} \\ 334 \end{array}$$

- 62 Let  $x$  years be his present age, his total salary for 13 years after he was 20 was

$$\text{Rs } (1000 \times 3 + 1200 + 1400 + 1600 + 1800 + 2000 + 2200 + 2400 + 2600 + 2800 + 3000),$$

or Rs 24000 The no of years his salary is stationary will be  $(x - 13 - 20)$ , or  $x - 33$  During this time he receives Rs 3000 a year But his salary during this whole period,  $(x - 20)$  years, averaged Rs 2500

$$24000 + 3000(x - 33) = 2500(x - 20), \text{ and dividing by } 500, \\ 48 + 6(x - 33) = 5(x - 20), \text{ whence } x = 50$$

- 63  $3425 = 5^2 \cdot 137$ ,  $1829 = 31 \times 59$ ,  $3245 = 5 \cdot 11 \cdot 59$

Let  $x$  be the third given number

Thus the candidate found the LCM of

$$5 \cdot 11 \cdot 59, 31 \cdot 59 \text{ and } x, \quad (1)$$

instead of the LCM of

$$5^2 \cdot 137, 31 \cdot 59 \text{ and } x \quad (2)$$

As the LCM for (1) and (2) was the same,  $x$  must have had as factors

11, which does not occur in (2),

$5^2$  and 137, which do not occur in (1),

also 2, since  $x$  is an even number

$x$  must be a multiple of 2 5<sup>2</sup> 11 137, or 75350  
 But 75350 is the only 5 figure multiple of 75350,  
 the third given number = 75350

$$64 \quad 1 \text{ interval on the first} = \frac{3}{10} \text{ in, on the second} = \frac{2}{8} \text{ ,}$$

$$\text{req'd diff} = \left( \frac{2}{8} - \frac{3}{10} \right) \text{ in} = \frac{0}{40} \text{ in} = \frac{3}{400} \text{ in}$$

Shortest distance req'd = LCM of  $\frac{2}{8}$  in and  $\frac{3}{10}$  in,  
 or  $\frac{1}{400}$  in and  $\frac{1}{400}$  in

$$\text{This LCM} = \frac{5040}{400} \text{ in} = 14\frac{1}{10} \text{ in}$$

$$65 \quad \text{We have } 35 \text{ ft} = \frac{35 \times 32}{160} \text{ m} = 3\frac{2}{5} \text{ m} \quad \text{Similarly } 4 \text{ ft} = \frac{128}{10} \text{ m}$$

Since the depth is the same in each case, if  $x$  gals are contained  
 in the first trench,

$$\frac{x}{2400} = \frac{48 \times 2}{32 \times 128}, \text{ or } x = \frac{48 \times 2 \times 3 \times 105 \times 2400}{32 \times 128}$$

Now 1 gal = 8 pints =  $\frac{4}{7} \times 8$  litres,

$$\text{req'd no of litres} = \frac{48 \times 2 \times 3 \times 105 \times 2400}{32 \times 128} \times \frac{4 \times 8}{7} = 81000$$

$$66 \quad \text{Suppose B walks 1 mi. in } x \text{ min. When they meet, B has walked } 5\frac{1}{2} \text{ mi in } 5\frac{1}{2}x \text{ min, A has walked } 3\frac{1}{2} \text{ mi in } 3\frac{1}{2} \times 18 \text{ min}$$

These times differ by 3 min,  $5\frac{1}{2}x = 3\frac{1}{2} \times 18 + 3$ , and  $x = 12$

B's speed is 1 mi in 12 min, or 5 mi per hr

$$67 \quad \text{Suppose B, instead of supplying the } £500 \text{ later on, supplied } £100x \text{ at the beginning. Then, since the time is the same in each case, A's proportion of the profits would be}$$

$$\frac{1100}{1100 + (1300 + 100x) + 1700} \text{ (see Art 281), or } \frac{11}{41+x}$$

$$\frac{11}{41+x} \text{ of } £2513\frac{7}{10} = £623\frac{7}{10}, \text{ or } \frac{11}{41+x} = \frac{6237}{25137} \left( \text{or } \frac{33}{133} \right),$$

$$\frac{41+x}{11} = \frac{133}{33}, \text{ and } x = 3\frac{1}{3} \quad 100x = 333\frac{1}{3},$$

so that B would have supplied  $£333\frac{1}{3}$  at the beginning

Now  $£333\frac{1}{3}$  lent for 12 months =  $£500$  lent for  $\frac{12 \times 333\frac{1}{3}}{500}$  months,  
 or 8 months Hence req'd time is 4 months later

$$\begin{aligned}
 68 \quad 15 \text{ lbs. per } 1 \text{ sq in} &= 15 \times 7000 \text{ grs per } \frac{1}{(39.37)} \text{ sq m} \\
 &= \frac{15 \times 7000}{15.43} \text{ gm per } \frac{1}{(39.37)} \text{ sq m} = \frac{15 \times 7000 \times (39.37)^2}{15.43} \text{ gm per } 1 \text{ sq m.} \\
 &= \frac{15 \times 7000 \times (39.37)^2}{15.43 \times 1000} \text{ Kg per } 1 \text{ sq m} \\
 &= \frac{15 \times 7000 \times (39.37)^2}{15.43 \times 1000 \times 100} \text{ Kg per } 100 \text{ sq cm} \\
 &= \frac{105 \times 1519.9969}{1513} \text{ Kg per } 100 \text{ sq cm} \\
 &= 105 \times 1.0045 \text{ Kg per } 100 \text{ sq cm} \\
 &= 105.47 \text{ Kg per } 100 \text{ sq cm} \\
 \text{excess} &= 5.47 \text{ in } 100, \text{ or } 5.5\%
 \end{aligned}$$

$$\begin{aligned}
 69 \quad \text{Let his income for } 1900 \text{ be } \text{£}x, \text{ then his income for } 1901 &= \text{£}x \times \frac{9}{10}, \\
 \text{and therefore his income for } 1902 &= \text{£}x \times \frac{9}{10} \times \frac{9}{10} \quad \text{Hence his} \\
 \text{expenditure for } 1902 \text{ is } x \times \frac{9}{10} \times \frac{9}{10} \times \frac{1}{4} \\
 x \times \frac{9}{10} \times \frac{9}{10} \times \frac{1}{4} &= x + 50 \quad \text{and } x = \text{£}4000
 \end{aligned}$$

$$\begin{aligned}
 70 \quad \text{Let req}^1 \text{ price per gallon be } \text{£}x \quad \text{Outlay} &= 10s \times 200 = \text{£}100 \\
 \text{£}100 \text{ amounts to } \text{£}100 \times (1.05)^4 \text{ (see Art } 313) \\
 175x &= 100 \times (1.05)^4, \\
 \text{and } x &= \frac{100 \times (1.05)^4}{175} = \frac{100 \times (1.025)^8}{175} = \frac{100 \times 1.2155}{175} = \frac{4 \times 1.2155}{7} \\
 &= \text{£}0.6916 = 13s \ 11d
 \end{aligned}$$

$$\begin{aligned}
 71 \quad \text{In } 1 \text{ cu cm of the mixture, there are } 0.7 \text{ cu cm of sulphuric} \\
 \text{acid and } 0.3 \text{ cu cm of water. This weighs} \\
 (0.7 \times 1.842 + 0.3 \times 1) \text{ gm, or } 1.5894 \text{ gm} \\
 \text{Hence } 1 \text{ cu cm of mixture weighs (1) } 1.5894 \text{ gm, (2) } 1.615 \text{ gm,} \\
 1 \text{ gm occupies (1) } \frac{1}{1.5894} \text{ cu cm (2) } \frac{1}{1.615} \text{ cu cm,} \\
 \text{thus loss of volume} &= \left( \frac{1}{1.5894} - \frac{1}{1.615} \right) \text{ cu cm in } \frac{1}{1.5894} \text{ cu cm} \\
 &= \frac{1.615 - 1.5894}{1.5894 \times 1.615} \text{ cu cm in } \frac{1}{1.5894} \text{ cu cm} \\
 &= \frac{0.0256}{1.615} \text{ cu cm in } 1 \text{ cu cm} \\
 &= \frac{0.0256}{1.615} \text{ per cent, or } 1.585\%
 \end{aligned}$$

- 72 Since the course is 1760 × 3 yds, or 5280 yds, the second man will have run  $\frac{3}{4}$  of 5280 yds, or 4950 yds, the third will have run  $\frac{2}{3}$  of 5280 yds, or 3630 yds. Now

$$4950 = 528 \times 10 - 330, \text{ and } 3630 = 528 \times 7 - 66,$$

giving as answer 330 yds and 66 yds respectively, from the starting point

- 73 His net income = Rs 1550 - Rs 140 = Rs 1410. Rs 1410 is interest on Rs 28000, giving Rs  $\frac{141}{280}$  per Rs 100, or 5.04 %

For second part of question, net income

$$= \text{Rs } 1410 - \frac{21}{100} \text{ of Rs } 1550 = \text{Rs } 1371.25$$

Now Rs 1371.25 interest on Rs 28000 = Rs  $\frac{1371.25}{280}$  interest on Rs 100

$$\text{Now } \frac{1371.25}{280} = \frac{34.28125}{7} = 4.897, \text{ giving } 4.90 \%$$

- 74 234 m cost £1 642, 234 × 1760 × 0.9144 m cost 1 642 × 25 17 fr, or 234 × 1 76 × 0.9144 Km cost 1642 × 25 17 c,

$$1 \text{ Km costs } \frac{1642 \times 25 \times 17}{234 \times 176 \times 0.9144} \text{ c}$$

$$\log \text{ numerator} = \log 1642 + \log 2517 = 2.2153 + 1.4009 = 3.6162,$$

$$\log \text{ denominator} = \log 234 + \log 176 + \log 0.9144$$

$$= 2.3692 + 0.2455 + \bar{1}.9611 = 2.5758,$$

$$\log \text{ fraction} = 3.6162 - 2.5758 = 1.0404,$$

$$\text{and fraction} = 10.97 \text{ (centimes per Km)}$$

Answer As above we get 1 Km costs  $\frac{1642 \times 25 \times 17}{234 \times 176 \times 0.9144} \text{ c}$

$$\text{This} = \frac{41329}{37658}$$

$$= 10.97 \text{ (centimes per Km)}$$

2517	234
<u>1642</u>	<u>176</u>
2517	234
15102	1638
10068	<u>1404</u>
503	41184
<u>413291</u>	<u>09144</u>
37659	370656
<u>413291</u> (10.97	<u>4118</u>
36701	<u>1647</u>
2803	<u>164</u>
	<u>376585</u>

- 75 Take a scale of 0.1" to 5 min horizontally and 0.1' to 2 miles vertically. Assume P is at the origin. Join the origin to the point (60, 40), and we obtain the graph of the motor car. The train goes at the rate of 1 mile per minute. Join the points (15, 0) and (25, 10), and produce this line as far as the point (50, 35). (This will be the graph of the train as far as

the first stopping place) Join the points (50, 35) and (55, 35) by a horizontal line (This will be the train's graph while waiting at the station) Again join the points (55, 35) and (100, 80) (This will be the graph of the train after leaving the station)

- (i) It will be seen that when both are in motion, the train and motor-car are together 30 miles from P and 40 miles from P
- (ii) At the point on each graph where the ordinate is 150, the abscissae are 170 and 225 respectively Thus the req<sup>d</sup> length of time is (225 - 170) min or 55 min

76. After the first replacement,

there are  $(a-b)$  gals of spirit out of the  $a$  gals of the mixture, the quantity of spirit is  $\frac{a-b}{a}$  of the whole mixture, or  $\frac{a-b}{a}$  of  $a$  gals

Also, since the water and spirit are supposed to be mixed *uniformly*, the amount of spirit in *any quantity of the mixture* is  $\frac{a-b}{a}$  of that quantity

when  $b$  gals are taken out of the mixture,

$\frac{a-b}{a}$  of  $b$  gals of spirit are removed.

$$\begin{aligned}\text{spirit left} &= \left( \frac{a-b}{a} \text{ of } a - \frac{a-b}{a} \text{ of } b \right) \text{ gals} \\ &= \frac{a-b}{a} \text{ of } (a-b) = \frac{(a-b)^2}{a} = a \times \left( \frac{a-b}{a} \right)^2\end{aligned}$$

[If the student finds any difficulty in the above solution, let him put  $a=5$  and  $b=2$ , and then work through the question with these values]

77 A, B and C together do  $\frac{1}{10}$  of the work in 1 day,

they do  $\frac{1}{10} \times 10$ , or  $\frac{1}{10}$  of the work in 10 days, and  $\frac{5}{8}$  of the work remains to be done

Now C does  $\frac{1}{12}$  of the work in 1 day

After A's withdrawal he works for 20 days,

and after B's withdrawal he works for  $96(1 + \frac{1}{3})$ , or 128 days,

he works for (20+128), or 148 days, and does  $\frac{148}{12}$  or  $\frac{37}{3}$  of the work

Thus B, after A's withdrawal, must have done  $(\frac{5}{8} - \frac{37}{3})$  of the work, or  $\frac{1}{3}$  of the work in 20 days,

he would do the whole work alone in  $20 \times 6$ , or 120 days

78 Let req<sup>d</sup> sum be £P and  $r$  the rate per cent

The Amt on £P at C I for 2 years at  $r$  per cent

$$= P \left( 1 + \frac{r}{100} \right)^2 \quad (\text{See Art 313})$$

$$\begin{aligned} \text{the C I on £P for 2 years at } r \text{ per cent} &= P \left( 1 + \frac{r}{100} \right)^2 - P \\ &= \frac{2Pr}{100} + \frac{Pr^2}{100^2} \end{aligned}$$

$$\text{Now the S I on £P for 2 years at } r \text{ per cent} = \frac{2Pr}{100}$$

$$\begin{aligned} \text{diff of C I and S I for 2 years} &= \frac{2Pr}{100} + \frac{Pr^2}{100^2} - \frac{2Pr}{100} = \frac{Pr^2}{100^2} \\ &= 13s \ 4d, \text{ or } £\frac{2}{3} \end{aligned}$$

Similarly,

$$\begin{aligned} \text{the C I on £P for 3 years at } r \text{ per cent} &= P \left( 1 + \frac{r}{100} \right)^3 - P \\ &= \frac{3Pr}{100} + \frac{3Pr^2}{100^2} + \frac{Pr^3}{100^3} \end{aligned}$$

$$\text{Now S I on £P for 3 years at } r \text{ per cent} = \frac{3Pr}{100}$$

$$\begin{aligned} \text{diff of C I and S I for 3 years} &= \frac{3Pr}{100} + \frac{3Pr^2}{100^2} + \frac{Pr^3}{100^3} - \frac{3Pr}{100} \\ &= \frac{3Pr^2}{100^2} + \frac{Pr^3}{100^3} \\ &= £2 \ 0s \ 4d, \text{ or } £2\frac{1}{6} \end{aligned}$$

$$\text{We have to solve} \quad \frac{Pr^2}{100^2} = \frac{2}{3}, \quad (1)$$

$$\text{and} \quad \frac{3Pr^2}{100^2} + \frac{Pr^3}{100^3} = 2\frac{1}{6} \quad (2)$$

Substituting from (1) in (2) we get

$$3 \times \frac{2}{3} + \frac{Pr^3}{100^3} = 2\frac{1}{6}, \text{ or } \frac{Pr^3}{100^3} = \frac{1}{60} \quad (3)$$

$$\text{Divide (3) by (1), then } \frac{r}{100} = \frac{1}{40}, \text{ and } r = 2\frac{1}{2}$$

$$\text{Substituting in (1) this value for } r, \frac{P \times (2\frac{1}{2})^2}{100^2} = \frac{2}{3},$$

$$\text{and } P = £1066\frac{2}{3} = £1066 \ 13s \ 4d$$

79  $\sqrt{25 + \sqrt{125}}$   
 $= \sqrt{25 + 11\ 1803399}$   
 $= \sqrt{36\ 1803399}$   
 $= 6\ 015$

21	125, (11 1803399
221	25
2228	400
2236,03	17900
	760000
	89191
	22110
	2076

36, 18,03 39,9 (6 015
1201
12015
18 03
60239
164

- 80 Let Rs  $7x$  and Rs  $5x$  be the respective capitals  
 The elder now has Rs  $7x \times (1 + \frac{1}{10})$ , or Rs  $10x$ , the younger  
 Rs  $(5x - 707)$

$$\frac{5x - 707}{10x} = \frac{43}{100}, \text{ or } 50x - 7070 = 43x,$$

$x = 1010$  Hence the elder has Rs 7070, the younger Rs 5050

81 Req<sup>d</sup> weight =  $(1054 \times 820 \times 0.8 \times 11.35)$  gm  
 $= (861280 \times 9.08)$  gm  
 $= 7848$  Kg

861	28
7778	9 08
69	5
7847	1
	6

82 £1 17s 5 $\frac{1}{2}$ d = £1 872 91666 = value of 1 cwt

	10, 3	
	187 291 66	= value of 100 cwt
	5 618 75	= " 3 cwt
14 lbs = $\frac{1}{4}$ cwt	234 11	= " 14 lbs
7 lbs = $\frac{1}{8}$ of 14 lbs	117 05	= " 7 lbs
3 $\frac{1}{2}$ lbs = $\frac{1}{4}$ of 7 lbs	058 52	= " 3 $\frac{1}{2}$ lbs
2 lbs = $\frac{1}{8}$ of 14 lbs	033 41	= " 2 lbs
	193 373 53	= total value
	= £193 7s 1d	

- 83 Take 12 o'clock at the origin, and 0.2" to 1 min horizontally  
 Take 1" to 5 miles vertically. Plot the points given in the table  
 and join them successively by straight lines. It will be seen  
 that the speed is greatest between 5 and 10 miles from the  
 start, since between these two points the graph has its  
 greatest *slope* or *gradient*. It then travels 5 miles in 5 min,  
 or 60 miles per hour

84  $(20 - 5)$  rolls cost Rs 90 - Rs 70, or 15 rolls cost Rs 20

Hence 5 rolls cost Rs  $\frac{20}{3}$

But gramophone and 5 rolls cost Rs 70,

gramophone costs Rs 63 $\frac{1}{3}$

gramophone and 50 rolls cost Rs 63 $\frac{1}{3}$  + Rs 66 $\frac{2}{3}$ , or Rs 130



- 85 (i)  $2\ 204\text{ lbs} = 1\text{ Kg}$ , or  $2\ 204 \times 16\text{ oz} = 1\text{ Kg}$

$$1\text{ oz} = \frac{1}{16 \times 2\ 204}\text{ Kg} = \frac{0\ 4537\ 05}{16}\text{ Kg} = 0\ 0283575\text{ Kg}$$

$$\text{But } 1\text{ oz} = \frac{8\ 5}{3}\text{ gm} = 28\ 3333\text{ gm}$$

$$= 28\ 3575\text{ gm}$$

error is  $(28\ 3575 - 28\ 3333)$  in  $28\ 3333$ , or  $0\ 0242$  in  $28\ 3333$ ,

$$\text{reqd percentage} = \frac{0\ 0242 \times 100}{28\ 3333} = 0\ 085, \text{ or } 0\ 09\%$$

- (ii)  $1\text{ lb} = \frac{1}{2\ 204}\text{ Kg} = 0\ 4537205\text{ Kg}$  correctly

$$1\text{ lb} = \frac{1}{2\ 2}\text{ Kg} = 0\ 4545455\text{ Kg}$$
 roughly

As in (i), we find error is  $0\ 000825$  in  $0\ 4537205$

$$\text{reqd percentage is } \frac{0\ 000825 \times 100}{0\ 4537205}, \text{ or } 0\ 18\%$$

86 A produces  $\frac{22\frac{1}{2}}{132}\text{ lb} = \frac{19}{110} = 0\ 172\text{ lb}$

B "  $\frac{139}{794\frac{1}{2}}\text{ lb} = \frac{278}{1589} = 0\ 175\text{ lb}$

C "  $\frac{12}{61}\text{ lb} = 0\ 197\text{ lb}$

D "  $\frac{112}{632}\text{ lb} = \frac{14}{79} = 0\ 177\text{ lb}$

E "  $\frac{20\frac{1}{2}}{118}\text{ lb} = \frac{81}{472} = 0\ 172\text{ lb}$

C produces the most butter per gallon, then follow in order of merit D, B, A, E

- 87 Converting to cm and gm,

$$\text{reqd thickness} = \frac{1\ 470\ 000}{1000 \times 100 \times 11\ 4}\text{ cm} = \frac{1\ 470\ 000}{114\ 000}\text{ cm} = 7\ 4\text{ mm}$$

- 88 The respective rates of interest are as 1 2 3

Then the total respective interests are as 1 4 9, giving respectively  $\frac{1}{4}$ ,  $\frac{1}{4}$  and  $\frac{9}{4}$  of Rs 2450

- 89 In every hour the minute hand traverses 60 minute spaces, while the hour-hand traverses  $2\frac{1}{2}$  of such spaces. Thus in every 60 min of time the minute hand will gain  $57\frac{1}{2}$  minute spaces on the hour-hand, it will therefore gain 15 minute spaces in  $15 \times \frac{60}{57\frac{1}{2}}$ , or  $15\frac{1}{2}\frac{5}{11}$  min of time past noon

The hands will first be together after 1 o'clock when the minute

hand has gained  $2\frac{1}{2}$  min on the hour-hand, i.e. in  $2\frac{1}{2} \times \frac{60}{57\frac{1}{2}}$ , or  $2\frac{1}{2}\frac{4}{11}$  min past one

- 90 Since they all *alter their speeds proportionally*, there will be no difference in the *respective positions* of the runners at the end of the different races, *whatever these speeds may be*. Hence in our calculation we need take no account of the ratio 7 8 9

Now A runs 100 yds while B runs 96 yds,

and A „ 429 yds „ C „ 440 yds,

A runs  $429 \times 100$  yds, while B runs  $96 \times 429$  yds, and C runs  $440 \times 100$  yds,

B runs  $96 \times 429$  yds while C runs 440 yds,

B runs  $4 \times 96 \times 429$ , or 164736 yds while C runs 1760,

C can beat B by 11264 yds

- 91 Cost of digging per sq ft of surface =  $\frac{1}{8}d = \frac{1}{8}d$

Cost of rubble „ „ =  $\frac{1}{8}d \times \frac{1}{8}d = \frac{1}{64}d$

Cost of gravel „ „ =  $\frac{1}{8}d \times \frac{1}{8}d = \frac{1}{64}d$

Cost of rolling „ „ =  $\frac{1}{8}d$

total cost per sq ft. of surface =  $(\frac{1}{8} + \frac{1}{64} + \frac{1}{64} + \frac{1}{8})d = \frac{5}{8}d$

total cost for the whole surface =  $\frac{5}{8}d \times 880 \times 3 \times 26$

= 145200d = £605

- 92 Value of total coinage = £19198815

Percentage of gold =  $\frac{£18598000}{£19198815} \times 100$   
= 96.8

Percentage of silver =  $\frac{£510490}{£19198815} \times 100$   
= 2.66

Percentage of bronze =  $\frac{£100325}{£19198815} \times 100$   
= 0.52

$$\begin{array}{r}
 96.8 \\
 192 \overline{) 18598} \\
 \underline{1808} \\
 518 \\
 \underline{516} \\
 266 \\
 1920 \overline{) 5105} \\
 \underline{3840} \\
 1265 \\
 \underline{1152} \\
 113 \\
 1920 \overline{) 10032} \\
 \underline{9600} \\
 432 \\
 \underline{432} \\
 0
 \end{array}$$

- 93 Cash received on sale = £  $\left( \frac{95\frac{1}{2}}{100} \text{ of } 850 + \frac{107\frac{1}{2}}{100} \text{ of } 1300 \right)$

= £  $\left( \frac{191 \times 17}{4} + \frac{421 \times 13}{4} \right)$  = £2180

Stock bought at 272½ = £  $\frac{2180 \times 100}{272\frac{1}{2}}$  = £  $\frac{2180 \times 200}{545}$  = £800,

total brokerage =  $\frac{1}{800}$  of £(850 + 1300 + 800) = £  $\frac{2950}{800}$   
= £  $3 \frac{13}{8}$  = £3 13s 9d

- 94 Let Rs  $x$  be his outlay on each sword his total outlay is Rs  $2000x$ , and, since he gains 15 %, his total receipts would be Rs  $2000x \times \frac{115}{100}$ . But he sells each sword at Rs  $12\frac{1}{2}$ , and there are  $(\frac{95}{100} \times 2000)$  swords

$$2000x \times \frac{115}{100} = \frac{95}{100} \times 2000 \times 12\frac{1}{2},$$

$$\text{and } x = \frac{95 \times 2000 \times 207 \times 100}{100 \times 16 \times 2000 \times 115} = \text{Rs } 10\frac{11}{16}$$

Now  $\frac{35}{100}$  of 2000, or 700 swords, prove worthless

his total receipts are Rs  $(12\frac{1}{2} \times 1300)$ , or Rs 16818 $\frac{3}{4}$

But his total outlay is Rs  $(10\frac{11}{16} \times 2000)$ , or Rs 21375 ,

his loss = Rs  $(21375 - 16818\frac{3}{4})$  = Rs 4556 4 a

- 95 By calculation we obtain the following table

Year	1850	1860	1870	1880	1890	1900
Pop <sup>n</sup>	5000	5400	5940	6702	7641	8787

Take scales of 1" to 10 years horizontally and 1" to 1000 population vertically. Take the point (1850, 5000) as origin. Plot the points as in Art 259, Ex. It will be seen that a smooth curve can be drawn through them, and the required results may then be read off.

Add 8%	5000
	400
	5400
Add 10%	540
	5940
Add 10%	594
" 3%	178 2
	6702 2
Add 10%	670 22
" 4%	268 083
	7640 5
Add 10%	764 05
" 5%	382 025
	8786 575

- 96 See Art 313

Last July's payment of Rs 1000 will amount to

Rs  $1000 \times (1.025)^3$  by next January

This year's January payment of Rs 1000 will amount to

Rs  $1000 \times (1.025)^2$  by next January

This year's July payment of Rs 1000 will amount to

Rs  $1000 \times (1.025)$  by next January

Next January's payment will be Rs 1000

total payment due next January

$$= \text{Rs } 1000 \times (1.025)^3 + \text{Rs } 1000 \times (1.025)^2 + \text{Rs } 1000 \times (1.025) + \text{Rs } 1000$$

$$= \text{Rs } 1076.8906 + \text{Rs } 1050.625 + \text{Rs } 1025 + \text{Rs } 1000$$

$$= \text{Rs } 4152.5156 = \text{Rs } 4152 \text{ 8 a } 3 \text{ p}$$

97 Let req<sup>d</sup> distance be  $x$  Km

From A to B I ride  $(x - \frac{3}{4})$  Km, and take  $\frac{x - \frac{3}{4}}{15\frac{3}{4}}$  hrs,

I walk  $\frac{3}{4}$  Km, and take  $\frac{\frac{3}{4}}{4\frac{1}{2}}$ , or  $\frac{1}{6}$  hrs

From B to A I ride  $x$  Km, and take  $\frac{x}{15}$  hrs

$$\frac{x - \frac{3}{4}}{15\frac{3}{4}} + \frac{1}{6} = \frac{x}{15}, \quad \text{or} \quad \frac{4x - 3}{63} + \frac{1}{6} = \frac{x}{15},$$

$$x = 10(4x - 3) + 105 = 42x, \quad \text{whence } x = 37.5 \text{ Km}$$

98 The LCM of 221 and 364 = 2912

In 2912 secs after starting  $A$  and  $B$  arrive for the first time at the starting point. In this time  $A$  will have gone round 13 times and  $B$  8 times,  $A$  passes  $B$  5 times

$$\begin{array}{r|l} 1031 & 485 \\ \hline & 1031485 \\ 1031 & 49 \\ 30 & 95 \\ 1 & 03 \\ & 41 \\ & 8 \\ \hline 1063 & 96 \end{array}$$

$$\begin{array}{r|l} 1063 & 96 \\ \hline & 1031485 \\ 1063 & 96 \\ 31 & 92 \\ 1 & 06 \\ & 43 \\ & 9 \\ \hline 1097 & 46 \end{array}$$

100 Req<sup>d</sup> area =  $127.35 \times 98.27$  sq. m

$$= \frac{127.35 \times 98.27}{(0.9144)^2} \text{ sq. yds}$$

$$= \frac{127.35 \times 98.27}{(0.9144)^2 \times 4840} \text{ ac}$$

$$= \frac{12.735 \times 0.9827}{0.83613 \times 4.84} \text{ ac}$$

$$\begin{array}{r|l} 12735 & 7 \\ \hline 982 & 7 \\ 11461 & 5 \\ 1018 & 8 \\ 25 & 5 \\ 8 & 9 \\ \hline 12514 & 7 \end{array} \quad \begin{array}{r|l} 0836 & 13 \\ \hline 481 & \\ 334 & 5 \\ 668 & 9 \\ 33 & 4 \\ \hline 4046 & 8 \end{array}$$

$$\begin{array}{r} 309 \\ 4047 \overline{) 12514} \\ \underline{373} \end{array}$$

$$101 \quad 1^{\text{st}} \text{ year's dividend} = \frac{\text{Rs } 3 \times 10000}{105} = \frac{\text{Rs } 2000}{7}$$

$$2^{\text{nd}} \text{ year's dividend} = \frac{\text{Rs } 2000}{7} + \frac{\text{Rs } 3 \times 2000}{107 \times 7} = \frac{\text{Rs } 2000}{7} \left( 1 + \frac{3}{107} \right) \\ = \frac{\text{Rs } 2000 \times 110}{7 \times 107}$$

$$3^{\text{rd}} \text{ year's dividend} = \frac{\text{Rs } 2000 \times 110}{7 \times 107} + \frac{\text{Rs } 3 \times 2000 \times 110}{108 \times 7 \times 107} \\ = \frac{\text{Rs } 2000 \times 110}{7 \times 107} \left( 1 + \frac{3}{108} \right) = \frac{\text{Rs } 2000 \times 110 \times 111}{7 \times 107 \times 108} \\ = \frac{\text{Rs } 2035000}{6741} = \text{Rs } 301 \text{ 14 a 3 p}$$

- 102 By investing Re 1 the respective interests received are Re 037, Re 036, Re 030, Re 046, and Re 042

the stocks arranged in relative order of merit as profitable investments stand thus — Bank of Madras, Bank of Bengal, Govt Paper (3), Govt Paper (3½), and Govt Consols

By investing Rs 97 3 the net interest received is Rs 3½ ×  $\frac{187}{10 \times 12}$   
by investing a lac of rupees the net interest received is

$$\text{Rs } \frac{100000}{973} \times \frac{7}{2} \times \frac{187}{10 \times 12} = \text{Rs } 3503 \text{ 7 a}$$

- 103 Since 27 gals = 216 pints, there are  $215\frac{1}{2}$  pints left after 1<sup>st</sup> day  
216 pints are reduced to  $215\frac{1}{2}$ , or 1 pint to  $\frac{215\frac{1}{2}}{216}$  pints

Thus the multiplying ratio (see Arts 269–271) is  $\frac{215\frac{1}{2}}{216}$  for each day

after 20 days  $216 \times \left( \frac{215\frac{1}{2}}{216} \right)^{20}$ , or  $216 \times \left( \frac{431}{432} \right)^{20}$  pints remain

$$104 \quad \text{Reqd no} = 16 \times \frac{60 \text{ secs}}{10 \text{ secs}} \times \frac{5 \text{ teeth}}{24 \text{ teeth}} = \frac{16 \times 60 \times 5}{10 \times 24} = 20$$

- 105 Suppose  $d_1$  inches fell in 1899,  $d_2$  in 1900, etc

$$\text{Then } d_1 + d_2 + d_3 = 74 \text{ 94, i.e. } (24 \text{ 98} \times 3), \quad (1)$$

$$d_4 + d_5 + d_6 = 88 \text{ 86, i.e. } (29 \text{ 62} \times 3), \quad (2)$$

$$-d_4 + d_5 = 4 \text{ 80}, \quad (3)$$

$$d_5 - d_6 = 6 \text{ 36}, \quad (4)$$

$$-d_2 + d_5 = 7 \text{ 47}, \quad (5)$$

$$d_2 - d_3 = 0 \text{ 17} \quad (6)$$

Adding (2), (3) and (4),  $3d_5 = 100.2$  and  $d_5 = 33.34$  in

Subst for  $d_5$  in (4) we find  $d_6 = 26.98$  in

Subst for  $d_1$  in (3) we find  $d_4 = 28.54$  in

Subst for  $d_6$  in (5) we find  $d_2 = 25.87$  in

Subst for  $d_2$  in (6) we find  $d_3 = 25.70$  in

Subst for  $d_2$  and  $d_3$  in (1) we find  $d_1 = 23.37$  in

106 Let req<sup>d</sup> length be  $x$  yds

$A$  runs  $x$  yds while  $B$  runs  $(x-15)$  yds and  $C$  runs  $(x-29)$  yds (1)

Again  $B$  runs  $x$  yds while  $C$  runs  $(x-15)$  yds (2)

From (1),  $\frac{B's \text{ rate}}{C's \text{ rate}} = \frac{x-15}{x-29}$ , from (2),  $\frac{B's \text{ rate}}{C's \text{ rate}} = \frac{x}{x-15}$

$$\frac{x-15}{x-29} = \frac{x}{x-15} \quad \text{Multiply by } (x-29) \times (x-15),$$

$$(x-15) \times (x-15) = x(x-29), \text{ or } x^2 - 30x + 225 = x^2 - 29x$$

Subtract  $x^2$  from each side, then  $-30x + 225 = -29x$ , and  $x = 225$

107 Req<sup>d</sup> value =  $\frac{\pounds 2000 \times 4\frac{1}{2}}{114\frac{1}{2}} \times \frac{229}{240}$  (see Ex. XVI d, No 21)

$$= \frac{\pounds 2000 \times 9 \times 229}{229 \times 240} = \frac{\pounds 600}{8} = \pounds 75$$

108 The 'remainder' =  $1 - \frac{1}{3} - \frac{3}{5} = \frac{1}{15}$

$$\text{his total profit} = \frac{14}{100} \text{ of } \frac{1}{3} + \frac{17\frac{1}{2}}{100} \text{ of } \frac{2}{5} + \frac{20}{100} \text{ of } \frac{1}{15}$$

$$= \frac{14}{300} + \frac{21}{200} + \frac{4}{300} = \frac{99}{600} = \frac{16\frac{1}{2}}{100}, \text{ giving } 16\frac{1}{2}\% \text{ profit}$$

109 Let  $\pounds x$  be his capital at first 7 months' interest at  $3\frac{3}{4}\%$  is

$$\pounds \frac{7}{12} \text{ of } \frac{3\frac{3}{4}x}{100}, \quad \text{capital + interest} = \pounds \left( x + \frac{7 \times 3\frac{3}{4}x}{1200} \right) = \pounds \frac{327}{320}x$$

He then invests  $\pounds \frac{327}{320}x$ , after six months he draws a dividend

$$\text{of } \pounds \frac{1}{2} \text{ of } \frac{3}{109} \text{ of } \pounds \frac{327}{320}x \text{ and realises by sale } \pounds \frac{106\frac{1}{2}}{109} \text{ of } \frac{327}{320}x$$

$$\text{the total return} = \pounds \left( \frac{1}{2} \times \frac{3}{109} \times \frac{327}{320}x + \frac{106\frac{1}{2}}{109} \times \frac{327}{320}x \right)$$

$$= \pounds \frac{3}{320} (1\frac{1}{2} + 106\frac{1}{2})x = \pounds \frac{81}{80}x,$$

net gain on  $\pounds x = \pounds \frac{x}{80}$ , giving  $1\frac{1}{4}\%$  gain

- 110  $A$  puts in 126 oven for 3 months, which costs him as much as  $126 \times 3$  oven for 1 month. Similarly the cost to  $B$  and  $C$  is that of  $162 \times 5$  oven and  $72 \times 12$  oven respectively for 1 month

Thus then expenses are as  $126 \times 3$   $162 \times 5$   $72 \times 12$ , or as 7 15 16

Thus  $A$  pays  $\frac{7}{38}$  of Rs 570,  $B$  pays  $\frac{15}{38}$  of Rs 570,  $C$  pays  $\frac{16}{38}$  of Rs 570 (see Art 281)

- 111 Let  $x$  be the no of children in town, out of 100 in town and country, then  $100 - x$  is the number in the country

The total cost is  $[69x + 39(100 - x)]$  pence it is also  $60 \times 100$  pence  
 $69x + 39(100 - x) = 6000$ , and  $x = 70$

Thus req<sup>d</sup> percentages are 70 and 30

- 112 The bookseller contributed Rs 8000 for 366 days

His partner contributed Rs 11500 for 108 days

then shares are as  $8000 \times 366$   $11500 \times 108$ , i.e. as 488 207

Thus the bookseller receives  $\frac{488}{885}$  of Rs 1654

$$= 488 \times \text{Rs } 2 \frac{3799}{1654} = \text{Rs } 1161 \frac{6}{1654}$$

His partner receives  $\frac{207}{885}$  of Rs 1654

$$= 207 \times \text{Rs } 2 \frac{3799}{1654} = \text{Rs } 492 \frac{10}{1654}$$

- 113 Let  $\pounds x$  be req<sup>d</sup> price, then the first bicycle realised  $\pounds \frac{60}{100}x$ ,

the second bicycle cost  $\pounds \left( \frac{60x}{100} + 5\frac{1}{4} \right)$  and realised

$$\frac{65}{100} \text{ of } \pounds \left( \frac{60x}{100} + 5\frac{1}{4} \right)$$

The third bicycle was bought with this and the addition of  $\pounds 5 \text{ } 15\text{s } 3\text{d}$ , or  $\pounds 5\frac{91}{80}$ ,

$$\frac{65}{100} \times \left( \frac{60x}{100} + 5\frac{1}{4} \right) + 5\frac{91}{80} = 16, \text{ or } \frac{65 \times 60x}{100 \times 100} + \frac{65 \times 21}{100 \times 4} = 10\frac{19}{80}$$

$$\text{Hence } \frac{39x}{100} + 3\frac{33}{80} = 10\frac{19}{80}, \text{ or } \frac{39x}{100} = 6\frac{33}{80}, \text{ and } x = \pounds 17\frac{1}{2}$$

- 114  $1 \text{ m} = 3 \text{ } 28 \text{ ft}$ ,  $1 \text{ sq m} = (3 \text{ } 28)^2 \text{ sq ft}$

Hence  $1 \text{ aie} = (3 \text{ } 28)^2 \times 100 \text{ sq ft}$

and  $1 \text{ Ha} = (3 \text{ } 28)^2 \times 100 \times 100 \text{ sq ft}$

$$= \frac{328 \times 328}{4840 \times 9} \text{ ac}$$

$$= \frac{41 \times 328}{605 \times 9} \text{ ac} = \frac{17448}{5445} \text{ ac}$$

$$= 2 \text{ } 470 \text{ ac nearly}$$

$$\begin{array}{r} 2 \text{ } 4698 \\ 5445 \overline{) 13448} \\ \underline{25580} \\ 3800 \\ \underline{533} \\ 43 \end{array}$$

115 True area =  $4 \times 3 \ 14159 \times (4000)^2$  sq m

Approx area =  $4 \times 3 \ 142857143 \times (4000)^2$  sq m [ $\frac{2}{7} = 3 \ 142857143$ ]

Difference =  $4 \times 4000^2 \times 0 \ 001267143$  sq m

=  $4 \times 4^2 \times 1267 \ 143$  sq m

=  $64 \times 1267 \ 143$  sq m

= 81097 sq m (more)

1267	143
6	4
76028	4
5068	6
81097	0

116 The company has to pay back altogether the Amt of Rs 1658775 at 4 % C I

This = Rs 1658775  $\times (1 \ 04)^1$  (see Art 313)

= Rs 1658775  $\times (1 \ 0816)^2$

= Rs 1658775  $\times 1 \ 16985856$

= Rs 1940532

1658775	
1	16985856
1658775	
165877	5
99526	50
14028	98
1327	02
82	94
13	26
	83
	10
1940532	13

Since they pay by annual instalments of Rs 456976, the first instalment paid at the end of the first year amounts to Rs  $456976 \times (1 \ 04)^3$  in 3 more years at 4 % C I

Similarly the second instalment paid at the end of the second year amounts to Rs  $456976 \times (1 \ 04)^2$  in 2 more years at 4 % C I

In the same way the third instalment amounts to Rs  $456976 \times 1 \ 04$  at the end of the time

With the last instalment the payments amount to

Rs  $456976(1 \ 04^3 + 1 \ 04^2 + 1 \ 04 + 1)$ ,

or Rs  $456976(1 \ 124864 + 1 \ 0816 + 1 \ 04 + 1)$ ,

or Rs  $456976 \times 4 \ 246164$ ,

or Rs 1940532, as above

456976	
4	246164
1827904	
91395	2
18279	0
2741	8
182	8
27	4
1	8
1940532	0

117 At the rate of 3 $\frac{1}{2}$  mi per hr, they row 308 ft in 63 secs. Since the next gun is fired when they have rowed for 60 secs, it follows that the report takes 3 secs to reach them. But in 3 secs, sound travels  $1100 \times 3$  ft, or 3300 ft

req<sup>d</sup> distance =  $(308 + 3300)$  ft, or 3608 ft

118 Then times for the whole journey are as 42 : 56, or as 3 : 4, their speeds are as 4 : 3 (see Art 271), and they meet at a point which divides the whole distance in the ratio of 4 : 3, i.e. they meet at  $\frac{4}{7}$  of the distance from Liverpool, and in  $\frac{4}{7}$  of 42 min or 24 min



Take 4 in vertically to represent the distance between Liverpool and Manchester. Mark the successive inch divisions along the  $y$ -axis from the origin inclusive, 0, 1, 2, 3, 4. For time take 1 in to represent 10 min along the  $x$ -axis. Assume Liverpool to be at the origin. Join the origin to the point (42, 4), to obtain the graph of the first train. The line joining the points (0, 4) and (56, 0) will be the graph of the second train. They cut at the point whose abscissa is 24.

- 119 Let each man do  $m$  units of work, and each boy  $b$  units of work per day

9 men and 6 boys in 2 days do  $(18m + 12b)$  units,

5 men and 7 boys in 3 days do  $(15m + 21b)$  units

But these sets of units are equal,

$$18m + 12b = 15m + 21b, \text{ or } m = 3b$$

the work consists of  $18m + 12b$ , or  $54b + 12b$ , or  $66b$  units

Again 2 men and 5 boys do  $(2m + 5b)$  units in 1 day

Now  $2m + 5b = 6b + 5b = 11b$  since  $11b$  units are done in 1 day,  
 $66b$  units will take 6 days

120	1 <sup>st</sup> Jar contains	2 <sup>nd</sup> Jar contains
At first	8 pints of brandy	8 pints of water
After 1 <sup>st</sup> operation	7 pints of brandy	$\left\{ \begin{array}{l} 8 \text{ pints of} \\ \text{water,} \\ 1 \text{ pint of} \\ \text{brandy,} \end{array} \right\}$ or $\left\{ \begin{array}{l} \frac{8}{9} \text{ water,} \\ \frac{1}{9} \text{ brandy} \end{array} \right\}$
After 2 <sup>nd</sup> operation	$\left\{ \begin{array}{l} (7 + \frac{1}{9}) \text{ pints of} \\ \text{brandy,} \\ \frac{8}{9} \text{ pint of} \\ \text{water} \end{array} \right\}$	$\left\{ \begin{array}{l} (8 - \frac{8}{9}) \text{ pints} \\ \text{of water,} \\ (1 - \frac{1}{9}) \text{ pints} \\ \text{of brandy,} \end{array} \right\}$ or $\left\{ \begin{array}{l} (7 + \frac{1}{9}) \text{ pints} \\ \text{of water,} \\ \frac{8}{9} \text{ pint} \\ \text{of brandy} \end{array} \right\}$

121 (1)  $\pi = 3\frac{1}{7} = \frac{22}{7}$

$$\begin{aligned} \frac{1}{\pi} &= \frac{7}{22} = \frac{56}{176} = \frac{44 + 12}{176} = \frac{1}{4} + \frac{1}{16} + \frac{1}{176} \\ \frac{576}{\pi} &= 576 \times \frac{1}{\pi} = 576 \times \left( \frac{1}{4} + \frac{1}{16} + \frac{1}{176} \right) \\ &= \frac{576}{4} \\ &\quad \frac{144}{16} \\ &\quad \frac{36}{176} \\ &\quad \frac{327}{18327} \end{aligned}$$

$18327 = 1833$  to two places of decimals

(2)  $\pi$  hours = 3 hours, 8 minutes, 30 seconds = 3 hours,  $8\frac{1}{2}$  minutes

$$= 3\frac{17}{2 \times 60} = 3\frac{17}{120} \text{ hours} = \frac{377}{120} = 3.1416 \text{ hours}$$

$$\pi = (1) 3.1416 = (n) 3\frac{17}{120}$$

- 122 Since external diameter = 7.5 in.,  
 Weight of hollow sphere  
 $= 0.5236(7.5^3 - 7^3) \times 4.48 \text{ oz}$   
 $= 0.5236(421.875 - 343) \times 4.48 \text{ oz}$   
 $= 0.5236(78.875) \times 4.48 \text{ oz}$   
 $= 185 \text{ oz.}$

78	875
4	48
315	50
31	55
6	31
373	36
	0.5236
176	68
7	07
1	06
	21
185	02

- 123 Let a third-class ticket cost  $x$  pence, then a second class ticket costs  $1\frac{1}{2}x$  pence, and a first class ticket costs  $1\frac{2}{3} \times 1\frac{1}{2}x$  pence

$$\text{Now } 16s \ 10\frac{1}{2}d = 202\frac{1}{2}d \quad \frac{x}{2} + 1\frac{1}{2}x + 1\frac{2}{3} \times 1\frac{1}{2}x = 202\frac{1}{2},$$

$$\text{or } x + 3x + 5x = 405, \text{ and } x = 45$$

the tickets are  $1\frac{2}{3} \times 1\frac{1}{2} \times 45d$ , or  $9s \ 4\frac{1}{2}d$ , etc.,

and the rates per mile are  $\frac{9s \ 4\frac{1}{2}d}{45}$ , or  $2\frac{1}{2}d$ , etc

- 124 The req<sup>d</sup> number must be odd, since it is a prime

it ends in 9, giving  $\{189, 279, 369, 459\}$  as possibilities, (1)

or " 7, "  $\{167, 257, 347\}$  " (2)

or " 5, "  $\{145, 235\}$  " (3)

or " 3, "  $\{123\}$  " (4)

Now every number in (1) is a multiple of 9, }  
 " " (3) " " 5, } and cannot be prime  
 " " (4) " " 3, }

From (2), we find  $527 = 17 \times 31$ ,

$$\text{and } 437 = 19 \times 23$$

257 and 347, although both prime, will not satisfy the conditions

We are thus left with 167 and 617, which are both prime

- 125 He rows at 4 miles an hour on still water

"  $1\frac{1}{3} \times 4$ , or  $5\frac{1}{3}$  miles an hour down stream,  
 the stream flows at  $1\frac{1}{3}$  miles per hour

He rows up stream at  $(4 - 1\frac{1}{3})$ , or  $2\frac{2}{3}$  miles per hour

$$\text{req<sup>d</sup> time} = \left(\frac{1\frac{1}{3}}{2\frac{2}{3}} + \frac{1}{4}\right) \text{ hrs} = \frac{3}{4} \text{ hr}$$

- 126 He has to pay back altogether the amount of Rs 25220 for 3 years at 5% C.I. This = Rs 25220  $\times (1.05)^3$  (See Art 313)

Suppose Rs P is the yearly instalment. Then Rs P paid at the end of the first year amounts to Rs  $P \times (1.05)^2$  in 2 more years at 5% C.I.

Similarly Rs P paid at the end of the second year amounts to Rs  $P \times 1.05$

With the last instalment, the payments settle the debt,

$$\text{and } P(1.05^2 + 1.05 + 1) = 25220 \times (1.05)^3$$

$$P \times 3.1525 = 25220 \times 1.157625,$$

$$\text{and } P = \frac{25220 \times 1.157625}{3.1525}$$

$$= \frac{25220 \times 4.6305}{12.61}$$

$$= \text{Rs } 9261$$

4	6305
252	2
9261	0
2315	3
92	6
9	3
1 26,1	11678
	2 (9261
	3292
	769
	12

- 127 Suppose  $x$  lbs are allowed free to each passenger

When they share the luggage  $2x$  lbs are not paid for,

and  $(345 - 2x)$  lbs cost them (Rs 3 2a + Rs 5), or 130a,

$$\therefore \frac{345 - 2x}{13} \text{ lbs cost } 10a$$

When one of them owns all the luggage,  $x$  lbs are not paid for,

and  $(345 - x)$  lbs cost him Rs 11 4a, or 180a,

$$\therefore \frac{345 - x}{18} \text{ lbs cost } 10a$$

$$\frac{345 - 2x}{13} = \frac{345 - x}{18}, \text{ or } 18(345 - 2x) = 13(345 - x), \text{ and } x = 75$$

- 128 Let  $x$  be the req<sup>d</sup> number. Expressing cost and receipts in annas, and equating them, we get

$$122x = 28x + 4800, \text{ and } x = 51\frac{3}{4}$$

Thus the least possible no. of copies is 52

To verify graphically, take scales of 50 rupees to the inch horizontally, and 10 photogravures to the inch vertically. Cost of producing 40 photogravures = Rs 370

Join the points (300, 0) and (370, 40), and we obtain the graph for the cost of production of any number of copies. Again 40 copies are sold for Rs 305

Join the origin to the point (305, 40), and the graph for the 'sales' is obtained

These graphs will be found to meet at a point whose ordinate lies between 51 and 52 in value

- 129 When he turns back to the starting point, 10' has gone. He arrives there in 10' more. Since the rest of the party will reach the station 1 hour after they started, he has now to walk 3 miles in 40', or at the rate of  $4\frac{1}{2}$  miles an hour.

To verify this result graphically, take 3 in horizontally to represent 1 hour, and 1 in vertically to represent 1 mile.

Join the origin to the point (1, 3), and we obtain the graph for the majority of the party. Next, join the points  $(\frac{1}{3}, \frac{1}{3})$ ,  $(\frac{1}{3}, 0)$  and (1, 3) successively, and we have the graph for the 'one' tourist after he leaves the rest of the party. The line joining the two last-mentioned points represents a rate of 3 miles in  $\frac{1}{3}$  of an hour, or  $4\frac{1}{2}$  miles an hour.

- 130  $x$  miles per hour = 5280  $x$  feet per 3600 seconds

$$= \frac{5280x}{3600} \text{ ft per sec.} = \frac{44x}{30} \text{ ft per sec.},$$

but by the rule we have  $\frac{45x}{30}$  " "

so that on  $\frac{44x}{30}$  ft we have an error of  $\frac{1}{30}x$  ft,

i.e. on 44 " " 1,

i.e. on 100 " "  $\frac{100}{44} = 2\frac{2}{11} = 2.27$ ,

i.e. 2% to the nearest integer

- 131 Maximum weight of sovereign = 123.47447 grains

Minimum " " = 122 " "

The heap of sovereigns weighs within the limits  $384 \pm 5 \times 180$  gr

$$\text{greatest no. of sovs} = \frac{384.5 \times 180}{122.5} = \frac{384.5 \times 180}{122.5} = 564.9,$$

$$\text{greatest no. of sovs} = 564$$

$$\text{The least no. of sovs} = \frac{384.5 \times 180}{123.47447} = 559.07,$$

$$\text{least no. of sovs} = 560$$

- 132 Let  $7x$  be the no. of teeth in the first wheel

then  $6x$  is " " second "

$5x$  " " third "

and  $4x$  " " fourth "

Now L.C.M. of  $7x$ ,  $6x$ ,  $5x$ ,  $4x$  is  $420x$

when  $420x$  teeth have passed on each wheel, each will have made an exact number of revolutions, and they will be simultaneously in their original positions

the first must revolve  $\frac{420x}{7x}$ , or 60 times, etc, etc

133 Rs (6760 - 6500), or Rs 260 is the interest on Rs 6500 for 1 yr

Rs  $\frac{260}{6500}$  or Rs 4     "     "     Rs 100     "

Thus if Rs P be the req<sup>d</sup> sum,

$$P(1.04) = 6500 \quad (\text{Art } 313)$$

$$P = \frac{6500}{1.04} = \text{Rs } 6250$$

134 Let req<sup>d</sup> length be  $x$  ft     Cost per sq ft =  $\frac{11}{9}$ ,

$$\text{Then total cost} = x \times 6 \times \frac{11}{9} = x \times 6 \times \frac{11}{9} \times \frac{25.20}{20} \text{ fr}$$

$$\text{But total cost} = 787.4 \times 1.53 \times 1.5 \text{ fr}$$

$$x \times 6 \times \frac{11}{9} \times \frac{25.20}{20} = 787.4 \times 1.53 \times 1.5,$$

$$\text{whence } x = \frac{787.4 \times 1.53 \times 1.5 \times 9 \times 20}{6 \times 11 \times 25.20} = \frac{787.4 \times 1.7}{0.7}$$

$$= \frac{1338.58}{0.7} = 1912 \text{ ft} = 637 \text{ yds}$$

135 In the first year profit = Rs (35,40,000 - 25,10,000) = Rs 10,30,000

In the second year receipts =  $\frac{961}{100}$  of Rs 35,40,000 = Rs 34,16,100,

"     "     expenses =  $\frac{1001}{100}$  of Rs 25,10,100 = Rs 25,22,550,

"     "     profit = Rs 8,93,550

Difference of the two years' profits = Rs 1,36,450,

$$\text{req<sup>d</sup> percentage} = 100 \times \frac{136450}{1030000} = \frac{13645}{103} = 13.2$$

136 Interest received =  $\frac{3}{117}$  of £9000 +  $\frac{21}{108}$  of £7200 (1)

Cash realised by sale =  $\frac{1101}{117}$  of £9000 +  $\frac{1101}{108}$  of £7200 (2)

Total cash obtained = (1) + (2) =  $\frac{1131}{117}$  of £9000 +  $\frac{1131}{108}$  of £7200

Hence req<sup>d</sup> gain =  $\left(\frac{1131}{117} - 1\right)$  of £9000 +  $\left(\frac{1131}{108} - 1\right)$  of £7200

$$= \frac{51}{108} \text{ of } £7200 - \frac{31}{117} \text{ of } £9000 = £\left(\frac{1100}{3} - \frac{1000}{4}\right) = £116 \text{ } 13s \text{ } 4d$$

$$\begin{aligned}
 137 \quad & \sqrt{1\ 0090} - \sqrt{0\ 091} \\
 & = 1\ 0044 - 0\ 3017 \\
 & = 0\ 7027 = 0\ 703
 \end{aligned}$$

$$\begin{array}{r}
 1\ 0090\ (1\ 0044 \\
 2004 \overline{) 00\ 9000} \\
 \underline{984} \\
 0\ 091\ (0\ 30166 \\
 601 \overline{) 1000} \\
 \underline{6026} \\
 39900 \\
 \underline{3744}
 \end{array}$$

- 138 Let  $2n$  workmen be engaged on the first house,  
and  $n$  " " second "

Suppose each is paid Re  $x$  per hour

Then the cost per day of the first house

$$= \text{Rs } 10x \times 2n + \text{Rs } 2 \times \frac{3x}{2} \times 2n = \text{Rs } 26nx,$$

and the cost per day of the second house  $= 10x \times n = \text{Rs } 10nx$

Again, the first house takes 4 months, or  $4 \times 30$  days,

and the second " 7 " or  $7 \times 30$  " ,

$$26nx \times 4 \times 30 + 10nx \times 7 \times 30 = 17400$$

Divide by 30, then  $104nx + 70nx = 580$ , or  $nx = \frac{1}{3}$

cost of first house  $= \text{Rs } 26nx \times 4 \times 30 = \text{Rs } 10400$  ,

cost of second house  $= \text{Rs } 10nx \times 7 \times 30 = \text{Rs } 7000$

- 139 Let  $V_1$  be the volume of water,  $V_2$  that of the other liquid  
Then  $V_1$  weighs  $V_1$  kilograms, and  $V_2$  weighs  $V_2 \times 1\ 340$  Kg  
The volume of the mixture is  $(V_1 + V_2)$ ,

and this weighs  $(V_1 + V_2) \times 1\ 270$  Kg

$$(V_1 + V_2) \times 1\ 270 = V_1 + V_2 \times 1\ 340, \text{ or } V_1 \times 0\ 27 = V_2 \times 0\ 07,$$

$$\frac{V_1}{V_2} = \frac{7}{27} \quad \text{Thus } \frac{7}{34} \text{ are water, } \frac{27}{34} \text{ other liquid}$$

- 140 Let req<sup>d</sup> distance be  $x$  miles, which the first train takes 50 min  
to travel Its rate in miles per min is therefore  $\frac{x}{50}$ , and at  
12 20 it has travelled  $20 \frac{x}{50}$  miles Similarly at 12 20 the  
second train has travelled  $15 \frac{x}{45}$  miles and the third  $10 \frac{x}{40}$   
miles

$$\frac{20x}{50} + \frac{15x}{45} + \frac{10x}{40} = 36\frac{1}{2}, \text{ or } \frac{2x}{5} + \frac{x}{3} + \frac{x}{4} = \frac{295}{8}, \text{ and } x = 37\frac{1}{2} \text{ miles}$$

141

Perimeter of room =  $2(a+b)$  ftArea of walls =  $2c(a+b)$  sq ftWidth of paper =  $d$  ftno of yds of paper req<sup>d</sup> =  $\frac{2c(a+b)}{3d}$ 

$$\text{Cost of paper} = \frac{2c(a+b)}{3d} \times \frac{n}{16 \times 12} \text{ rupees}$$

$$= \text{Rs } \frac{nc(a+b)}{288d}$$

142 Since 1 md = 40 srs, after the first replacement,

 $\frac{7}{8}$  md is milk After the second,  $\frac{7}{8}$  of  $\frac{7}{8}$ , or  $(\frac{7}{8})^2$  md is milkSimilarly, after the 3<sup>rd</sup> replacement,  $(\frac{7}{8})^3$  md is milkNow suppose 1 md cost  $x$  annas at firstThen  $(\frac{7}{8})^3$  md is sold for  $x$  annas, $\therefore$  1 md is sold for  $(\frac{8}{7})^3 x$ , or  $\frac{512}{343} x$  annasgain on  $x$  annas =  $(\frac{512}{343} x - x)$  annas =  $\frac{169}{343} x$  annas,gain per cent =  $\frac{16900}{343} = 49 \frac{27}{100}$ 

143 (i) Correcting to four significant figures,

the fraction becomes  $\frac{52 \ 45 \times 378 \ 4 \times 0 \ 0209}{87 \ 32 \times 0 \ 584}$ 

Numerator

Denominator

log 52 45 = 1 7197

log 378 4 = 2 5780

log 0 0209 =  $\bar{2}$  3201

log 87 32 = 1 9411

log 0 584 =  $\bar{1}$  7664

log numerator = 2 6178

log denominator = 1 7075

log fraction = 2 6178 - 1 7075 = 0 9103,

and antilog 0 9103 = 8 134

(ii) Denote the expression by  $x$ , thenlog  $x$  = 2 log 1 03 + 4 log 1 025 + log 1 05

= 0 0256 + 0 0428 + 0 0212 = 0 0896,

and antilog 0 0896 = 1 229

144 See Art 311 and the note to Ex, Art 313 From this we get

1 req<sup>d</sup> Amount = £500  $\times$  (1 03)<sup>2</sup>  $\times$  (1 025)<sup>4</sup>  $\times$  1 05= £500  $\times$  1 229 = £614 5 = £614 10s

145 Water in mixture =  $(\frac{11}{100} \text{ of 1 gal} + \frac{7}{100} \text{ of 3 gals} + \frac{1}{2} \text{ gal})$   
 $= (\frac{11}{100} + \frac{21}{100} + \frac{1}{2}) \text{ gals} = \frac{82}{100} \text{ gals}$  Also mixture =  $4\frac{1}{2} \text{ gals}$

Now  $\frac{82}{100} \text{ gals out of } 4\frac{1}{2} \text{ gals} = \frac{82}{4\frac{1}{2}} \text{ out of 100, or } 18\frac{2}{3} \%$

146. 712 lbs weigh more than 1728 cu in

1 lb weighs more than  $\frac{1728}{712} \text{ cu in}$   
 If  $x$  shot go to the lb,

then their volume  $\frac{\pi^2}{6 \cdot 8^3} > \frac{1728}{712} \text{ cu in,}$

i.e.  $x > \frac{1728 \times 6 \times 8^3}{712 \times \pi^2},$

or  $x > \frac{1728 \times 6 \times 512}{712 \times 3.14159}, \text{ or } 2373.2$

Similarly,

1 lb weighs less than  $\frac{1728}{708} \text{ cu in,}$

and the volume  $\frac{\pi^2}{6 \cdot 8^3} < \frac{1728}{708} \text{ cu in,}$

and  $x < \frac{1728 \times 6 \times 512}{708 \times 3.14159}, \text{ or } 2386.6$

Thus  $x$  lies between 2373 and 2387

$$\begin{array}{r} 1728 \times 6 \times 64 = 663552 \\ 3 \ 14159 \times 89 = 279 \ 60151 \\ \hline 2373 \ 2 \\ 2 \ 79602 \ ) \ 6635 \ 52 \\ \hline 1043 \ 48 \\ \hline 204 \ 67 \\ \hline 8 \ 95 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 864 \times 512 = 442368 \\ 3 \ 14159 \times 59 = 185 \ 35381 \\ \hline 2386 \ 6 \\ 1 \ 85354 \ ) \ 4423 \ 68 \\ \hline 716 \ 60 \\ \hline 160 \ 54 \\ \hline 12 \ 26 \\ \hline 1 \ 24 \end{array}$$

147 2 oz. of bluestone and  $1\frac{1}{2}$  oz of lime go to 1 gal or 10 lbs of water,

or 2 oz of bluestone and  $1\frac{1}{2}$  oz. of lime go to 160 oz of water,

2 gm of bluestone and  $1\frac{1}{2}$  gm of lime go to 160 gm of water,  
 or  $\frac{2000}{160} \text{ gm of bluestone and } \frac{1500}{160} \text{ gm of lime go to 1000 cc}$   
 of water,

i.e. 125 gm of bluestone and 781 gm of lime go to 1 litre of water

148 Let  $x$  = no of rupees lent

Then  $x(1.04)^4 - x(1.08)^2 = 650,$

i.e.  $x(1.16985856 - 1.1664) = 650,$

i.e.  $x \times 0.00345856 = 650$

$x = 187939$

Principal = Rs 187900 to the nearest hundred



- 149 Let one man's work in one day represent a unit of work  
 Since 150 men in 25 days complete  $\frac{1}{4}$  of the work,  
 $150 \times 25 \times 4$ , i.e. 15000, units of work are needed for the whole  
 100 men in 60 days complete  $\frac{2}{5}$  of the work  
 $\frac{7}{20}$  of the work, i.e. 5250 units of work, remain  
 But  $130 - (25 + 60 + 10)$ , i.e. 35, days only remain  
 $\frac{7 \times 50}{35}$ , i.e. 150, men are required
- 150 Let  $x$  Rs = cost of goods  
 Then  $\frac{3}{4}$  of stock sells for Rs  $\frac{x \times 140 \times 3}{100 \times 4}$   
 $= \text{Rs } \frac{105x}{100}$   
 The remaining  $\frac{1}{4}$  of the stock sells for Rs  $\frac{x \times 140 \times 60}{100 \times 100 \times 4}$   
 $= \text{Rs } \frac{21x}{100}$   
 whole stock sells for Rs  $\frac{126x}{100}$   
 gain = 26 %
- 151 Suppose the man rows  $x$  mi an hour on still water, and that the stream flows at  $y$  miles per hr  
 Then he rows up stream at  $(x-y)$  mi per hr, and down stream at  $(x+y)$  mi per hr  
 In 18 min, or  $\frac{3}{10}$  hr, he rows  $\frac{3}{10}(x-y)$  mi up stream. He rows this distance down stream at  $(x+y)$  mi per hr, and therefore takes  $\frac{3(x-y)}{10(x+y)}$  hrs  
 He rows the last  $1\frac{1}{2}$  mi down stream in  $\frac{1\frac{1}{2}}{x+y}$  hrs  
 the total time between his passing and overtaking the bottle  
 $= \left( \frac{3}{10} + \frac{3(x-y)}{10(x+y)} + \frac{1\frac{1}{2}}{x+y} \right)$ , or  $\frac{6x+15}{10(x+y)}$  hrs  
 But this is the time taken by the bottle to float down  $1\frac{1}{2}$  mi,  
 i.e.  $\frac{1\frac{1}{2}}{y}$  hrs  
 $\frac{6x+15}{10(x+y)} = \frac{1\frac{1}{2}}{y}$ , or  $(6x+15)y = 15(x+y)$ , i.e.  $6xy = 15x$   
 Divide by  $x$ , and  $y = 2\frac{1}{2}$

152 Substituting the given values  $\alpha = \frac{5000 \times (1.03)^{10} \times 0.03}{(1.03)^{10} - 1}$ ,

$$\begin{aligned}\log \text{ numerator} &= \log 5000 + 10 \log 1.03 + \log 0.03 \\ &= 3.6990 + 0.1280 + \bar{2}.4771 = 2.3041, \\ \text{numerator} &= \text{antilog } 2.3041 = 201.4\end{aligned}$$

Noting that  $10 \log 1.03 = 0.1280$ , and  $\text{antilog } 0.1280 = 1.343$ ,  
we see that  $\text{denominator} = 1.343 - 1 = 0.343$

$$\alpha = \pounds \frac{201.4}{0.343} = \pounds 587.2$$

153  $(6^2 \times 5000)$  cu cm are discharged per 1 min (see Art 247, Ex 3),

1000 cu cm, or 1 litre are discharged per  $\frac{1}{6^2 \times 5}$  min,

i.e. 30000 lit are discharged per  $\frac{30000}{36 \times 5}$  min, or  $166\frac{2}{3}$  min

154 Suppose the clients deposit Rs  $x$ . The interest from the preference stock = Rs  $\frac{4}{150} \times \frac{x}{2}$ , from the mortgages = Rs  $\frac{5}{100} \times \frac{x}{4}$ . He pays Rs  $\frac{2}{100} \times x$  as interest on deposits

$$\frac{4}{150} \times \frac{x}{2} + \frac{5}{100} \times \frac{x}{4} - \frac{2x}{100} = 10000$$

Multiply by 1200, and  $16x + 15x - 24x = 12000000$ ,

whence  $x = 17,14,286$  to nearest Re

155 Suppose there are  $x$  cu cm of the first and  $y$  cu cm of the second liquid. Then their weights are  $x \times 0.68$  gm, and  $y \times 1.04$  gm, respectively

Similarly the weight of the mixture =  $(x \times 0.82 + y \times 0.82)$  gm

$$x \times 0.68 + y \times 1.04 = x \times 0.82 + y \times 0.82, \text{ and } \frac{x}{y} = \frac{11}{7}$$

Hence  $x = \frac{11}{18}$ , and  $y = \frac{7}{18}$ , of the whole mixture

$$x = \frac{11}{18} \times 100\% = 61\frac{1}{3}\%, \text{ and } y = \frac{7}{18} \text{ of } 100\% = 38\frac{9}{10}\%$$

156 Since 15 sq in is increased to 1 sq ft, or 144 sq in,

1 sq in                   ,,                    $\frac{144}{15}$ , or 9.6 sq in,

1 in                       ,,                    $\sqrt{96}$  in, or 9.8 in

Hence 4 in               ,,               9.8  $\times$  4, or 39.2 in,

and 6 in                 ,,               9.8  $\times$  6, or 58.8 in

the least dimensions are 59 in and 40 in

- 157 Let a unit of work = work of one boy in one hour

A man =  $\frac{7}{4}$  boys

$$\frac{\{(14 \times \frac{7}{4}) + 13\} \times 3 \times 2}{5} = \text{no of units of work to be done}$$

$$= \frac{75}{2} \times \frac{6}{5} = 45 \text{ units}$$

If for this work there are  $x$  men,

there must be  $27 - x$  boys

$$\text{Then } x \times \frac{7}{4} + (27 - x) = 45$$

$$7x + 108 - 4x = 180,$$

$$3x = 72,$$

$$x = 24$$

So there are 24 men and 3 boys

10 boys are replaced by that no of men

- 158 Let  $x$  = no of Kms per hour travelled by motor car

$$\text{Then } \frac{120x}{100} = \quad \quad \quad \text{train}$$

$$\frac{75}{x} = \frac{75}{120x} + \frac{12\frac{1}{2}}{60},$$

$$\frac{75}{x} = \frac{750}{12x} + \frac{25}{120},$$

$$9000 = 7500 + 25x,$$

$$25x = 1500,$$

$$x = 60$$

- 159 Let Rs  $x$  be the retail price per gross Since a royalty of 10 % is paid and discount of 40 % is allowed,  $\frac{\text{Rs } 50}{100}x$ , or  $\frac{\text{Rs } x}{2}$  is the net receipt per gross by the manufacturer

But this allows a profit of 25 % on Rs 170 ,

$$\frac{x}{2} = 170 \times \frac{125}{100}, \text{ whence } x = \text{Rs } 425 \text{ per gross}$$

price per article = Rs 2 15 a 3 p

For second part of question, the net receipts on each article are  $\frac{50}{100}$  of Rs 3 But each article costs  $\frac{\text{Rs } 170}{144}$

$$\text{reqd profit} = \text{Rs } \frac{2}{7} \frac{3}{2} = 5 \frac{1}{6} \text{ a}$$

- 160 The line for the Latin marks is found by joining the points (51, 30) and (153, 120), and for the Greek marks by joining (56, 40) and (161, 100)

The most convenient scale is that of 10 marks to the inch for each case, beginning at 50 for the unreduced marks and at 30 for the reduced marks

The figure will however be large

The two lines cut at a point whose coords are approximately (74, 50) Hence an actual mark of 74, either in Latin or Greek, appears as 50 when reduced

- 161 Since 15 mi per hr = 22 ft per sec,  
in 50 min he is nearer the fort by  $50 \times 22 \times 60$  ft  
But the interval between the 1<sup>st</sup> and the 52<sup>nd</sup> report is 51 min,  
sound takes 1 min to travel  $50 \times 22 \times 60$  ft,  
giving a speed of 1100 ft per sec
- 162 The volume of water flowing through the pipe per sec is that of a cylinder whose base area is  $\pi 6^2$  in, or  $\pi \frac{1}{4}$  sq ft, and whose height is 1 ft, giving  $\frac{\pi}{4}$  cu ft
- If the req<sup>d</sup> time be  $\tau$  hrs, then in this time  $\left(\frac{\pi}{4} \times \tau \times 60 \times 60\right)$  cu ft are discharged
- But this must equal  $\tau$  cuboid of water whose base area is  $(20 \times 4840 \times 9)$  sq ft, and whose depth is  $\frac{1}{12}$  ft

$$\frac{\pi}{4} \times \tau \times 60 \times 60 = 20 \times 4840 \times 9 \times \frac{1}{12}$$

$$x = \frac{20 \times 4840 \times 9 \times 4}{3 \ 14159 \times 60 \times 60 \times 12} = \frac{242}{3 \ 14159 \times 3} = \frac{242}{9 \ 42477} = 25 \ 68$$

- 163 Income at first = Rs  $\frac{14625 \times 3\frac{1}{2}}{97\frac{1}{2}}$   
 $= \frac{14625 \times 7 \times 2}{2 \times 195}$   
 $= \text{Rs } 525$
- 2<sup>nd</sup> income = Rs  $\frac{14625 \times 98 \times 4}{97\frac{1}{2} \times 112}$   
 $= \frac{14625 \times 98 \times 4 \times 2}{112 \times 195}$   
 $= \text{Rs } 525$

Thus there is no change in income



If exactly £40000 is carried over, the amount for the dividend on ordinary stock would be

$$£1492156\ 5875 - [£40000 + £745746\ 75],$$

$$\text{or } £1492156\ 5875 - £785746\ 75,$$

$$\text{or } £706409\ 8375$$

On this supposition the req<sup>d</sup> percentage per half year would be

$$\frac{£706409\ 8375 \times 100}{£35538259}, \text{ or } \frac{£7064}{3554}$$

This is greater than  $1\frac{3}{4}$  and less than 2, and nearer to the latter value. But if 2% is taken, the sum carried over will clearly be not as much as £40000. Thus to satisfy the conditions of the question, we must take  $1\frac{3}{4}$ %

Now  $1\frac{3}{4}$ % on £35538259

$$= (2 - \frac{1}{4}) \text{ of } £355382\ 59$$

$$= £710765\ 18 - £88845\ 6475$$

$$= £621919\ 5325 \text{ (interest on ordinary stock)} \quad (1)$$

$$\text{The interest on the other stock} = £745746\ 75 \quad (2)$$

Adding (1) and (2) we obtain for total interest

$$£1367666\ 2825$$

$$\text{amount carried over} = £1492156\ 5875 - £1367666\ 2825$$

$$= £124490\ 3 = £124490\ 6s$$

170 From Art 284, III,

$$\frac{\text{Volume of statuette}}{\text{Volume of model}} = \frac{10^3}{48^3} = \frac{5^3}{24^3} = \frac{125}{13824}$$

Supposing the statuette were made of plaster, it would weigh  $\frac{125}{13824}$  of 150 lbs. Since, however, silver is 12.9 times as heavy as plaster, it weighs  $\frac{125}{13824}$  of  $150 \times 12.9$  lbs,

$$\text{or } \frac{125 \times 150 \times 12.9}{13824} \text{ lbs, or } 17.5 \text{ lbs}$$

171 36 in is correct length of unit, 36.18 is incorrect length of unit

Now the greater the measure of the unit, the smaller the number of units in any distance. Hence we have

$$\frac{\text{correct measurement of side}}{\text{incorrect measurement of side}} = \frac{36.18}{36} = \frac{1.005}{1}$$

$$\frac{\text{correct measurement of area}}{\text{incorrect measurement of area}} = \left( \frac{1.005}{1} \right)^2 = \frac{1.010025}{1}$$

Now  $1.010025 > 1$  by  $0.010025$ , i.e. by  $1.0025\%$

172 Let  $V_1, V_2$  be the volumes of the spheres

„  $s_1, s_2$  „ sp gravities „

„  $W_1, W_2$  „ weights „

„  $r_1, r_2$  „ radii „

Then from Art 247, Ex 1, we see that  $\frac{V_1 s_1}{V_2 s_2} = \frac{W_1}{W_2}$

But  $\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$ ,  $\frac{r_1^3 s_1}{r_2^3 s_2} = \frac{W_1}{W_2}$  Substituting given values,

$$\frac{r_1^3 \times 289}{r_2^3 \times 64} = \frac{8}{17}, \text{ and } \frac{r_1^3}{r_2^3} = \left(\frac{8}{17}\right)^3, \text{ or } \frac{r_1}{r_2} = \frac{8}{17}$$

173 Their invested capitals are in the proportion 6 : 4 : 3

A uses whole capital for 4 months,

and half „ 8 „

B and C each use whole capital for 12 months

then shares of the profits are in the proportion

$$(6 \times 4) + (3 \times 8) : 4 \times 12 : 3 \times 12,$$

$$\text{i.e. } 48 : 48 : 36,$$

$$\text{i.e. } 4 : 4 : 3$$

So A has  $\frac{4}{11}$  of Rs 2024 = Rs 736

174 The increase in first population is  $45682 \times 0.08$ , in the second,  $25408 \times 0.045$ , the decrease in the third is  $18960 \times 0.1$ . If  $x$  be the req<sup>d</sup> average increase per cent, this actual increase is

$$(45682 + 25408 - 18960) \times \frac{x}{100}, \text{ or } 90050 \times \frac{x}{100}$$

$$90050 \times \frac{x}{100} = 45682 \times 0.08 + 25408 \times 0.045 - 18960 \times 0.1,$$

$$\text{or } 900.5x = 3654.56 + 1143.36 - 1896,$$

$$\text{whence } x = \frac{2901.92}{900.5} = 3.2$$

175 Since 2s 8d per cwt = £2 13s 4d per ton,

each ton of ore costs him £8 16s 8d + £2 13s 4d, or £11 $\frac{1}{2}$

But only  $\frac{92}{100}$  of this is copper, of which only 92% can be extracted and sold. Hence if £ $x$  be req<sup>d</sup> price,

$$\frac{92}{100} \text{ of } \frac{92}{100} \text{ of } x = 11\frac{1}{2} \times \frac{117}{100} \text{ (see Art 292),}$$

$$x = \frac{23}{2} \times \frac{117}{100} \times \frac{100}{92} \times \frac{100}{92} = £62 \text{ } 10\text{s}$$

- 176 Suppose the thickness of a halfpenny is  $r$  in,  
 then the thickness of a penny is  $1.23r$  in  
 volume of halfpenny  $= 0.7854 \times 1^2 \times r$  cu in,  
 and volume of penny  $= 0.7854 \times (1.23)^2 \times 1.23r$  cu in  

$$\text{req'd fraction} = \frac{0.7854 \times 1^2 \times r}{0.7854 \times (1.23)^2 \times 1.23r} = \frac{1}{(1.23)^3} = \frac{1}{1.861} = 0.54$$

- 177 Marked price  $=$  nominal cost price  $\times \frac{100}{90}$  (see Art. 289)

But because of the fraudulent balance,

$$\frac{\text{nominal cost price}}{\text{actual cost price}} = \frac{1 + \frac{1}{15}}{1} = \frac{16}{15},$$

$$\text{nominal cost price} = \text{actual cost price} \times \frac{16}{15}$$

$$\text{Hence marked price} = \text{actual cost price} \times \frac{16}{15} \times \frac{100}{90}$$

$$= \text{actual cost price} \times \frac{160}{135},$$

$$\text{and req'd gain} = 12\%$$

178

I	II	III
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">           8 wine, 8 water         </div>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">           9 wine, 15 water         </div>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">           4 wine, 26 water         </div>

In II there is room for 8 more gallons  
 after filling up II from III there are

$$\left\{ \left( 9 + \frac{4}{3} \text{ of } 8 \right) \text{ wine, } \right\} \text{ i.e. } \left\{ 10 \text{ wine, } \right\}$$

$$\left\{ \left( 15 + \frac{2}{3} \text{ of } 8 \right) \text{ water, } \right\} \left\{ 22 \text{ water } \right\}$$

In I there is room for 16 more gallons  
 after filling up I from II there are

$$\left\{ \left( 8 + \frac{1}{2} \text{ of } 16 \right) \text{ wine, } \right\} \text{ i.e. } \left\{ 13 \text{ wine, } \right\}$$

$$\left\{ \left( 8 + \frac{2}{3} \text{ of } 16 \right) \text{ water, } \right\} \left\{ 19 \text{ water } \right\}$$

179

$$\text{Area of enclosure} = (10 \times 4840 \times 9) \text{ sq ft}$$

$$\text{length of its side} = \sqrt{(48400 \times 9)} \text{ ft}$$

$$= 660 \text{ ft}$$

$$\text{length of street} = (4 \times 660) \text{ ft}$$

and its width is 30 ft

$$\text{no. of paving stones req'd} = 4 \times 660 \times 30 \times \frac{4}{3}$$

$$= 110400$$



180 Let £ $v$  = present value of bill,

$$\text{then } v \frac{304}{300} = 513$$

$$v = \frac{513 \times 300}{304}$$

$$= 506\frac{1}{4}$$

$$= £506 \ 5 \ a$$

$$\text{total assets} = £1422 \ 10s \ 4d$$

$$\text{and debts} = £2134 \ 10s \ 6d$$

$$\text{he pays in the pound } £ \frac{1422 \ 516}{2134 \ 525}$$

$$= £666$$

$$= 13s \ 4d$$

181 The decimal evidently terminates when the denominator is  
(i) any power of 10, (ii) any power of 2, (iii) any power of 5,  
(iv) a product of any power of 2 and any power of 5  
And (i) is included in (iv)

But, after the fraction has been reduced to its lowest terms,  
with any of the other nos, 3, 6, 7, 9, 11, etc, as denominator,  
or as a factor in the denominator, the decimal does not  
terminate

$$\frac{119}{138} = \frac{7}{8}, \text{ where } 8 = 2^3 \quad \text{the decimal terminates}$$

182 (i)  $A+B+C$  do  $\frac{1}{18}$  of the work in 1 day

$$(ii) \ B+C+D \text{ „ } \frac{1}{20} \quad \text{„} \quad \text{„}$$

$$(iii) \ C+D+A \text{ „ } \frac{1}{24} \quad \text{„} \quad \text{„}$$

$$(iv) \ D+A+B \text{ „ } \frac{1}{27} \quad \text{„} \quad \text{„}$$

$$\text{combining (i) and (ii), } A-D = \frac{1}{18} - \frac{1}{20} = \frac{10-9}{180} = \frac{1}{180}$$

$$\text{„ (i) „ (iii), } B-D = \frac{1}{18} - \frac{1}{24} = \frac{8-6}{72} = \frac{1}{36}$$

$$A+B-2D = \frac{1}{180} + \frac{1}{72} = \frac{2+5}{900} = \frac{7}{900}$$

$$\text{But (iv) } A+B+D = \frac{1}{27}$$

$$3D = \frac{1}{36} - \frac{7}{900}$$

$$= \frac{40-21}{900}$$

$$= \frac{19}{900}$$

$$D \text{ does } \frac{19}{900} \text{ of the work in 1 day}$$

$$\begin{aligned}
 A+B+C+D \text{ do } \frac{1}{18} + \frac{1}{3} + \frac{1}{40} \text{ in 1 day} \\
 = \frac{130+12}{360} \\
 = \frac{142}{360}
 \end{aligned}$$

$$\begin{aligned}
 A+B+C+D \text{ do the work in } \frac{360}{142} \text{ days,} \\
 \text{i.e. in } 16\frac{5}{142} \text{ days}
 \end{aligned}$$

183 Let Rs  $x$  = marked price of article

$$\text{Then Rs } \frac{95x}{100} = \text{price obtained from customer}$$

$$\text{But Rs } 712\frac{1}{2} \times \frac{4}{3} = \quad \quad \quad \text{''} \quad \quad \quad \text{''} \quad \quad \quad \text{''}$$

$$\frac{95x}{100} = 712\frac{1}{2} \times \frac{4}{3}$$

$$\frac{19x}{20} = \frac{1425}{2} \times \frac{4}{3}$$

$$19x = 19000,$$

$$x = 1000$$

$$\text{marked price} = \text{Rs } 1000$$

184

$$\begin{aligned}
 1^{\text{st}} \text{ income} &= \text{Rs } \frac{44100}{19} \times \frac{7}{5} \\
 &= \text{Rs } 1575
 \end{aligned}$$

$$\begin{aligned}
 2^{\text{nd}} \text{ income} &= \text{Rs } \frac{44100 \times 98\frac{1}{2} \times 5}{98 \times 110\frac{1}{16}} \\
 &= \text{Rs } \frac{44100 \times 197 \times 5 \times 16}{98 \times 2 \times 1773} \\
 &= \text{Rs } 2000
 \end{aligned}$$

$$\begin{aligned}
 \text{increase in income} &= \text{Rs } (2000 - 1575) \\
 &= \text{Rs } 425
 \end{aligned}$$

185 The loser runs  $9\frac{1}{11}$  yds in 1 sec

$$\begin{aligned}
 \text{he runs 100 yds in } \frac{100}{9\frac{1}{11}} \text{ secs,} \\
 \text{i.e. 11 secs}
 \end{aligned}$$

The winner takes 1 sec less

$$\text{he runs 100 yds in 10 secs}$$

$$\begin{aligned}
 100 \text{ yds in 10 secs} &= \frac{100 \times 60 \times 60}{1760 \times 10} \text{ miles per hour} \\
 &= 20\frac{5}{11} \quad \quad \quad \text{''} \quad \quad \quad \text{''}
 \end{aligned}$$

$$\begin{aligned}
 100 \text{ yds in 11 secs} &= \frac{100 \times 60 \times 60}{1760 \times 11} \text{ miles per hour} \\
 &= 18\frac{7}{22} \quad \quad \quad \text{''} \quad \quad \quad \text{''}
 \end{aligned}$$

- 186 When  $B$  has once completed the course  $A$  is not half way round  
 So  $B$ , while going round the second time, will catch up  $A$   
 Let  $x$  = no. of secs from start before they are together again

In  $x$  secs  $A$  goes  $\frac{x}{224}$  of the course

"  $B$  goes once round and  $\frac{x}{224}$  of the course,

$$x = \frac{224 + x}{224}$$

But in  $x$  secs  $B$  goes  $\frac{x}{88}$  of the course

$$\frac{x}{88} = \frac{224 + x}{224},$$

$$224x = 19712 + 88x,$$

$$136x = 19712,$$

$$x = 144\frac{1}{4} \text{ secs}$$

- 87 Interest paid = Rs  $\frac{250 \times 3 \times 3}{4 \times 100} = \frac{45}{8}$  = Rs 5 10 a

Interest obtained = Rs 5 10 a + Rs 2 13 a = Rs 8 7 a.

Let  $x$  = rate of interest obtained

Then 
$$\frac{250 \times 3 \times x}{4 \times 100} = 8\frac{7}{16},$$

$$\frac{15x}{8} = \frac{135}{16},$$

$$x = \frac{9}{4} = 4\frac{1}{4}\%$$

188

Original value = Rs 54321

Less 5 % 2716 05

After 1 yr = 51604 95

Less 5 % 2580 2475

After 2 yrs = 49024 7025

Less 5 % 2451 235125

After 3 yrs = 46573 467375

Less 5 % 2328 673368

After 4 yrs = 44244 794

Less 5 % 2212 2397

After 5 yrs = 42032 5543

Less 5 % 2101 6277

After 6 yrs = 39930 9266

i.e. value to the nearest rupee = Rs 39931

189 Let  $\tau$  = no of minute spaces covered by hour hand before hands coincide

Then  $12\tau$  = no of minute spaces covered by minute hand

$$12\tau - 60 = \tau,$$

$$11\tau = 60,$$

$$\tau = \frac{60}{11} = 5\frac{5}{11}$$

So that hands should coincide after  $65\frac{5}{11}$  mins

But they coincide after 66 mins of correct time

in 66 mins. of correct time clock is  $\frac{60}{11}$  min slow

$$\begin{aligned} \text{" } 60 \times 24 \text{ " " " } & \frac{60 \times 24 \times 60}{11 \times 60} \text{ min slow} \\ & = 144\frac{4}{11} \text{ " } \\ & = 111\frac{0}{11} \text{ " } \end{aligned}$$

190 While  $D$  goes 440 yds,  $C$  goes 121 yds

$$\text{" " " } B \text{ " } \frac{440 \times 440}{4 \times 6} \text{ yds}$$

$$\begin{aligned} \text{" " " } 1 \text{ " } & \frac{440 \times 440 \times 440}{17 \times 11 \times 6} \text{ yds} \\ & = 132\frac{7}{11} \text{ yds} \end{aligned}$$

$$D \text{ wins by } 7\frac{8}{11} \text{ yds}$$